Solid Model Creation for Materially Graded Objects*

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1.0 Introduction

Materially graded objects are objects composed of different constituent materials and could exhibit continuously varying composition and/or microstructure [1][2]. Such continuous changes results in gradation in their properties and distinguish them from objects made of conventional composites. They are also known as heterogeneous objects, functionally graded/gradient materials (FGM) and multi-material objects/structures [1]. In this paper, we use the terms "materially graded objects" and "heterogeneous objects" interchangeably. Recently, heterogeneous objects have found use in several engineering applications. The fabrication process that has shown potential to manufacture heterogeneous objects is called Solid Freeform Fabrication (SFF), also known as Layered Manufacturing (LM) [3][4]. SFF is a material deposition process in which the material deposition can be controlled to vary the material composition throughout an object, thus fabricating a materially graded object.

All SFF technologies are computer-based and require the CAD model of the object to be manufactured. However, current CAD systems are capable of representing only the geometry/topology information. Therefore, heterogeneous objects are fabricated using SFF by manually feeding the material information along with the geometry data. This is a cumbersome process and leads to the under-utilization of the SFF process. An assessment of existing representations for SFF is presented in [5] highlighting the need for CAD models which represent material information along with the geometry data of the object.

2.0 Previous Work

Traditional geometric/solid modeling has focussed on modeling objects based on their geometry and topology [6][7][8]. There is no additional information in the solid model regarding the material of the object. The geometry of the object is modeled by considering the mathematical space $\mathbf{T} = \mathbf{R}^3$. Certain subsets of this space called r-sets and manifold solids are widely accepted as valid mathematical models of physical objects [7][8]. The most commonly used representation schemes for this model are the Constructive Solid Geometry (CSG), Boundary Representation (B-Rep) or a hybrid [6][7][8].

In our earlier work [9], we proposed an approach to model objects composed of finite number of domains with each domain made of a single material or the domain being a single embedded component. Examples of these multiple material objects are shown below in Figure 1.

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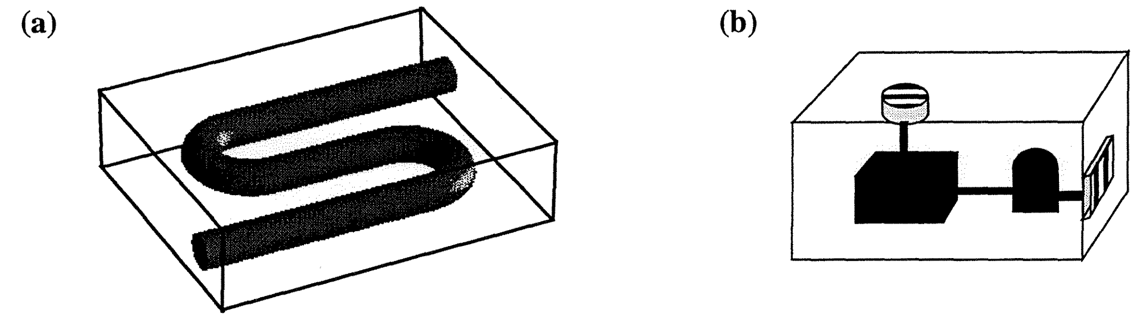

FIGURE 1. Objects with distinct material domains (a) Two material object: copper cooling channel in steel block (b) Object with embedded electromechanical systems

In order to represent these objects, the mathematical space **T** was expanded to include a material space (**M**) along with the spatial dimensions $\mathbf{R}^3$. The choice $\mathbf{M} = \mathbf{Z}$, the set of integers, was sufficient to model objects made of a finite number of unique materials. Thus, the product space $\mathbf{T} = \mathbf{R}^3 \times \mathbf{Z}$ with the product topology formed the new modeling space for representing these objects. Subsets of this space **T** called r_m-sets were proposed to model a single material domain of a multiple material object. In r_m-sets, the geometry was still modeled by the traditional r-sets and the additional integer identifies the material of the domain. A set of r_m-sets called r_m-classes was then defined to model objects made of these material domains. Boolean operations were defined to create and manipulate these models. Computer representation to implement this model was developed by including additional data structure on the B-Rep scheme. The main advantage of this approach is that the geometry is still modeled using traditional solid modeling methods and hence, the internal structure of the representation is unaffected. Additional structure has to be included to implement the material dimension of the proposed model.

