Validation of Rapid Prototyping Material for Rapid Experimental Stress Analysis

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ABSTRACT

The paper will detail the validation work carried out on various Rapid Prototyping (RP) materials to determine their suitability for the application of Thermoelastic Stress Analysis. The overall objective is to drastically reduce the product design cycle, by providing "real experimental data" for correlation with Finite Element Analysis (FEA), prior to any expensive manufacturing process. In order to achieve this the homogeneity of the Rapid Prototyping material has to be established to ensure a valid transfer of results from model to actual part.

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1. INTRODUCTION AND THEORY

Especially in the automotive industry, there is a requirement to fully optimise the design in order to produce a lightweight but durable product. The work described here, is to determine the feasibility of using models produced by Rapid Prototyping machines for experimental stress analysis techniques (Calvert, 1994), in this case Thermoelastic Stress Analysis. Using this method, the FEA can be checked at the concept stage and the design validated early reducing the need for actual testing and subsequent design changes ,i.e. producing a high integrity product at reduced cost and time.

Thermoelastic Stress Analysis is a non-destructive technique which can be used to obtain a stress field over the whole surface of a component or structure. Its use on metallic parts is well established and proven (Stanley and Chan, 1988), but its effectiveness on RP materials is unknown. The stress measurement is based on the sensitive detection of the part surface temperature change induced in materials under dynamic stress conditions. Under adiabatic conditions there is the following linear relationship (Thomson, 1855)

$$\Delta T = -K_m \cdot T \cdot \Delta\sigma \qquad (1)$$

where ΔT is the peak to peak temperature change [K], K_m is the thermoelastic material constant [mm^2/N], T is the absolute surface temperature of the solid part [K] and $\Delta\sigma = \sigma_1 + \sigma_2$ is the peak to peak change in the sum of principal stresses [N/mm^2].

This relationship is only valid for homogeneous and isotropic material in the linear elastic region. The thermoelastic constant K_m is expressed

$$K_m = \alpha / \rho \cdot c_p \qquad (2)$$

where α is the heat coefficient of linear thermal expansion [1/K], ρ is the density [g/cm^3] and c_p is the specific heat capacity at constant pressure (under constant stress) [J/g·K].

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For the detection of this cyclic temperature change any kind of infrared camera system can be used, but SPATE is especially designed to make stress measurements. The SPATE system equation to make a theoretical calibration (Ometron, 1996) is

$$\Delta\sigma = F_{th} \cdot S = \frac{D \cdot G \cdot R}{T \cdot e \cdot 2048 \cdot K_m} \cdot S \qquad (3)$$

where F_{th} is a theoretical calibration factor [N/mm^2], S is the measured signal [-], D is the temperature response [K/volt], G is the Correlator sensitivity [volt], R is a temperature correction factor [-] and e is the emissivity of the paint. This relation is dependant on equation (1) and the measurement equipment. The parts are normally painted to get a high and consistent emission.

2. SUITABILITY CRITERIA

The following criteria can be defined for the suitability of a Rapid Prototyping material for Thermoelastic Stress Analysis.

2.1 Homogeneous and isotropic behaviour

Equation (1) is only valid for an homogeneous and isotropic material. Because of the manufacturing process (the test part is created by building one layer on top of another) the material may show different strength behaviour in different directions, also there could be defects in the form of voids inside the material.

2.2 Linear elastic deformation and adiabatic conditions

Only the linear elastic temperature change is proportional to the sum of principle stresses. But in reality there will also be other reasons which may cause a temperature change

$$\Delta T = \Delta T_{le} + \Delta T_{ve} + \Delta T_{pl} + \Delta T_{flow} \qquad (4)$$

where ΔT_{le} is due to linear elastic effect, ΔT_{ve} is due to visco-elastic behaviour, ΔT_{pl} is due to plastic deformation and ΔT_{flow} is the heat flow due to temperature gradient. A correlator can be used to filter all the changes that are not periodic with dynamic load, but some of the terms may still have some influence:

ΔT_{ve} increases with the differential of strain after time $d\varepsilon/dt$. This differential $d\varepsilon/dt$ is periodical to the test frequency (f). Therefore a low f would be positive. ΔT_{pl} is low, if there is less plastic deformation. Therefore the offset yield stress should be small. ΔT_{flow} is proportional to the thermal conductivity k. Under adiabatic conditions a part under stress will have infinitesimal low heat flow between areas of different stress and temperature. Therefore, there is a need for a high f, especially if k is high. But on the other side, the paint is like a thermal resistor. For a good heat flow from the stress area through the paint, a low f would be an advantage, especially if the paint thickness is thick.

