Agile Product Testing with Constrained Prototypes

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Abstract

The means to acquire reliable functional information is a critical factor that differentiates product development time and cost. Thanks to advances in solid freeform fabrication techniques, industries can produce geometrically complex parts within dramatically reduced time and cost. Even though industries can save significant efforts by performing functional tests rapid prototypes, they still prefer full-scale product tests, especially in later design phases, due to inherent limitations in traditional similarity methods (TSM). This paper describes a new method to perform reliable functional tests with rapid prototypes that cannot be properly handled by the TSM.

1 Introduction

The means to acquire and manage system information, which guides design activities, is influential to the performance of design processes. Our ancestors created primitive devices mainly through natural experiences. Recognizing the need to obtain necessary information with efficiency, people started to perform planned observations (active observations), reflecting on them to exploit specific phenomena. Thanks to such exploits and advances in virtual modeling techniques, contemporary designers are able to obtain necessary functional information far more efficiently. However, there are still needs to obtain reliable functional information in time, as the phenomena that can be described purely by mathematics are still very limited.

Thanks to advances in various rapid prototyping techniques, industries can fabricate geometrically complex parts with dramatically reduced effort (Jacobs, 1992; Aubin, 1994). Considering the significant time and cost spent on product testing to verify or improve final product quality, it is natural to expect various industrial applications of rapid prototypes for function testing. However, industries utilize rapid prototypes mostly in early design phases, and very limited case studies are reported on functional testing with rapid prototypes to verify product performance or tune design parameters (Dornfield, 1995; O'Reilly, 1993), mainly due to material issues (Wall et al., 1991). This paper introduces a new method to correlate distorted systems that cannot be handled by traditional methods, especially when systems are composed of multiple parts.

2 Similarity Methods for Prototype Testing

To predict full-scale product behavior using economic scaled (rapid) prototypes, mainly two types of similarity methods can be applied:

- Dimensional analysis that designs scaled models, and correlates product and scaled model behavior on the basis of dimensions of dominant system parameters, through the Buckingham Π theorem (Bridggman, 1931; Sedov, 1959; Szucs, 1980; Baker et al., 1991).
- Analytical fractional analysis that mathematically correlates the solutions of two known equations, by comparing and manipulating the equations (Sedov, 1959; Kline, 1966).

In dimensional analysis, one should design scaled models so that all of corresponding dimensionless parameters of the scaled model and target system should be identical. Especially when prototyping materials are limited, the chance to fabricate well-scaled models is very low. Besides, one should precisely know system parameters beforehand. Such limitations are clarified in our previous publications (Cho et al. 1998(a)(b)). In comparison to dimensional analysis, analytical fractional analysis can provide more accurate scale testing results (Kline, 1965). However, one should know precise governing equations, material properties, and loading conditions that are not well informed in many circumstances.

In order to overcome such limitations in traditional methods, we introduced a novel empirical similarity method (ESM), in which geometrically simple specimens are utilized as additional information sources. One can refer to (Cho and Wood 1997, Cho et al. 1998(a)(b)) for fundamentals of the method, and this paper will describe the concept through an archery bow example focusing on correlation of systems composed of multiple parts.

3 System Correlation through Relative State Representation

When products are composed of multiple parts, the TSM requires to design scaled models to satisfy the following material proportionality condition:
Proportionality of material properties - The material property ratio of corresponding model and product components should be kept identical, in order to perform scale testing with the TSM.
The proportionality condition is very difficult to be satisfied, when available prototyping materials are limited. If two systems do not satisfy this proportionality, they are defined

to have distorted configurations, and there exists no method to correlate systems with distorted configurations within the authors' knowledge. As an initial attempt, we test the possibility to predict behaviors of systems composed of two materials, utilizing a scaled model fabricated from a single prototyping material.

The product specimen[1], the scaled model, and the product system parameters can be represented in terms of the model specimen[2] parameters, through the scale and form factors. One should notice that the form factors and the state scale factors should satisfy

$$\lambda_{X1} \cdot \phi_p = \lambda_{X2} \cdot \phi_m, \tag{1}$$

where the state scale factors λ_{X1} and λ_{X2} can be mathematically derived through dimensional analysis, and ϕ_m and ϕ_p are unknown form factors. From the equation, the form factors ϕ_m and ϕ_p, which represent the amplifications of the model and product states under pure geometrical changes, should be identical, if $\lambda_{X1} = \lambda_{X2}$, viz. the model and the model specimen are designed from the same similarity constraints.