3.0 Modeling of Materially Graded Objects

In this paper, we propose a method to model heterogeneous objects which have continuous variation in material composition. Refer Figure 2 for examples. The material composition of a heterogeneous object can be fully specified by specifying the volume fractions of all its constituent materials. From the volume fraction information, the density of the object at any point can be calculated.

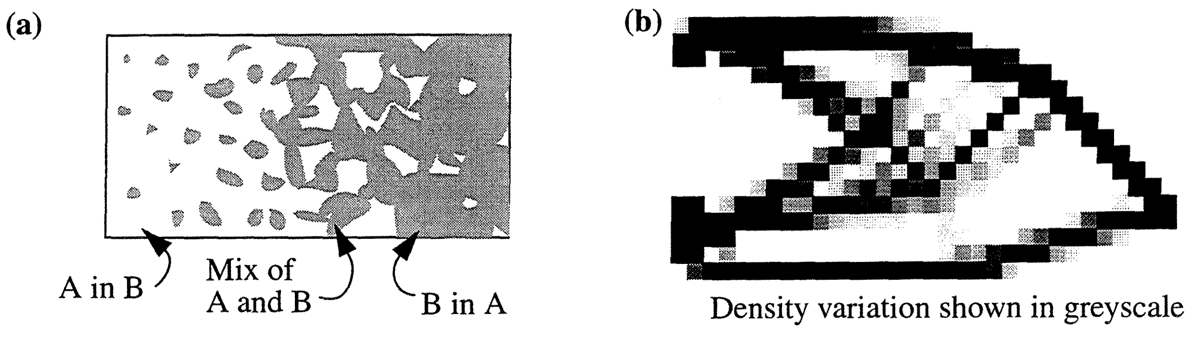

FIGURE 2. Materially Graded Objects (a) Two material object with varying composition (b) Object made of single material with varying density [13]

Modeling: To model objects with continuous material variation, the material space (**Z** in Section 2.0) must be expanded. A suitable choice for the new mathematical space is $\mathbf{T} = \mathbf{R}^3 \times \mathbf{R}^n$, n being the number of constituent materials (also referred to as primary materials). $\mathbf{R}^3$ is the geometry space, the space where the geometry and topology of the object is defined. $\mathbf{R}^n$ is the material space with each dimension representing a primary material. Each point in the object can be composed of either a single primary material or a combination of several. Thus, the material at any point can be identified by volume fractions of each of the primary materials. Noting that these volume fractions must sum to 1, we can precisely define the space of volume fractions **V** as:

$$\mathbf{V} = \left\{ \underline{v} \in \mathbf{R}^n \middle| \, \|\underline{v}\|_1 \equiv \sum_{i=1}^{n} v_i = 1 \text{ and } v_i \geq 0 \right\} \qquad \text{(EQ 1)}$$

where v_i (i-th component of $\underline{v}$) represents the volume fraction of material i. An underline is used to denote a vector in the corresponding space, such as $\underline{v}$. Note that porosity of a local region can also be modeled by including void as one of the primary materials. A set of n points ($\underline{m}^1$, $\underline{m}^2$,..., $\underline{m}^n$) called the primary points can be defined in **V** to represent the n primary materials. The coordinates of these primary points in **V** are defined as $v_i(\underline{m}^j) = \delta_{ij}$.