It should therefore be possible to find a test frequency, where the last 3 terms are low and the signal due to the linear elastic effect is high. Additionally it is necessary to define a linear elastic limit stress σ_θ.

2.3 Repeatability and calibration

It should be possible to calibrate the test equipment for a specific material enabling every test to give directly, good stress values. Therefore the material itself should always have the same mechanical properties throughout. If the environmental conditions or the load test procedure are different, the influence of those differences will need to be established. Otherwise the test conditions must remain the same and not vary from test to test. Especially, it should be ensured that the following should not change: material, surface finish, test frequency, load waveform and test surface temperature.

2.4 Correlation with theoretical solution in a wide region of stress

In the case of a known stress calibration the thermoelastic signal can be multiplied by a constant calibration factor F_{known}. Therefore the stress value will be exact for a certain signal. It is necessary to investigate the effect of signal variation on stress values.

2.5 Accuracy and correlation with theoretical calibration

For the SPATE system a theoretical calibration after the equation (3) is possible. The variation between the theoretical and the known stress calibration should be investigated. The size of the thermal emission from a material has an influence on the minimum measurable stress value $\Delta\sigma_{min}$. To obtain a high degree of accuracy the quotient of $\sigma_\theta/\Delta\sigma_{min}$ should be as high as possible

$$\frac{\sigma_\theta}{\Delta\sigma_{min}} = \frac{\sigma_\theta}{\dfrac{\Delta T_{min}}{K_m \cdot T}} = \frac{\sigma_\theta \cdot K_m \cdot T}{\Delta T_{min}} = \frac{\sigma_\theta \cdot \alpha \cdot T}{\rho \cdot c_p \cdot \Delta T_{min}} \qquad (5)$$

where ΔT_{min} is the minimum measurable temperature change of the camera system.

2.6 Transferability

The RP model will be of a different material to the real component, but its elastic stress distribution (or stress field) should be similar to that of the metal prototype for similar loading conditions. In simple cases, the stress field is only dependant on the geometry. But for isotropic materials there can also be an influence due to material constants:

a) different Poisson's Ratio ν: can cause different stress distribution, if the lateral deformations are suppressed (examples: rigid boundaries, friction in contact surfaces)

b) different density ρ: can cause different stress distribution, because centrifugal body forces are dependant on density and the self-weight of a model will cause body forces as well

c) different Young's modulus E: will cause different deformation under similar load condition (examples: distortions become appreciable and therefore the deformation is not linear any more, with instability problems.the different distortion may drastically change the magnitude of forces).

3. INVESTIGATIONS AND EXPERIMENTS

To prove the suitability of a Rapid Prototyping material, the following experiments were carried out. At the IKP in Stuttgart experiments were conducted to determine the material constants $\alpha = \alpha(T)$ and $c_p = c_p(T)$. Tensile tests were conducted with a horizontal and a vertical extensiometer. Brazilian Disc tests were conducted to establish the materials thermoelastic performance.

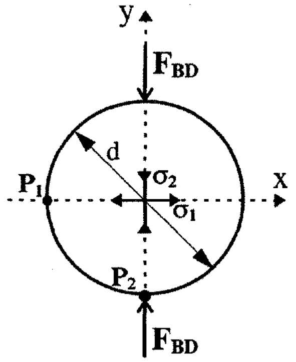


Figure 1 Brazilian Disc Test

The disc under compression (see figure 1) has a well known stress field (Timoshenko, 1951 and J.M. Dulieu-Smith,1995). At the point P_1 for example the sum of principal stresses is zero while at the point P_2 it is unlimited.

Test specimens were manufactured using SLA technology (Stereolithography): lying (layers parallel to the load direction) and standing (layers vertical to the load direction) for the tensile tests, lying discs for the Brazilian Disc test. For nylon (SLS technology, Selective Laser Sintering) material details were taken from reference books.

