

FUNDAMENTALS OF STEREOLITHOGRAPHY

July 1992

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1. INTRODUCTION

It has only been a little over four years since the introduction of the first StereoLithography system, the SLA-1. From early 1988 until June 1992 over 300 SLA-1, SLA-250, SLA-190 and SLA-500 units have been sold by 3D Systems. These machines, currently operating in 20 countries on five continents, amount to about 90 percent of all the rapid prototyping systems now in use.

Notwithstanding these figures, the field of rapid prototyping is still quite young. The author has been surprised to note that even among existing users, knowledge of the most basic relationships of this new technology is at best uncertain. For newcomers, even less is known.

It is therefore appropriate, on the occasion of the 1992 Solid Free-Form Fabrication Conference in Austin, for us to develop those fundamental relationships which form the foundation of this technology. In the transition of StereoLithography from an art to a science, it is natural that we should attempt to develop a model of the process. Although the mathematics may seem formidable to some readers, the physical model is actually quite simple. While requiring only three key assumptions, we shall derive seven fundamental relationships, leading to nineteen important conclusions.

The interaction of actinic photons with reactive photopolymer involves some very complex physics and chemistry. Nonetheless, the theoretical predictions of the model described herein are in good agreement with numerous experimental measurements, at least to first order. The primary benefit of this analysis is the development of a good physical understanding of the basic phenomena. This description avoids the extreme complexity and inevitable loss of generality that would likely result from either analytical or numerical attempts at an even more accurate model. While advanced studies continue, it was felt that the material presented is certainly appropriate at the operational level.

2. GAUSSIAN LASER SCANNING

Consider an actinic laser beam being scanned in a straight line, at constant velocity, over the free surface of a vat of liquid photopolymer. We shall make three key assumptions:

1. The photopolymer resin obeys the Beer-Lambert law of exponential absorption.
2. The laser irradiance distribution is Gaussian.
3. The resin transitions from the liquid phase to the solid phase at the so-called "gel point".

Let us now define a coordinate system as shown in Figure 1. Here the laser is scanned directly along the x axis, in the direction of increasing x. The y coordinate is laterally orthogonal to the laser scan axis, with positive y defined by the right hand rule. Thus, y = 0 directly under the centerline of the laser scan axis. Finally, the positive z coordinate extends downward into the resin, and is measured normal to the x-y plane of the free resin surface, where z = 0. To define the origin, consider some arbitrary point Q(x,y,z) within the resin. Let Q'(0,y,0) be the projection of Q onto the resin surface. We shall arbitrarily select the origin such that both the x and z coordinates of Q' are zero.

From the Beer-Lambert law (Reference 1),

$$H(x,y,z) = H(x,y,0) \exp[-z/D_p] \quad (1)$$

where $H(x,y,z)$ is the irradiance at any arbitrary point, $H(x,y,0)$ is the surface irradiance at any point $x,y,0$, and D_p is the "Penetration Depth" of the resin at the laser wavelength. D_p is defined as that depth of resin which will reduce the irradiance to $1/e$ (about 37 %) of the surface irradiance.

Further, if we assume that the laser irradiance distribution is Gaussian (as seen in Figure 2, this is generally a reasonably good approximation), then from Reference 2,

$$H(x,y,0) = H(r,0) = H_0 \exp[-2r^2/W_0^2] \quad (2)$$

where W_0 is the radius of the Gaussian beam, defined per Reference 2 at the $1/e^2$ point (i.e. at that location where the local irradiance equals about 13.5 percent of the peak irradiance, H_0). Note that since the Gaussian function is circularly symmetric, it is also convenient to perform some of the calculations in cylindrical coordinates.

3. TOTAL LASER POWER

To determine H_0 , we recognize that the total laser power incident on the resin surface, P_L , must equal the integral of the laser irradiance distribution over the entire resin surface (i.e. from $r = 0$ to $r = \infty$). Thus,

$$\begin{aligned} P &= \int_0^{\infty} H(r,0) 2\pi r dr \\ &= 2\pi H_0 \int_0^{\infty} \exp[-2r^2/W_0^2] r dr \end{aligned} \quad (3)$$