Each point in an object S can now be characterized in product space **T** as ($\underline{x}$, $\underline{v}(\underline{x})$) where $\underline{x} \in$ S is a point in the object and $\underline{v}(\underline{x})$ represents the material at that point such that $\underline{v}(\underline{x}) \in \mathbf{V}$. The material $\underline{v}(\underline{x})$ of any point $\underline{x}$ in S can be considered as a mapping **F** from the geometric points $\underline{x}$ to the material space **V**. The geometry of the object S can be modeled as an r-set P and the material distribution for the r-set P can be represented by the set B in **V** which is defined by function **F**. Thus, an object having varying material distribution can then be modeled as an r_m-*object*:

$$S = (P \in \mathbf{A}, B \subseteq \mathbf{V}) \text{ where } B = \{\underline{v}(\underline{x}) \equiv \mathbf{F}(\underline{x}) \in \mathbf{V}, \forall \underline{x} \in P\} \qquad \text{(EQ 2)}$$

Here, **A** denotes the class of r-sets. It might not always be possible to define the mapping **F** (i.e., $\underline{v}(\underline{x})$) using a single function for each $v_i(\underline{x})$ to characterize the material distribution for the entire object. Instead, **F** can be composed of a finite number of piecewise C^∞ functions:

$$\underline{v}(\underline{x}) \equiv \mathbf{F}(\underline{x}) = \{v_i^j(\underline{x}), j = 1 \ldots k \text{ (finite)}\}, \forall i = 1 \ldots n \qquad \text{(EQ 3)}$$

In this case, the geometric domain of each function $\underline{v}^j(\underline{x})$ has to be prescribed separately and explicitly which is equivalent to specifying C^0 and C^1 discontinuities of **F**. The r_m-*object* can then be defined as:

$$S = (P_j \in \mathbf{A}, B_j \subseteq \mathbf{V}), j = 1 \ldots k \text{ (finite)}$$

$$\bigcup_{j=1}^{k} P_j = P, \bigcup_{j=1}^{k} B_j = B \text{ where } B_j = \{\underline{v}^j(\underline{x} \in P_j)\} \equiv (v_1^j(\underline{x}), \ldots, v_n^j(\underline{x})) \qquad \text{(EQ 4)}$$

The material distribution B_j for each r-set P_j can be defined with respect to a local coordinate system L_j attached to P_j. This would make the definition of the functions $\underline{v}^j(\underline{x})$ simple. A point $\underline{x}$ that does not belong to the object is assigned $\underline{v}(\underline{x}) = \underline{0}$.

Boolean Operations: In order to create and manipulate these models, boolean operations are necessary. Consider two r_m-objects $D = \{D_t\} = \{(P_t, B_t), t = 1...k\}$ and $G = \{G_w\} = \{(P_w, B_w), w=1...m\}$. The boolean operations on any two regions D_t and G_w can be defined as:

$$D_t \square_m^* G_w = \left\{P_t \square^* P_w, B\right\}, B = \left\{\underline{w}(\underline{x}) \in \mathbf{V}, \forall \underline{x} \in P_t \square^* P_w\right\} \qquad \text{(EQ 5)}$$
$$\underline{w}(\underline{x}) = \text{normalize}(\underline{c}_u(\underline{x})(\underline{u}(\underline{x}) \in B_t) + \underline{c}_v(\underline{x})(\underline{v}(\underline{x}) \in B_w))$$

where $\square$ represents the three boolean operations $/$, $\cap$ and $\cup$. The "normalize" is the normalization operation with respect to the L_1 norm as in (EQ-1) and is performed only if the norm of $\underline{w}(\underline{x})$ exceeds one. The weight functions $\underline{c}_u(\underline{x})$ and $\underline{c}_v(\underline{x})$ can be used to manipulate $\underline{w}(\underline{x})$. Alternate functions can also be used to define the material in the intersecting region as long as $\underline{w}(\underline{x})$ lies in **V**. Now the difference operation on the two r_m-objects D and G is defined as:

$$D /_m^* G = \left\{ \cap_m^* {}_{\forall(G_w \in G)} (D_t /_m^* G_w) \middle| \forall D_t \in D \right\} \qquad \text{(EQ 6)}$$

where the difference operation between each D_t and G_w is now defined using (EQ-5). Similarly, the intersection operation can be defined as:

$$D \cap_m^* G = \bigvee \left\{ D_t \cap_m^* G_w \middle| \forall D_t \in D, G_w \in G \right\} \qquad \text{(EQ 7)}$$

where the intersection between D_t and G_w is now defined using (EQ-5). The join operator ($\bigvee$) is defined as the union of two r-sets if their corresponding material distribution is constant and equal. Finally, the union operation is defined as:

$$D \cup_m^* G = \bigvee \{(D \cap_m^* G), (D /_m^* G), (G /_m^* D)\} \qquad \text{(EQ 8)}$$

with the union between D_t and G_w defined using (EQ-5).

Representation: The computer representation of the r_m-object model can be implemented as shown in Figure 3.

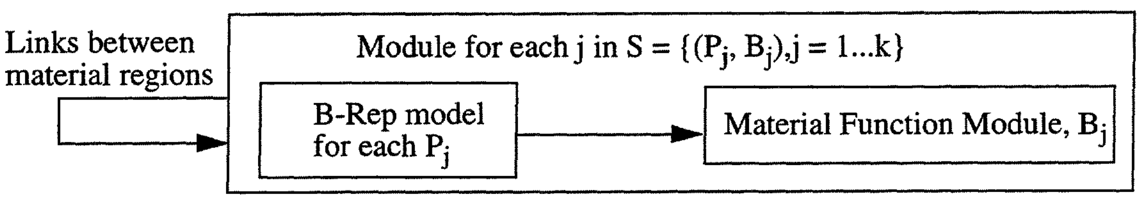

FIGURE 3. Computer representation of r_m-object

4.0 Example

We will illustrate the proposed modeling strategy through an example shown in Figure 4. The example object is a simplified model of a valve seat and is made of three regions. The outer shape (L) is made of one material (Al33) and the inner deposit (M) is made of another material (brass). The interfacial region (N) between the two is a mixture of two materials with the composition gradually changing from one material to another.