In the following the RP material will be evaluated due to the criteria in section 2. The criterion is identified in brackets { } before each evaluation.

3.1 Evaluation of Basic material and tensile test results

{ 2.1 }

Results for lying and standing dogbones:

E_S/E_L [-]	D_E [%]	$\sigma_{maxS}/\sigma_{maxL}$ [-]	$D_{\sigma max}$ [%]	ν_S/ν_L [-]	D_ν [%]
1.028	2.8	0.976	-2.4	0.966	-3.4

Notation: D_E is the Deviation of Young's modulus, $D_{\sigma max}$ is the Deviation of maximum stress and D_ν is the Deviation of Poisson's ratio.

Notation of the indices: L for lying and S for standing

{ 2.2 }

The material had in all directions a region with very low plastic strain. So it was possible to determine σ_θ.

{ 2.3 }

The following table presents the standard deviation in % for each series of specimens:

SD_{E_L}	SD_{E_S}	$SD_{\sigma max\_L}$	$SD_{\sigma max\_S}$	$SD_{\sigma 0.2\_L}$	$SD_{\sigma 02\_S}$
2.2	1.2	1.0	1.1	3.4	2.8

Notation: SD_E is the Standard Deviation of Young's modulus, $SD_{\sigma max}$ is the Standard Deviation of maximum stress and $SD_{\sigma 0.2}$ is the Standard Deviation of offset yield stress at offset strain 0.002 mm/mm.

Notation of the indices: L for lying and S for standing

All results are less than 5%, hence the mechanical repeatability of part manufacturing is good.

For the repeatability of Thermoelastic Stress Analysis it is important, that the calibration of the camera signal is only constant, if the part surface temperature doesn't change. But in reality it is very difficult and expensive to test always at the same temperature. Therefore it would be useful to know the influence of the temperature on the calibration factor. A temperature influence is given due to the measurement system and equation (1), but in this particular case it is also due to the thermoelastic constant K_m.

By using equation (3) the calibration factor can be described as below

$$F_{th}(T) = A \cdot \frac{R(T)}{T \cdot e(T) \cdot K_m(T)} = A \cdot \frac{R(T) \cdot \rho(T) \cdot c_p(T)}{T \cdot e(T) \cdot \alpha(T)} \qquad (6)$$

where A is a constant factor. The temperature influence of e and ρ is very low, while the investigations have shown, that the influence of $\alpha(T)$ and $c_p(T)$ are quite considerable. The glass temperature of the RP material is quite low and near to room temperature and therefore α for example will increase, if temperature is increasing.

The whole influence of temperature to the calibration factor can then be described as

$$Q_{F20}(T)=\frac{F_{th}(T)}{F_{th}(20^{\circ}C)}=Q_{MC}(T)\cdot Q_{Km}(T) \qquad (7)$$

where $Q_{MC}(T)$ is the influence of measurement conditions and $Q_{Km}(T)$ is the influence of the thermoelastic material constant. The figure 2 below presents the results of equation (7).

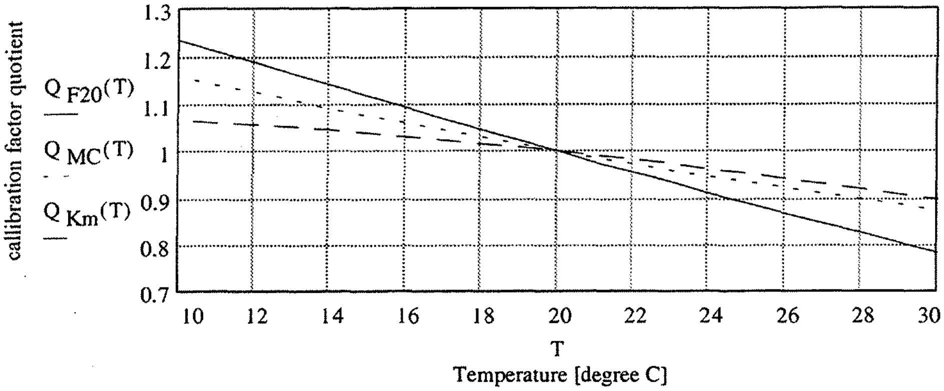


Figure 2 Influence of temperature to the calibration factor

{ 2.5 }
With the help of the material constants it is possible to calculate K_m, $\Delta\sigma_{min}$ and $\sigma_\theta/\Delta\sigma_{min}$. The table below for $\sigma_\theta/\Delta\sigma_{min}$ compares two Rapid Prototyping materials with a steel plate at 20 degree Celsius.