In our new approach, each part is designed following the Π theorem in order to maintain the identity of the form factors of the model and the product, and the scale factors of the specimen pair and mode-product pair. Through the identical factors, one can represent the lumped model of the product specimen, the scaled model and the product, in terms of the model specimen parameters. As a result, one can represent the product state as a function of the states of the specimens and the model. The step by step procedure of this new approach named as lumped ESM is summarized below.
Procedure of the Lumped ESM:
(1) Build lumped parameter models: Describe the states of the model pair (the model and the model specimen) and the product pair (the product and the product specimen) as lumped models with unknown effective lumped parameters of each part of the systems.
(2) Determine the scale factor of each component: Considering the material/loading parameters and size scaling, design scaled models through dimensional analysis, and mathematically derive the scale factors of lumped parameters (the ratio of the product and model parameters) of each component.
(3) Relatively describe lumped models: Employing the derived scale factors and an unknown form factor (between the model specimen and the model), represent the product specimen, the scaled model, and the product in terms of lumped parameters of the model specimen.
(4) Represent product states in terms of the model and specimen states: By comparing and manipulating the four lumped models, describe the unknown product states as a function of the measured states of the scaled model and the specimen pair.

In summary, the lumped ESM correlate the states of model and product pairs by implicitly deriving the unknown lumped parameters through the measured states of a scaled model, and a specimen pair.

[1]Geometrically simplified version of product
[2]Geometrically simplified version of scaled model

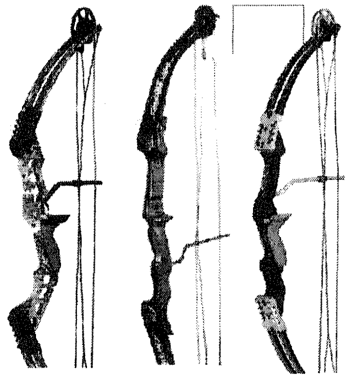

Figure 1: Exemplary Archery Bows

Numerical Archery Bow Example

In designing archery bows, the shooting force, viz. the force required to fully deform a bow to shooting position, is a typical system behavior that draws our attention. One should design bows so that both ergonomic (e.g., required shooting forces) and functional (e.g., arrow speed at a distance) requirements are satisfied.

Virtual bow models may not be accurate enough to design quality bows, mainly due to geometrical and material nonlinearity. Large bow deformation causes significant geometrical nonlinearity, and the stress-strain behavior may not be linear within the wide strain range, considering popular bow materials (e.g., fibers and woods). For these reasons, designers may need to fabricate several test bows to verify or improve product quality. Considering the geometrical complexity of bow frames shown in Figure 1, significant cost and time to fabricate test-bows are expected. We illustrate and validate the lumped ESM to predict the required (or resultant) force of a product bow in an effective manner. As a preliminary study for future physical testing, we test lumped ESM using numerically simulated bow behaviors. In this bow example, the TSM requires the identity of the Young's modulus ratios of the bow frames and strings of models and products. However, the identity of the ratios cannot be satisfied in many cases, as the prototyping material choices to fabricate the string and the frame of model bows are limited.

Shooting forces were simulated with ANSYS^{TM}, and the finite element bow models of the target (product and scale model) and specimen bows of interest are shown in Figure 2. In the finite element models, the string is modeled as a two-dimensional elastic link (Spar element), and the bow frame as triangular solid elements (Plane2). The width of the bow frame is set to $35mm$, and the cross section area of the string to $10mm^2$. As shown in Table 1, we use the same bow string for both model and product bows, considering the difficulty in finding proper frame (or string) materials that satisfy the condition required

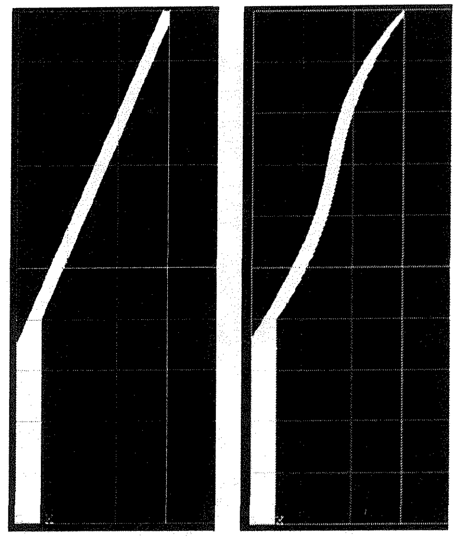

Figure 2: Symmetric Half of the Specimen and Product Bows

Parameters	Polymer Bows (Models)	Fiber Bows (Products)
Frame E^b (GN/m^2)	7.46 (Mean)	16.0 (Constant)
String E^s (GN/m^2)	5	5
String Pre-Strain	1E-8	1E-8

Table 1: System Parameters of the Bows

by the TSM,

$$\frac{E_p^b}{E_m^b} = \frac{E_p^s}{E_m^s}, \tag{2}$$

where the superscripts b and s denote bow frame and string, and the subscripts m and p denote model and product materials, respectively. In this simulation, we consider full-size models for the purpose of clear comparisons. However, one can reduce the size of the model bows without loss of generality, as long as the model systems are designed from the same scale laws.