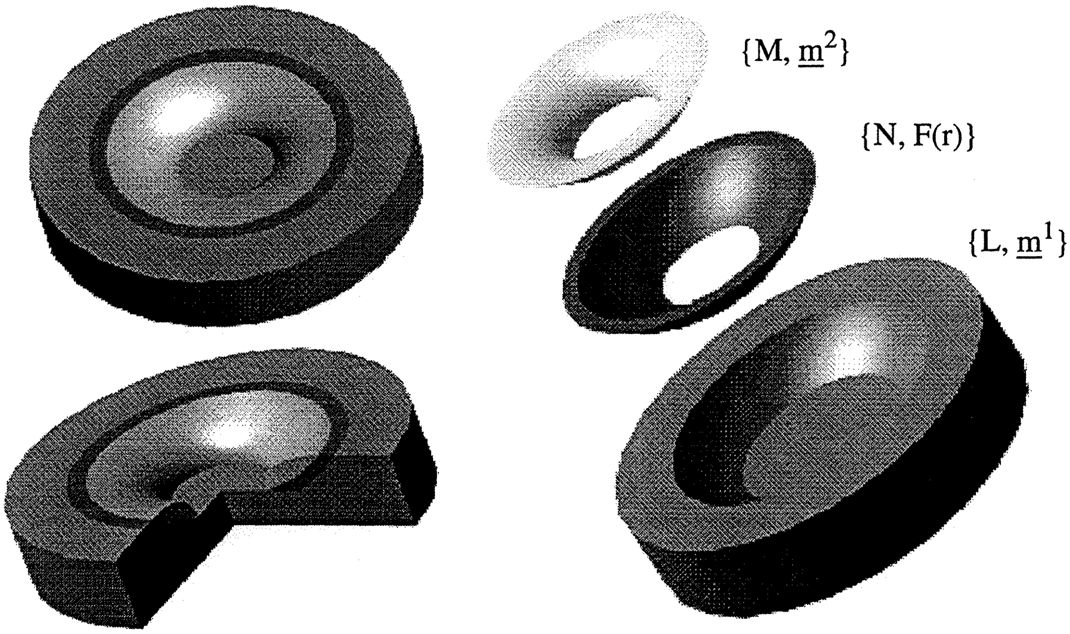

FIGURE 4. Sample Heterogeneous Object

We model this object in $\mathbf{R}^3 \times \mathbf{R}^3$ with both the geometry space and the material space being $\mathbf{R}^3$. The space of volume fractions $\mathbf{V}$ is a plane in the first quadrant of the $\mathbf{R}^3$ joining the three material points $\underline{m}^1 = (1, 0, 0)$ and $\underline{m}^2 = (0, 1, 0)$ and $\underline{m}^3 = (0, 0, 1)$. The point $\underline{m}^3$ is the void space which is used for constructing objects as shown in this example. The region L is modeled as a cylinder, the region M as a elliptical torus and the region N as a spherical shell.

The sequence of operation we used to model this object is shown in Figure 5. The primitives used are cylinder (C, $\underline{m}^1$), spheres (S1, $\underline{m}^3$) and S2, and an elliptical torus (T, $\underline{m}^2$). The material distribution of each primitive is defined with respect to its local coordinate system. Note that S2 is not assigned any material because it is used only for the difference operation. When two regions are united, the material for the intersecting region is chosen using the "matl" function (as shown in the figure). The spheres are split by two planes P1 and P2 to obtain (S1c, $\underline{m}^3$) and S2c. The object (S1c, $\underline{m}^3$) is then united with the cylinder C by choosing the material for the intersecting region as $\underline{m}^3$. From the resulting object, S2c is subtracted followed by the union of object (T, $\underline{m}^2$). Finally, the material distribution for N is changed to $F(r) = \underline{m}^1 + t(\underline{m}^2 - \underline{m}^1)$ where t is the thickness of N and r is the radius in the local co-ordinate system of N. Although the final model does not contain any region made of material $\underline{m}^3$, the sphere S1 is assigned that material in order to construct the model. The final object contains three regions and is represented by an r_m-object as $\{(L, \underline{m}^1), (N, F(r)), (M, \underline{m}^2)\}$.

