

THE PREDICTION OF THE THERMAL CONDUCTIVITY  
OF POWDERS

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**Abstract**

A revised view factor for the prediction of the thermal conductivity of powder beds at high temperatures that includes a radiation contribution to the conductivity is presented. Comparison of predictions by this equation with 424 measured values shows the predictions to be accurate to within a  $\pm 30\%$  relative error.

**Introduction**

Several equations are available in the literature that can predict the thermal conductivity of powder beds at high temperatures. These include the Yagi-Kunii (YK) equation [1] and the Zehner-Schlünder (ZS)[2] equation. At last year's SFF Symposium [3], we presented a rederived and corrected ZS equation that was modified by the direct addition of a Damköhler [4] radiation term,

$$k_R = 4F\sigma T^3 x_R \tag{1}$$

where  $k_R$  = the conductivity by thermal radiation (W/m-K),

$F$  = view factor,

$$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4},$$

$T$  = the mean absolute temperature of the powder bed, in K, and

$x_R$  = the effective length for radiation between particles, or the particle diameter of the powder, in m.

The resulting Zehner-Schlünder-Damköhler (ZSD) equation,

$$\frac{k}{k_g} = (1 - \sqrt{1 - \epsilon})(1 + \frac{\epsilon k_R}{k_g}) +$$

Free fluid

$$+ \sqrt{1 - \epsilon} \left[ \frac{2}{1 - \frac{Bk_g}{k_s}} \left( \frac{B}{\left(1 - \frac{Bk_g}{k_s}\right)^2} \left(1 - \frac{k_g}{k_s}\right) \ln \frac{k_s}{Bk_g} - \frac{B+1}{2} - \frac{B-1}{1 - \frac{Bk_g}{k_s}} \right) + \frac{k_R}{k_g} \right] \tag{2}$$

Core heat transfer

where  $k$  = effective thermal conductivity of the powder bed, W/m-K;  
 $k_g$  = thermal conductivity of the continuous gas phase, W/m-K;  
 $k_s$  = thermal conductivity of the skeletal solid, W/m-K;  
 $\epsilon$  = porosity of the powder bed;  
 $k_R$  = thermal conductivity part of the powder bed owing to radiation, denoted by the  
Damköhler's equation above, W/m-K; and  
 $B$  = deformation parameter of the particle.

was compared with limited data and found to work quite well, relative to the YK Equation [3].

In this paper, the ZSD Equation is further enhanced by an improved estimate for the view factor,  $F$ , in Equation (1).

### Modifications to Model

Numerous estimates of the view factor for heat transfer by radiation within the powder bed have been made. For example, Damköhler [4] took 1/3 for the view factor in his equation. Bosworth [5] also took it to be 1/3. Argo and Smith [6] took the view factor,  $F$ , to be  $e/(2-e)$ , where  $e$  is the emissivity of the powder. Schotte [7] took the view factor,  $F$ , to be equal to the emissivity,  $e$ . Wakao and Kato [8] took the view factor,  $F$ , to be equal to  $2/(2/e - 0.264)$ .

In last year's paper, we adopted Damköhler's view factor of 1/3. This year, in adopting the Wakao-Kato's approximation for the view factor, we found that our calculations of thermal conductivity to more nearly approach experimental values in all cases. This improvement occurs because the Wakao-Kato view factor is substantially larger than the other approximations.

For example, in Figure 1, the data of the thermal conductivity of alumina powder bed of the porosity of 0.71 [9] is plotted together with the equation of prediction with the Wakao-Kato's view factor  $F$ , the same equation with Damköhler's view factor of 1/3 (the form we adopted in last year's symposium paper), and with the Yagi-Kunii's equation. In Figure 1, it may be noticed that Equation (2) with Damköhler's  $F$  of 1/3 gives an average error of deviation between the reported measured values and the predicted values of -8.41%. After the change of using the Wakao-Kato's view factor,  $F$ , (the emissivity values in the expression came from our own measurement and extrapolation of the data of our measurement; see "Emissivity of Powder Beds" in the proceedings of this symposium, by the same authors of this article), the average error of deviation between the reported measured values and the predicted values decreases to -2.03%.

Comparison of  $k$  of alumina powder (porosity = 0.71)  
 by measurement, by the proposed equation  
 with Wakao-Kato's  $F$ , by using Damköhler's  $F$  of  $1/3$ ,  
 and by the Yagi-Kunii's equation