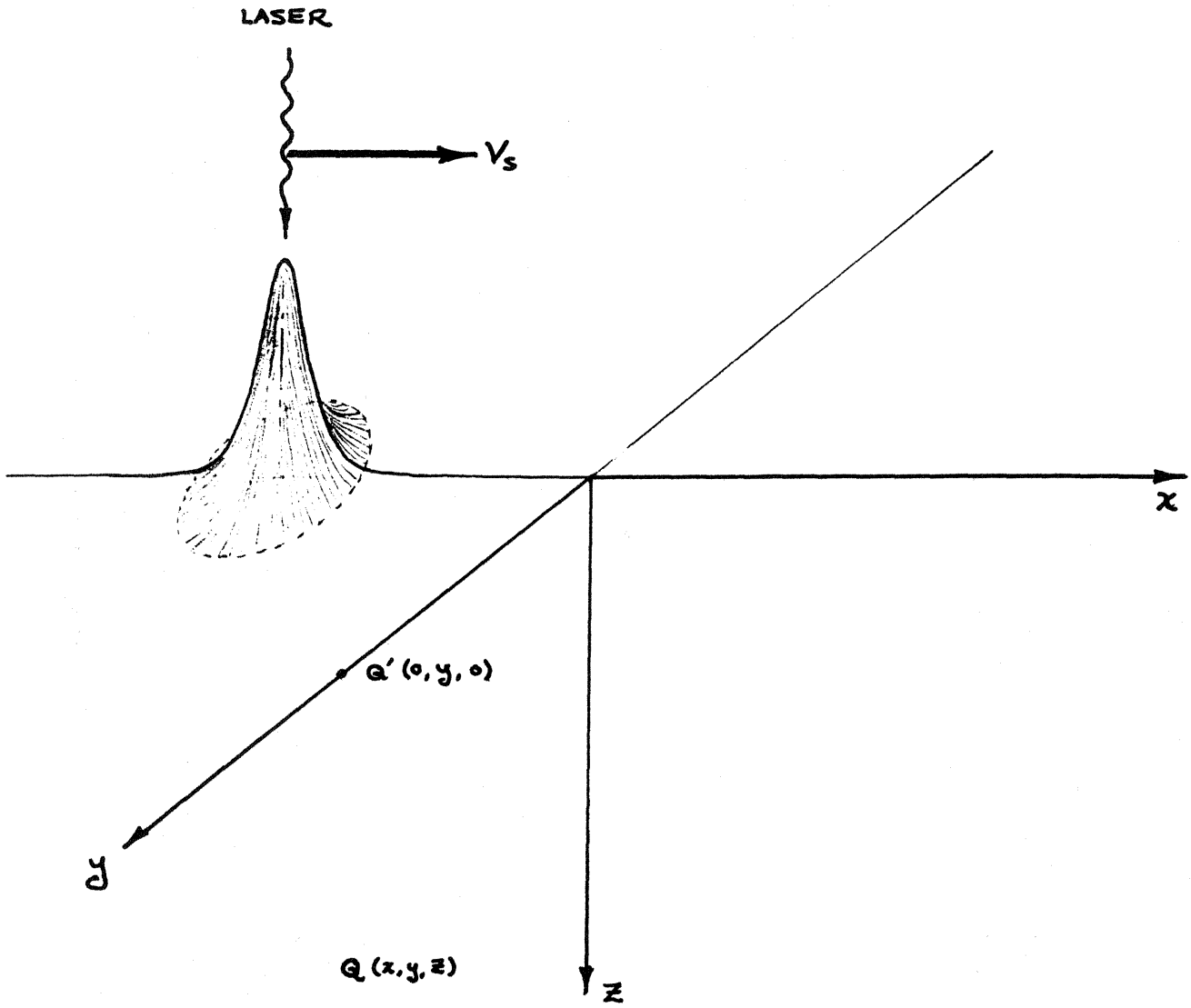


FIGURE 1

ID: BETA4 slit#2 140 mw @ head

Date: 2/12/92

Position (data) : 2704.3

Position (gauss) : 2703.7

Goodness Of Fit : 0.012

Clip Level	Beam Dia.	Gauss Dia.
5.0%	322.9	282.8
13.5%	239.8	231.2
36.8%	167.8	163.4
50.0%	140.7	136.0
60.7%	119.7	115.5