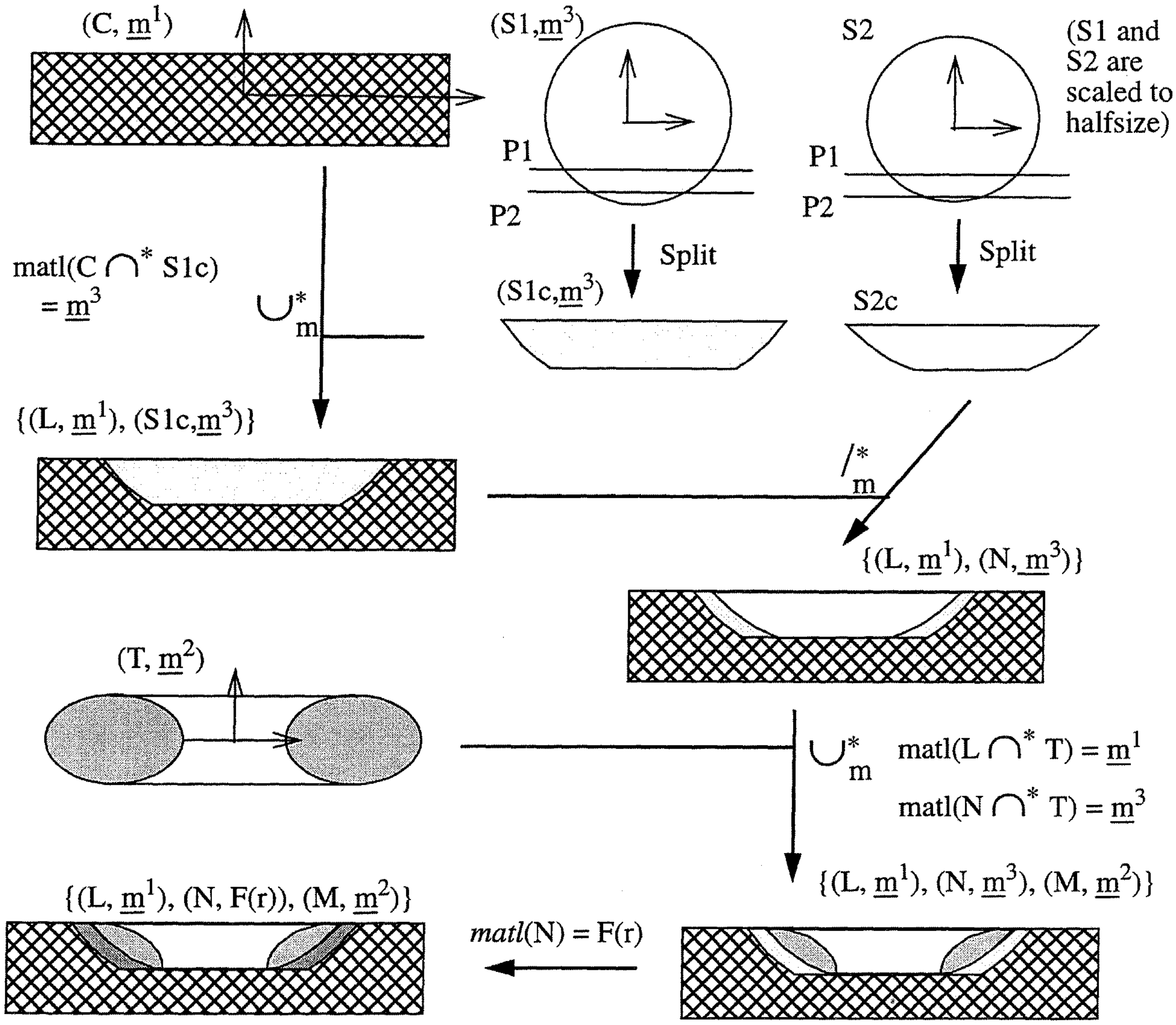

FIGURE 5. Steps in the creation of the valve model

5.0 Fabrication

Fabrication of materially graded objects by SFF involves additional processing compared to fabrication of homogeneous objects. These are identification of material distribution in each layer after performing the slicing procedure and material based tool path generation for deposition in each layer. To illustrate the steps involved, consider an r_m-object S = {(P_i, F_i), i=1...k} (equivalent definition of S= {(P_i, B_i), i=1...k} where F_i is the material distribution defined in the local coordinate system of P_i. The r_m-object S = {(P_i, B_i), i= 1...k} is processed by performing the operations on (P_i, F_i).

Orientation: Denoting the transformation by OT, the oriented r_m-object can be defined as:

$$S_z = OT(S) = \{(OT(P_i), F_i), \forall i = 1 \ldots k\} \qquad \text{(EQ 9)}$$

Slicing: The oriented r_m-object S_z is sliced by the slice planes SP(z) to obtain the layers L(z):

$$L(z) = S_z \cap SP(z) = \{(OT(P_i) \cap SP(z), F_i \cap OT^{-1}(SP(z))), \forall i = 1...k\} \quad \text{(EQ 10)}$$

The material distribution function for each layer obtained in (EQ-10) (denoted by $F_z(x,y)$) has to be approximated depending on the resolution of the fabricating process. An example of a 1D single material distribution $F_z(x)$ and its approximation is shown in Figure 6.