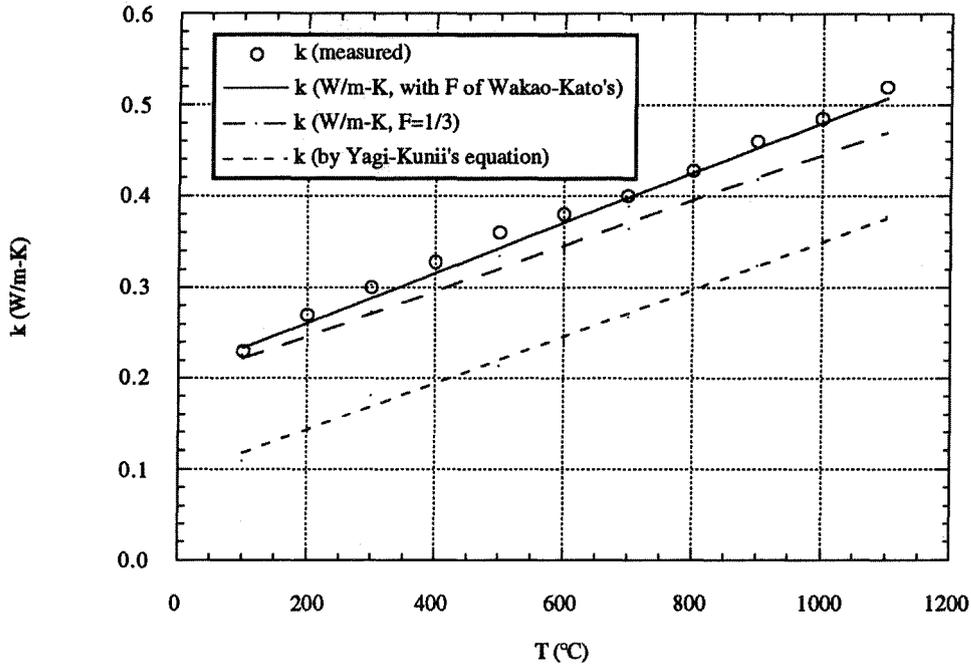


Figure 1.  $k$  of alumina powder (porosity = 0.71) by measurement, compared with the predicted values by Equation (2) with Damköhler's  $F$  of  $1/3$ , with Wakao-Kato's  $F$ , and by Yagi-Kunii's Equation

### Comparison with Experiments and Conclusions

Equations (1) through (2) have been tested with 424 data points taken from the literature for a variety of powders [9, 10, 11,12]. When emissivity data for the powder are available, Wakao-Kato's expression has been used for the view factor  $F$ . Otherwise we have set  $F=0.5$  because we believe it to be closer than the  $1/3$  value used by Damköhler. In all the calculations,  $B$  is assumed to be 1 (i.e. the particles are spherical in shape and there is no flattening of contact surfaces). The comparison is shown in Figure 2.

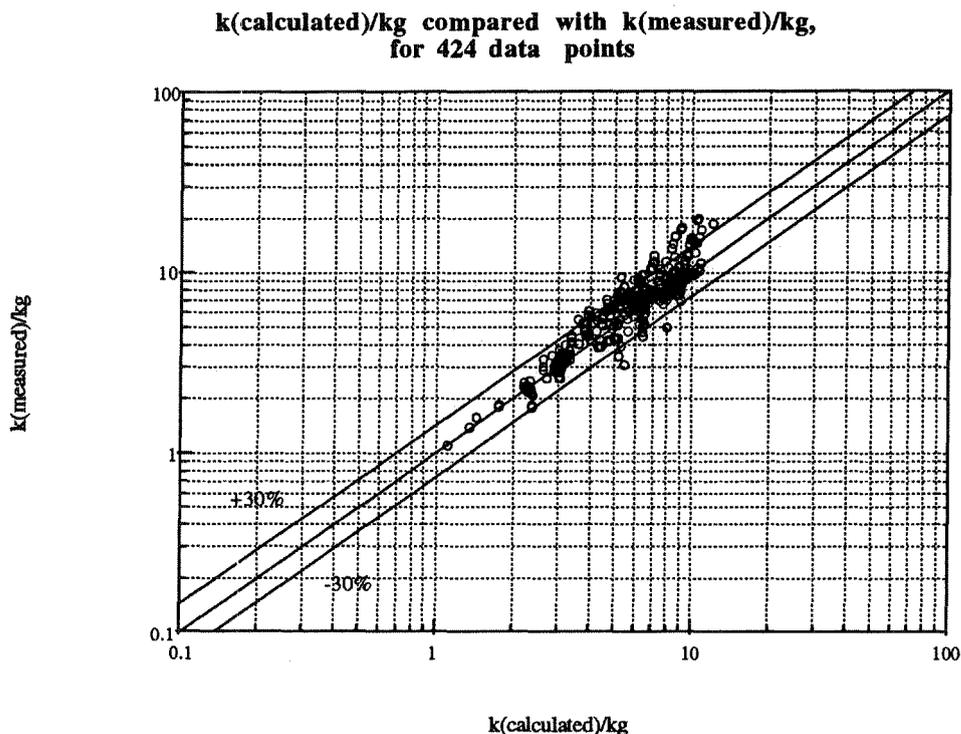


Figure 2. The Normalized Data of the Thermal Conductivity of Powder Beds ( $k/kg$ ) Compared to the Normalized Predicted Thermal Conductivity Values according to Equation (2)

We may conclude that the Wakao-Kato's expression for the view factor of the radiation term proved to be more consistent with the measured values than the Damköhler's value of view factor. The revised equation for the prediction of the thermal conductivity of powder beds has been compared with 424 sets of data from literature and measurements of the authors. The comparison showed to be within an error range of +30% to -30%.

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