Material	$\sigma_\theta/\Delta\sigma_{min}$ [-]
Rapid Prototyping material 1 (SLA)	351
Rapid Prototyping material 2 (Nylon, SLS)	44
steel plate material 3	198

{ 2.6 }
The table below shows a comparison of ρ, E and ν with a common steel material ($\rho_{steel}\approx7.8$ g/cm^3, $\nu_{steel}\approx0.3$, $E_{steel}\approx200000$ N/mm^2).

Material	ρ/ρ_{steel} [-]	ν/ν_{steel} [-]	E_{steel}/E [-]
Rapid Prototyping material 1 (SLA)	0.16	1.37	66
Rapid Prototyping material 2 (Nylon, SLS)	0.15	(?)	157

3.2 Evaluation of Brazilian Disc test results

{ 2.2 }
Figure 3 below presents the measured signal over a range of test frequencies.

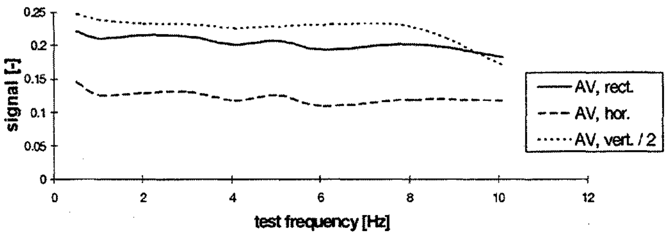


Figure 3 Measured signal over different test frequencies

In figure 3 the Brazilian Disc test results have been averaged. Notation of the legend: AV, rect. is the averaged signal in an rectangle area near the centre, AV, hor. over the horizontal diameter and AV, vert. over the vertical diameter of the disc.

The signal was high and quite constant at a low frequency comparing well with the necessary frequency of steel for example. This is reasonable, because the thermal conductivity k of the RP material is much lower than k of steel.

{2.3}

A calibration factor can now be defined by making a known stress calibration.

$$F_{\text{known}}(T_{\text{disc}}) = \frac{\sigma_{\text{sum\_C}}}{\text{signal}_{\text{centre}}(T_{\text{disc}})} \qquad (8)$$

where $\sigma_{\text{sum\_C}}$ is the theoretical sum of principle stresses in the centre of the disc (known stress), $\text{signal}_{\text{centre}}$ is the measured signal in the centre of the disc and T_{disc} is the surface temperature of the test specimen used for determining F_{known}.

Because the signal was a little noisy, a filter was used to obtain the signal for calibration. For tests with another test body temperature, this calibration factor can be corrected with the help of figure 2.

{2.4}

After calibration it is possible to compare the whole measured stress field with the theoretical solution.