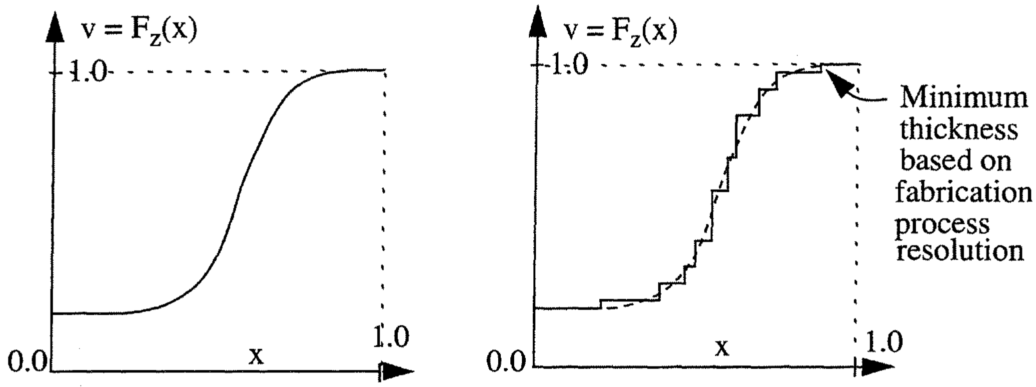

FIGURE 6. Approximation of a distribution function by series of step functions

Toolpath Generation: Once the material distribution for each layer is obtained, the tool paths for material deposition have to be generated. Currently, there does not exist any automated way of generating optimal tool paths for a given material distribution in a layer and this issue has to be addressed. Figure 4 illustrates these steps for the example discussed in Section 4.0.