INTENSITY

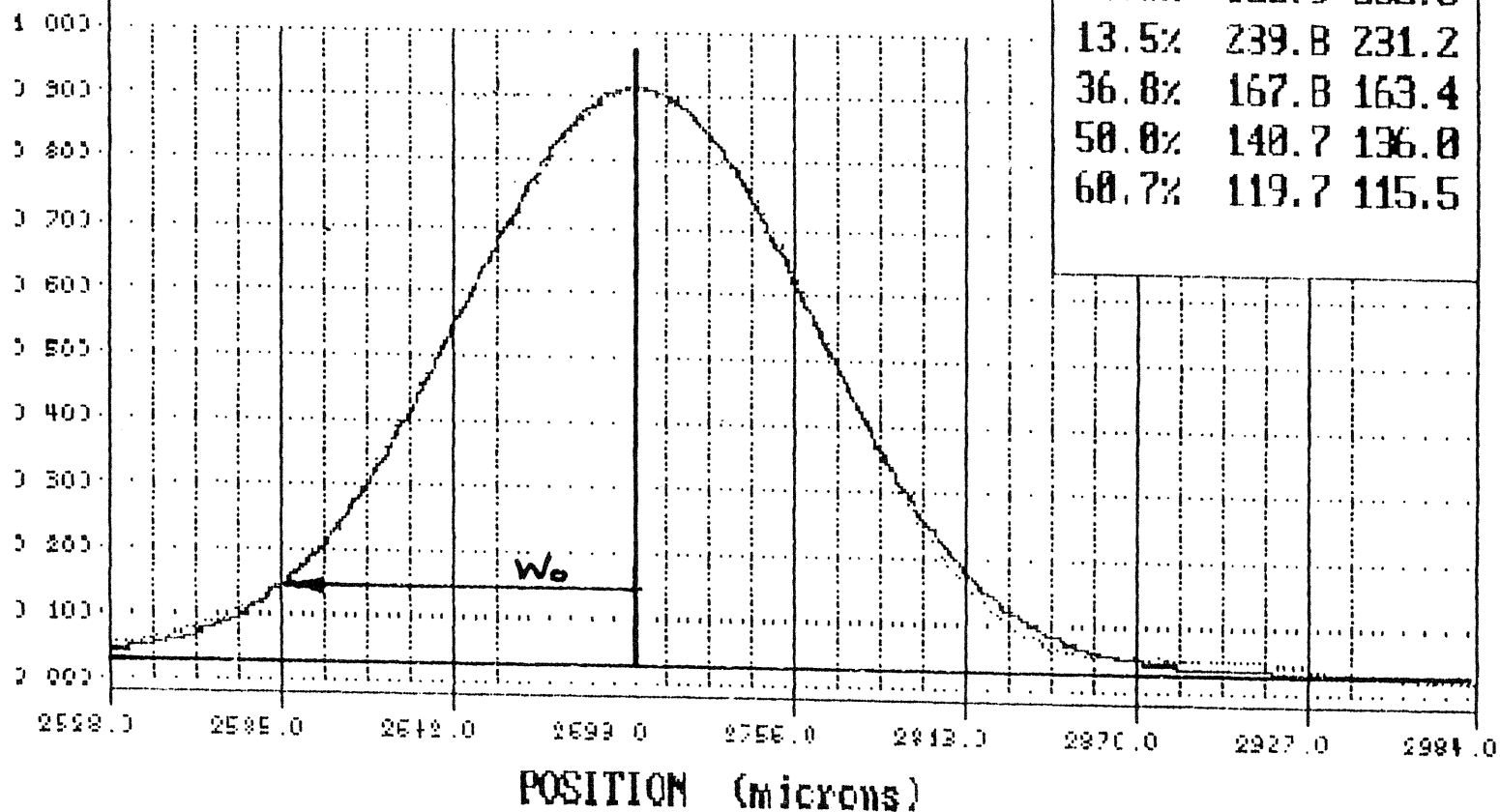


FIGURE 2

$$\text{Define } u \equiv 2r^2 / Wo^2 \quad \text{or} \quad rdr = (Wo^2 / 4) du \quad (4)$$

Substituting equation (4) into equation (3),

$$P_L = \frac{\pi}{2} Wo^2 Ho \int_0^{\infty} \exp[-u] du = \frac{\pi}{2} Wo^2 Ho \quad (5)$$

Solving for the peak irradiance at the free resin surface, Ho , we obtain the result,

$$Ho = 2 P_L / \pi Wo^2 \quad (6)$$

Substituting this result into equation (2), and then into equation (1), we obtain the Gaussian laser irradiance distribution function in cylindrical coordinates,

$$H(r,z) = (2 P_L / \pi Wo^2) \exp[-z/Dp - 2r^2 / Wo^2] \quad (7)$$

4. THE EXPOSURE FUNCTION

For StereoLithography photopolymers, the extent of reaction depends upon the number of actinic photons absorbed per unit volume. This quantity can be shown to be directly proportional to the actinic exposure, E , which has the units of energy per unit area (e.g. millijoules per square centimeter). By definition, the actinic exposure is the time integral of the actinic irradiance. Hence,

$$E = \int H dt \quad (8)$$

Since the laser is being scanned at constant velocity, Vs , along the x axis, with x increasing, then

$$Vs = dx/dt \quad \text{or} \quad dt = dx / Vs \quad (9)$$

Substituting equations (7) and (9) into equation (8),

$$E(r,z) = (2 P_L / \pi Wo^2 Vs) \exp[-z/Dp] \int_{-\infty}^{\infty} \exp[-2r^2 / Wo^2] dx \quad (10)$$

where the factor $\exp[-z/Dp]$ can be moved outside the integral sign since z is not a function of x . From the Pythagorean theorem,