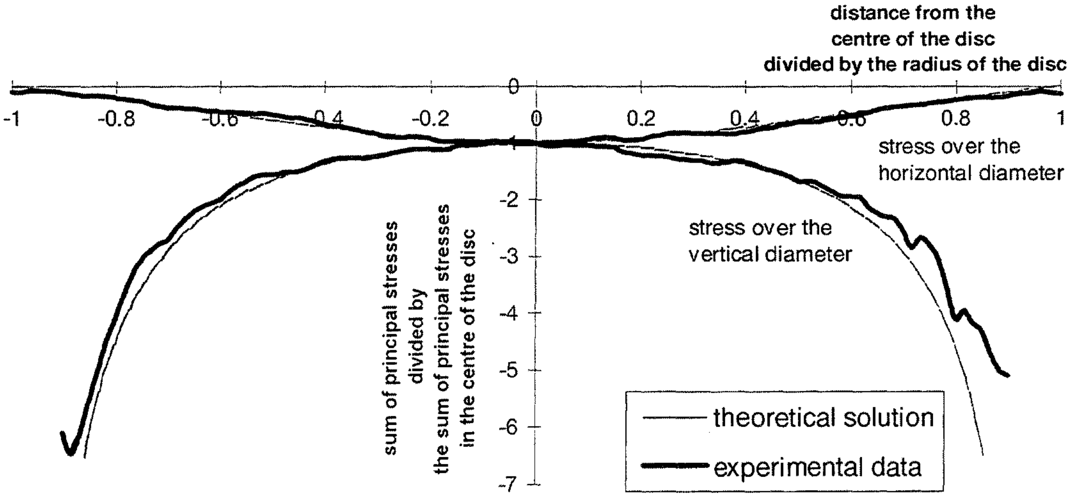
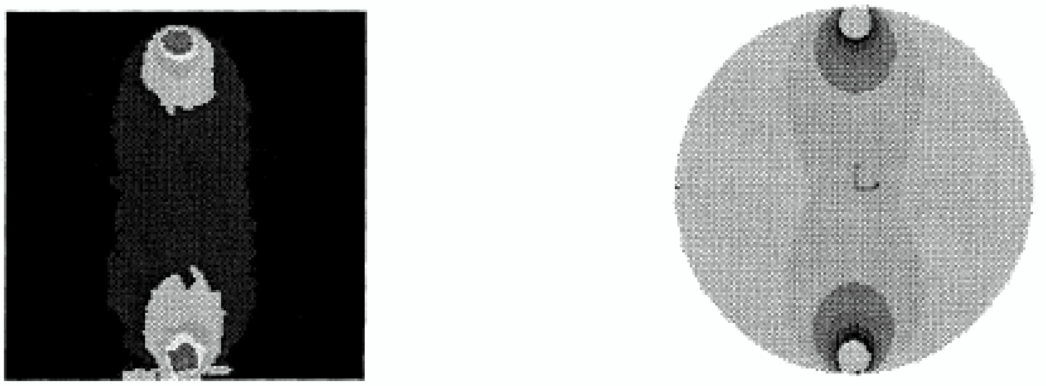


Figure 4 Comparison of experimental data with the theoretical solution for a machined disc



Scan image:

FE image:

Figure 5 Comparison of the measured stress field with theoretical FE solution of $\Delta\sigma$.

In figure 4 the upper curves are the stresses over the vertical disc diameter and the lower curves are the stresses over the horizontal diameter. Figure 4 and 5 show a very good correlation below and over the calibration point. The deviation outside the vertical diameter can be explained by the method of clamping. First of all the theoretical solution goes to infinity. This is not reasonable, because in reality there is an area pressing instead of a point load. Therefore it is correct, that the measured results are below the theoretical solution.

{2.5}
Using the SPATE system the following quotient should be near to one:

$$\frac{F_{th}(T_{disc})}{F_{known}(T_{disc})} \approx 1$$

The table below shows the results for the RP material and a steel plate.

Material	$\frac{F_{th}(T_{disc})}{F_{known}(T_{disc})}$ [-]
Rapid Prototyping material 1 (SLA)	0.68
steel plate material 3	0.61

The signal for the RP materials is about 30% and for steel about 40% lower as expected, but the results are in the right region. The potential for errors in equation (3) should be noted.
The deviation of the experimental results with the theoretical solution can be calculated as shown in figure 6.