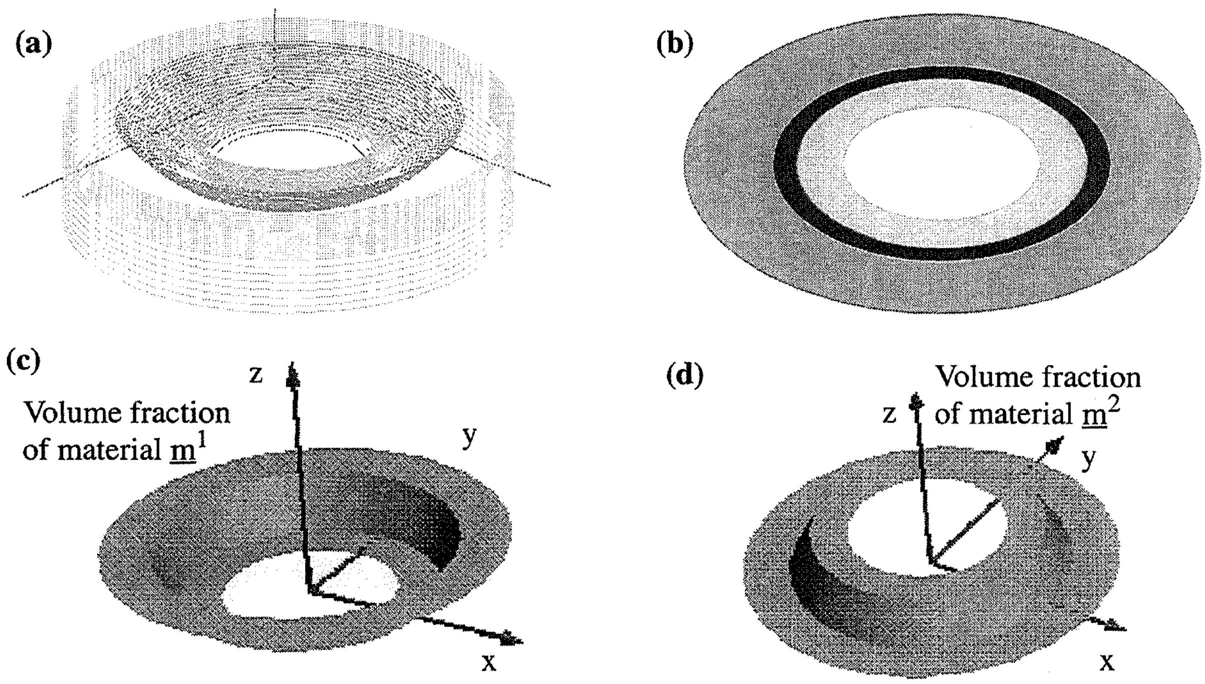

FIGURE 7. (a) Adaptive slicing (b) Material Distribution at slice z=0.8 (c) Variation of volume fraction of material-1 in slice z=0.8 (d) Variation of volume fraction of material-2 in slice z=0.8

Figure 7(a) shows the adaptive slicing [10] of the object and Figure 7(b) shows the material regions of a slice at z=0.8. The variation of volume fraction of material $\underline{m}^1$ and material $\underline{m}^2$ are shown in Figure 7(c) and Figure 7(d) respectively. The slice geometry is in the x-y plane and the variation of volume fraction of each material is shown along the z-axis.

6.0 Summary

In this paper, we proposed a solid modeling scheme for materially graded objects by extending beyond geometry/topology representation (based on r-sets) to include the material variation of the object. For a more detailed treatment of this problem, refer to [12]. The r_m-object model enhances the theory of r-sets and is compatible with traditional solid models. The computer representation of this model was built on existing B-Rep scheme and hence, can easily be adapted into existing solid modeling systems. Finally, fabrication of objects modeled as r_m-objects using SFF was also discussed.

7.0 References

1. Special Issue on Functionally Gradient Materials, MRS Bulletin, January 1995.
2. Functionally Gradient Materials, Ceramic Transactions, Editors: J. Birch Holt, Mitsue Koizumi, Toshio Hirai, Zuhair A. Munir, Vol. 34, 1993.
3. J. Mazumder, J. Koch, K. Nagarathnam and J. Choi, *Rapid Manufacturing by Laser Aided Direct Deposition of Metals*, preprint, Dept. of Mechanical Engineering, University of Michigan, Ann Arbor, 1996.
4. R. Merz, Shape Deposition Manufacturing, Dissertation, Technical University of Vienna, Austria, 1994.
5. V. Kumar and D. Dutta, *An Assessment of Data Formats for Layered Manufacturing*, Advances in Engineering Software, Vol. 28, No. 3, pp. 151-164, April 1997.
6. C. M. Hoffmann, Geometric & Solid Modeling, Morgan Kaufmann Publishers, 1989.
7. M. Mäntylä, An Introduction to Solid Modeling, Computer Science Press, 1988.
8. A. Requicha, *Representations for Rigid Solids: Theory, Methods and Systems*, Computing Surveys, Vol. 12, No. 4, 1980.
9. V. Kumar and D. Dutta, *An Approach to Modeling Multiple Material Objects*, Proceedings of 4th ACM Solid Modeling Symposium, Atlanta, May 1997.
10. P. Kulkarni and D. Dutta, *An Accurate Slicing Procedure for Layered Manufacturing*, Computer Aided Design, Vol. 28, No. 9, pp. 683-697, 1996.
11. J. R. Zuiker, *Functionally Graded Materials: Choice of Micromechanics Model and Limitations in Property Variation*, Composites Engineering, Vol. 5, No. 7, pp. 807-819, 1995.
12. V. Kumar and D. Dutta, *An Approach to Modeling & Representation of Heterogeneous Objects*, ACM Transactions on Graphics, in review, available as technical report UM-MEAM-97-05, Department of Mechanical Engineering, University of Michigan, Ann Arbor, 1997.
13. M. P. Bendsoe and N. Kikuchi, *Generating Optimal Topologies in Structural Design using a Homogenization Method*, Computer Methods in Applied Mechanics and Engineering, Vol. 71, pp. 197-224, 1988.