$$r^2 = x^2 + y^2 \quad (11)$$

thus,

$$\exp[-2r^2/Wo^2] = \exp[-2x^2/Wo^2] * \exp[-2y^2/Wo^2] \quad (12)$$

Substituting equation (12) into equation (10), moving the factor $\exp[-2y^2/Wo^2]$ outside the integral, as it is also not a function of x , and further recognizing that since the integral of equation (10) is symmetric about $x = 0$ (viz. the contribution from $-\infty$ to 0 is exactly equal to that from 0 to ∞) then the total integral must be twice the value from 0 to infinity. Therefore,

$$E(y,z) = (4P_L / \pi Wo^2 Vs) \exp[-z/Dp - 2y^2/Wo^2] \int_0^{\infty} \exp[-2x^2/Wo^2] dx \quad (13)$$

$$\text{Define } v \equiv \sqrt{2} x / Wo \quad (14)$$

Taking differentials of equation (14),

$$dv = (\sqrt{2} / Wo) dx \quad \text{or} \quad dx = (Wo / \sqrt{2}) dv \quad (15)$$

Thus, evaluating the integral in equation (13), by substituting from equations (14) and (15), we obtain,

$$\int_0^{\infty} \exp[-2x^2/Wo^2] dx = (Wo / \sqrt{2}) \int_0^{\infty} \exp[-v^2] dv \quad (16)$$

This integral is related to the error function. From Reference 3 we find that

$$\int_0^{\infty} \exp[-v^2] dv = \sqrt{\pi} / 2 \quad (17)$$

Substituting equations (16) and (17) into equation (13), we obtain after some algebra,

$$E(y,z) = (2 / \pi)^{1/2} \{ P_L / Wo Vs \} \exp - [z/Dp + 2y^2/Wo^2] \quad (18)$$

From equation (18) it is evident that the exposure reaches its maximum value $E = E_{max}$ when $y = 0$ (i.e. on the laser scan axis) and $z = 0$ (i.e. on the free resin surface). This maximum laser exposure value is given by the wonderfully simple expression;

$$E_{\max} = (2/\pi)^{1/2} P_L / W_o V_s \quad (19)$$

Equation (19) is our first important result. It shows that the maximum actinic laser exposure is:

- * Directly proportional to the laser power.
- * Inversely proportional to the product of the beam radius and the scan velocity.
- * The proportionality constant is simply $(2/\pi)^{1/2} = 0.7979\dots$, a pure number involving no empirical quantities whatever.

5. THE PARABOLIC CYLINDER

For photopolymer resins, when the exposure is less than a critical value, E_c , the resin remains liquid. When $E > E_c$, the resin undergoes at least partial polymerization. However, if $E = E_c$, the resin is at the so-called "gel point", corresponding to the transition from the liquid phase to the solid phase. Hence, we may solve for the locus of points $y = y^*$, and $z = z^*$, which are just at the gel point. All points inside this locus will be at least partially solidified while all points outside this boundary will still be liquid. Clearly, the resulting boundary will then define the cross-sectional shape of a single laser cured photopolymer "string".

Thus, setting $y = y^*$ and $z = z^*$ when $E = E_c$ in equation (18), after some algebra we find,

$$\exp [2y^{*2} / W_o^2 + z^* / D_p] = (2/\pi)^{1/2} \{ P_L / W_o V_s E_c \} \quad (20)$$

Taking natural logarithms of equation (20), and substituting from equation (19), we obtain the result,

$$2 y^{*2} / W_o^2 + z^* / D_p = \ln [E_{\max} / E_c] \quad (21)$$

This is our second important result. We may write equation (21) in the form

$$A y^{*2} + B z^* = C$$

where A, B, and C are all positive constants. From Reference 4, this is the equation, in three dimensions, of a parabolic cylinder whose axis is the x axis, which, by definition of our coordinate system is precisely the laser scan axis.

Figure 3 shows a parabolic cylinder which results from simply scanning an actinic laser in a straight line at constant velocity over the surface of a vat of liquid photopolymer, provided that $E_{max} > E_c$. Thus, the fundamental building elements of StereoLithography are actually parabolic cylinders, often referred to as "strings".