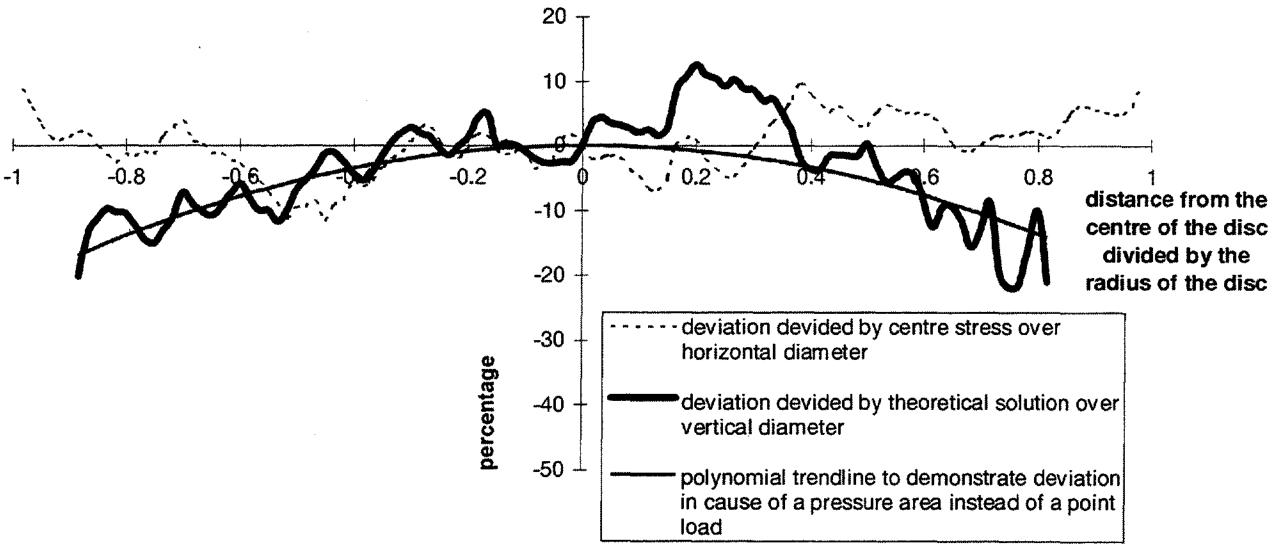


Figure 6 Deviation of experimental data from theoretical solution

Except near the load input the deviation is below or about 20%.

3.3 Summary

The results for the criteria {2.1} to {2.5} show good results and the RP material can be validated for Thermoelastic Stress Analysis, while the following facts should be highlighted:

- The theoretical accuracy $\sigma_\theta/\Delta\sigma_{min}$ of the measurement is about 2 times higher than for the steel plate material, which is a common metallic material in the automotive industry.
- The test can be conducted at about 10 times lower test load and about 10 times lower frequency. That means the test can be conducted on cheaper testing machines and probably under office conditions.

- The low necessary test frequency reduces the influence of the paint thickness and the losses of the emission are lower.
- The finishing and preparation of the disc specimens hasn't been costly. Any other test prototype could be built within hours from the CAD-file.

Because E, ρ and ν are different from a metallic material, the transferability (criterion {2.6}) depends on the geometry and on the loading of the part. Curiously enough, if no elastic deformation is suppressed, the load is much higher than body force due to self weight and the deformations are small, then practical examples have shown, that the stress fields of two different materials are very similar. But also the absolute value of σ_1 (positive) + σ_2 (negative) has been smaller than the von Mises stress σ_{Mises}, which is often used as a comparison stress with the unidirectional limit stress σ_{max}.

4. CASE STUDY AS FUTURE WORK

The transferability and the interpretation of the results could be investigated theoretically by FEA within a case study by defining the following two criteria.

4.1 Similar stress distribution

The influence of ρ, E and ν can be checked and estimated theoretically by a comparison of the von Mises stress fields as follows:

$$\sigma_{Mises} \text{ of FEA}(\rho, E, \nu \text{ of RP material; testload})$$

$$\Updownarrow$$

$$\sigma_{Mises} \text{ of FEA}(\rho_C, E_C, \nu_C \text{ of the case study material, critical load}).$$

If the two calculations produce a similar stress field for a linear static problem, the influences of ρ and ν can be ignored. The influence of E can be estimated with the theoretical deformation.
It is then possible to convert the results (measured stresses) directly to the sum of principal stresses in the metal part by simply scaling the loads from RP model to series part.

4.2 Interpretation of measuring results

The interpretation could be done by a comparison of the measured results with $\Delta\sigma$ of an FEA postprocessing as follows:

$$\text{Dynamic Test and Measurement(RP material, testload)}$$

$$\Updownarrow$$

$$\Delta\sigma \text{ of FEA}(\rho_C, E_C, \nu_C \text{ of the case study material, critical load})$$

and by calculating the following stress quotient: $\left|\frac{\Delta\sigma}{\sigma_{Mises}}\right|$.

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