6. THE WORKING CURVE

We may now define the maximum cure depth of a single laser cured string by the symbol C_d . From either Figure 3, or equation (21), it is evident that $z^* = z(\max) = C_d$ when $y^* = 0$. Or, simply stated, the maximum cure depth will occur directly under the laser scan axis. Thus, setting $z^* = C_d$ when $y^* = 0$ in equation (21), we finally obtain the fundamental "Working Curve" equation of StereoLithography, as discussed in Reference 5;

$$C_d = D_p \ln [E_{max} / E_c] \quad (22)$$

This is our third important result, and is absolutely fundamental to an understanding of this technology. Equation (22) shows that:

- * The cure depth should scale as the natural logarithm of the maximum actinic laser exposure.
- * A semi-logarithmic plot of C_d vs $\ln E_{max}$ should result in a straight line relationship, known as the Working Curve.
- * The slope of the Working Curve is exactly the penetration depth, D_p , of the resin, at the laser wavelength.
- * Since $\ln(1) = 0$, the intercept of the Working Curve (i.e. the value of E_{max} where $C_d = 0$) is precisely the critical exposure, E_c , of the resin, at the laser wavelength.
- * Since D_p and E_c are purely resin parameters, then within the limits of this model, both the slope and the intercept of the Working Curve should be independent of either the laser power, P_L , the laser spot size, W_o , or the laser scan velocity, V_s .

Figure 4 shows an actual Working Curve for Ciba-Geigy resin XB 5149. Note the excellent linearity of this semi-logarithmic plot. The resulting values of D_p and E_c are indicated.

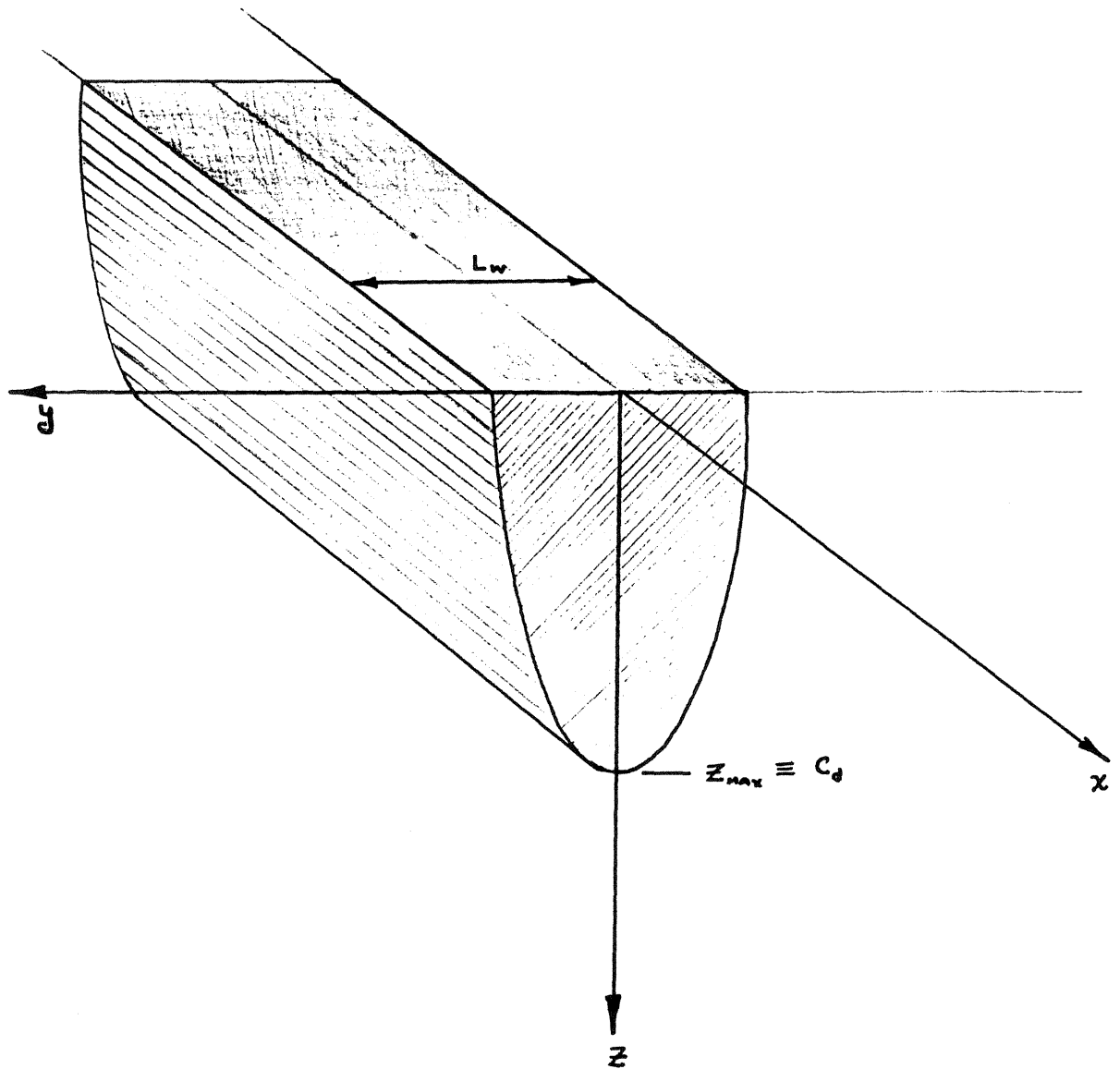


FIGURE 3

WORKING - CURVE

Resin : XB 5149

$D_p = 5.8$ mils

$E_c = 6.8$ mJ/cm²

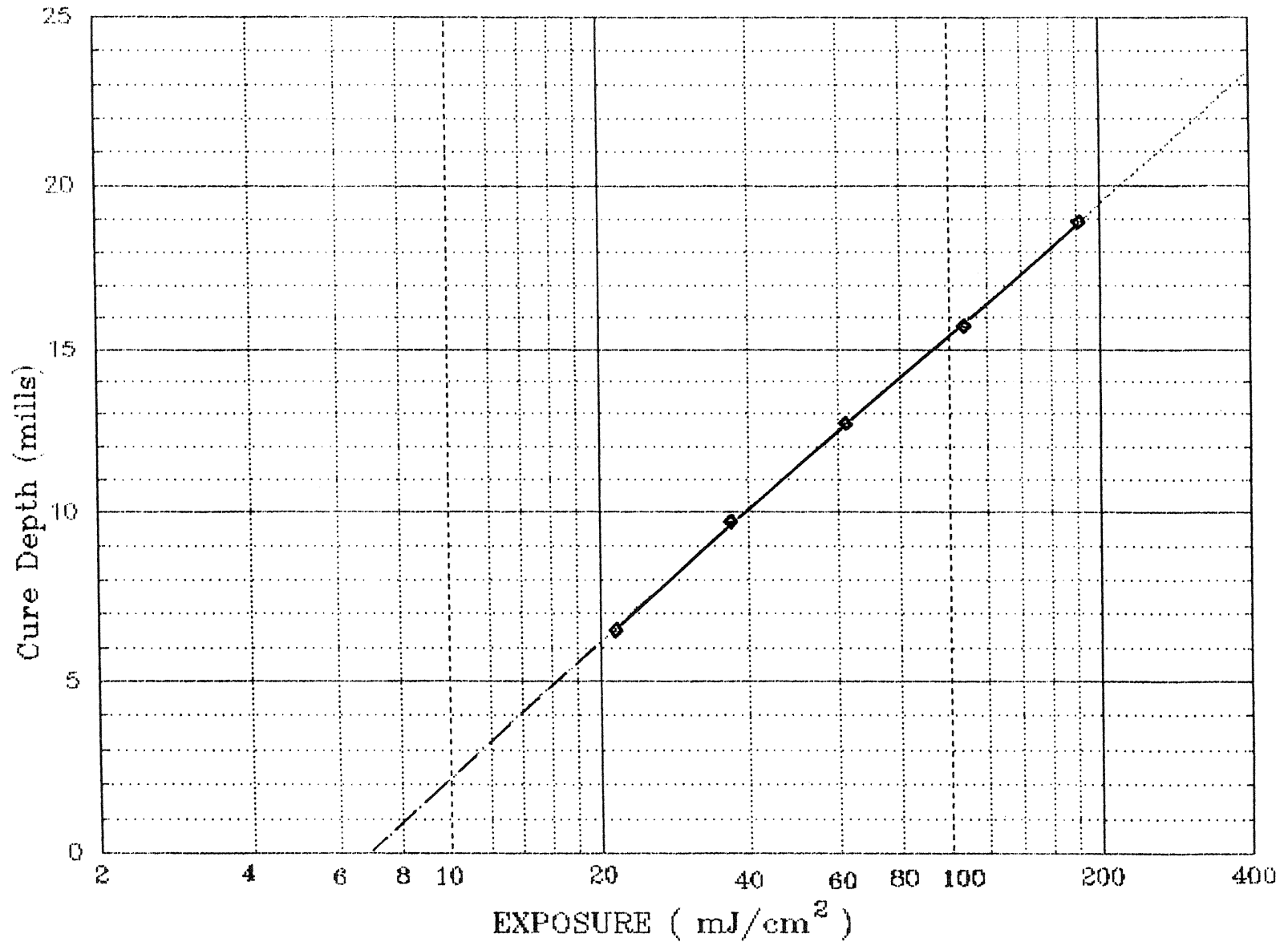


FIGURE 4

7. THE CURED LINEWIDTH

Returning to equation (21) and Figure 3, it is also clear that the maximum cured linewidth, L_w , will occur at the resin surface, where the parabolic cylinder has its greatest width. Therefore, setting $y^* = y(\max) = L_w / 2$, when $z^* = 0$, we obtain after some algebra,

$$L_w = \sqrt{2} W_o \{ \ln [E_{\max} / E_c] \}^{1/2} \quad (23)$$

Substituting for $\ln [E_{\max} / E_c]$ from equation (22), we obtain the basic "Cured Linewidth" equation,

$$L_w = W_o \sqrt{2 C_d / D_p} \quad (24)$$

This is our fourth important result. It shows that:

- * The cured linewidth of a string is directly proportional to the laser spot size at the plane of the resin surface. In calculating numerical values, remember that W_o is the radius of the laser spot, not the diameter.
- * The cured linewidth is also proportional to the square root of the ratio of the cure depth to the resin penetration depth. Thus strings of greater cure depth will also be wider, but their width will not increase linearly with C_d .
- * Even if W_o and C_d are held constant, the cured linewidth will depend upon the resin penetration depth. This is important to remember whenever one changes resins.

8. LASER SCAN VELOCITY

A modified form of equation (22) may be written as follows:

$$E_{\max} = E_c \exp [C_d / D_p] \quad (25)$$

Substituting for E_{\max} from equation (19) we find,

$$E_{\max} = (2 / \pi)^{1/2} \{ P_L / W_o V_s \} = E_c \exp [C_d / D_p] \quad (26)$$

Solving for the laser scan velocity, V_s , we obtain the result

$$V_s = (2 / \pi)^{1/2} \{ P_L / W_o E_c \} \exp [-C_d / D_p] \quad (27)$$

This is our fifth important result. It illustrates the following:

- * The laser scan velocity is directly proportional to the laser power. Thus, subject to system servo limits, the higher the laser power the faster the scan speed for a given resin, laser spot size, and cure depth.
- * The laser scan velocity is inversely proportional to the laser spot size. Thus, increasing the spot size decreases the laser scan velocity for a given laser power, resin and cure depth.
- * The laser scan velocity decreases in an exponential manner with an increase in the ratio C_d / D_p . For a given resin, this is the reason why increased cure depths draw much more slowly than shallow cure depths.
- * Again, the constant of proportionality predicted by this model is the same pure number, $(2 / \pi)^{1/2} = 0.7979....$

Equation (27) is currently incorporated into 3D Systems software. It is the basis for all automatic laser scan velocity calculations on the SLA-190, the SLA-250, and the SLA-500. Experimentally measured cure depths, generated using the automatic laser scan velocity algorithm, are typically within a few percent of the desired values. Further, the residual errors are approaching the limits of the experimental technique (viz. standard deviations of +/- 0.15 mil or about +/- 4 microns).

9. DRAWING TIME PER UNIT AREA

In StereoLithography, the great majority of the laser drawing time is spent "hatching" to solidify the regions of parts interior to their borders. Except for very tiny parts, the time required to draw the borders is generally a small fraction of the time required to complete the hatching process. For complete generality we could analyze any arbitrary cross section. However, to simplify the calculations, let us consider a rectangle of length L , in the scan direction, and width w , perpendicular to the scan direction. Figure 5 shows this rectangle, whose area is simply $A = Lw$.

Now, let us perform the hatching operation by drawing straight parallel vectors with a hatch spacing, h_s , as occurs with the advanced building techniques WEAVE™ and STAR-WEAVE™. Neglecting finite acceleration and deceleration effects at the ends of each hatch vector, the time required to draw a single string of length L is simply:

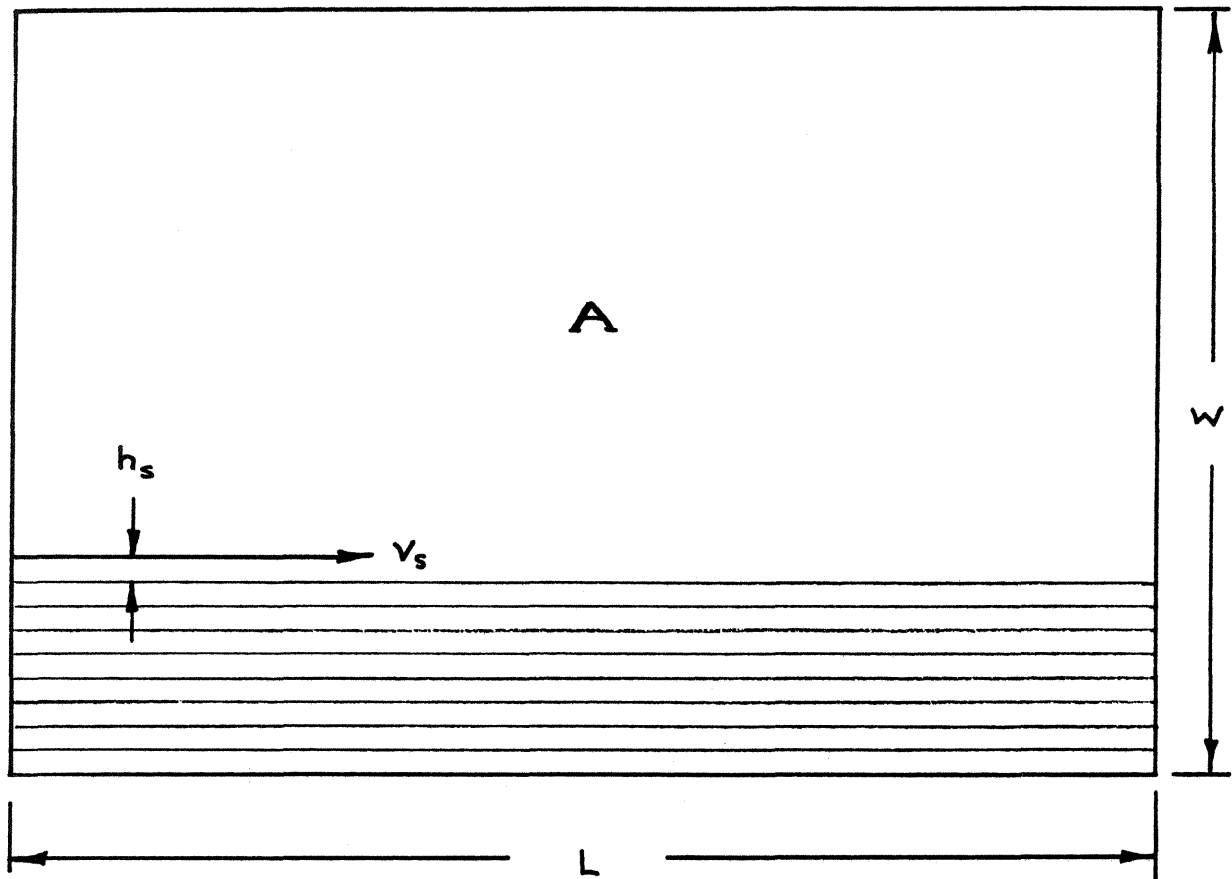


FIGURE 5

$$t = L / V_s \quad (28)$$

Within integer round-off, the number of such vectors, N , is given by

$$N = w / h_s \quad (29)$$

Thus, to close approximation the laser drawing time, t_d , is given by the following:

$$t_d = N t = (w / h_s)(L / V_s) = A / h_s V_s \quad (30)$$

Finally, in the advanced build methods WEAVE™ and STAR-WEAVE™, the hatch spacing is taken proportional to the laser spot size W_o . Thus,

$$h_s = k W_o \quad (31)$$

where k is a constant, generally near 2, so that the optimum hatch spacing is of the order of the beam diameter. Substituting equations (27) and (31) into equation (30) we obtain after some algebra:

$$t_d / A = (1/k) (\pi / 2)^{1/2} \{ E_c / P_L \} \exp [C_d / D_p] \quad (32)$$

This is our sixth important result. Equation (32) indicates that within the accuracy of the approximations we have made;

- * The drawing time per unit cross-sectional area is directly proportional to the resin critical exposure, E_c . Resins with higher values of E_c will draw more slowly.
- * The drawing time per unit area is inversely proportional to the laser power, P_L . As one might expect, higher power lasers will reduce draw time, provided one has not reached the system servo limit.
- * The drawing time per unit area increases exponentially with increased cure depth to penetration depth ratio. For a given resin with a specific value of D_p , increased cure depths draw much more slowly. Thus, in addition to improved part accuracy, because the advanced techniques WEAVE™ and STAR-WEAVE™ utilize reduced cure depths relative to the former Tri-Hatch method, they actually draw faster even though many more individual vectors are required.

- * Finally, and especially significant, is the observation that at least to first order, the laser drawing time per unit area is independent of the laser spot size. This result, which is probably not intuitively obvious, has been substantiated experimentally for all the approved StereoLithography resins.

The latter point leads to the seventh important result. Since laser drawing time is independent of laser spot size to first order, then dynamic optical zoom is of limited utility in such a system, unless one is always operating at the servo limit. Dynamic zoom (viz. the ability to vary the laser spot size over a wide dynamic range) would increase system cost, increase system complexity, reduce reliability, and, as the above analysis shows, provide productivity gain only for those laser power levels which exceed the system servo limit.

10. SUMMARY

We have derived some of the most fundamental relationships of StereoLithography. Each of these results has been experimentally confirmed to varying levels of accuracy. Higher order effects such as optical self-focusing, resin "bleaching", radiation scattering, non-Gaussian laser modes, finite acceleration and deceleration intervals, and system servo limitations will alter the trends somewhat. Nonetheless, the mathematical results discussed in this paper are predominantly valid, and should be considered fundamental to a solid understanding of this technology.

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