To predict shooting forces of the target fiber bow (product) in Figure 2 (b) through the ESM, it is assumed that bows can be modeled as lumped spring systems shown in Figure 4. Then, the shooting force of a model specimen (polymer bow with the simple geometry shown in Figure 2 (a)) can be represented as,

$$F_{ms} = \frac{k_{ms}^b \cdot k^s}{k_{ms}^b + k^s} \cdot \delta, \tag{3}$$

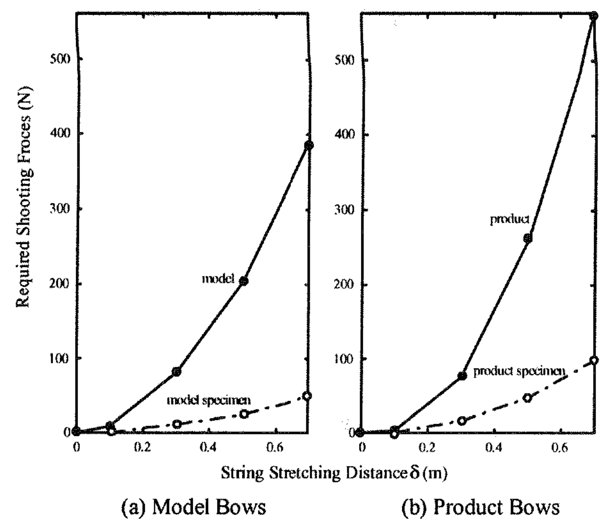

Figure 3: Simulated Shooting Forces of the Bows

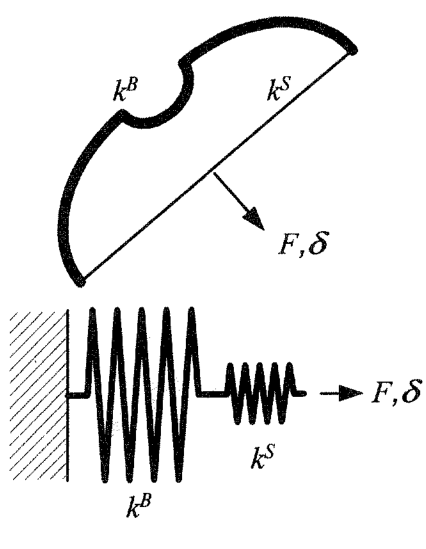

Figure 4: Lumped Model of a Bow

where F is the shooting force (or the force required to deform the bow), δ is the traveling distance of the bow string, and k^b and k^s are unknown effective spring constants of the bow frame and string respectively. By defining a new state variable X, the force equation can be equivalently represented as

$$X_{ms} = \frac{\delta}{F_{ms}} = c^b_{ms} + c^s, \tag{4}$$

where $c^b = \frac{1}{k^b}$ and $c^s = \frac{1}{k^s}$ represent effective compliance of the bow frame and string. Then the states of the scaled model, product specimen, and product bows can be represented as

$$\begin{aligned} X_{ps} &= \lambda_c c^b_{ms} + c^s, \\ X_m &= \phi c^b_{ms} + c^s, \\ X_p &= \lambda_c \phi c^b_{ms} + c^s, \end{aligned} \tag{5}$$

where $\lambda_c = \frac{c^b_p}{c^b_m} = \frac{c^b_{ps}}{c^b_{ms}}$ is the compliance scale factor, and $\phi = \frac{c^b_p}{c^b_{ps}} = \frac{c^b_m}{c^b_{ms}}$ is an unknown form factor of the bow frames.

The remaining task is to express X_p as $X_p = h(X_m, X_{ms}, X_{ps})$, so that we can predict X_p from the measured states, X_m, X_{ms}, and X_{ps}. From Equations (4) and (5), we can derive the following equations without bow string related parameters

$$\begin{aligned} X_p - X_{ps} &= \lambda_c(\phi - 1)c^b_{ms}, \\ X_m - X_{ms} &= (\phi - 1)c^b_{ms}. \end{aligned} \tag{6}$$

By combining the two equations, one can derive the following prediction equation without unknown form factor ϕ,

$$X_p = \lambda_c(X_m - X_{ms}) - X_{ps}, \tag{7}$$

where

$$\lambda_c = \frac{1}{\lambda_k} = \frac{E_m}{E_p}. \tag{8}$$

As a result, one can determine $X_p(\delta)$ from Equation (7) based on the measured $X(\delta)$ of the specimens and the model, to predict the bow force of the product $F_p(\delta)$ from the definition of the state X.

Figures 5 (a) and (b) show the shooting force predicted through the TSM and ESM. The TSM cannot predict the bow force with accuracy, as

$$\frac{E^b_p}{E^b_m} \neq \frac{E^s_p}{E^s_m}. \tag{9}$$

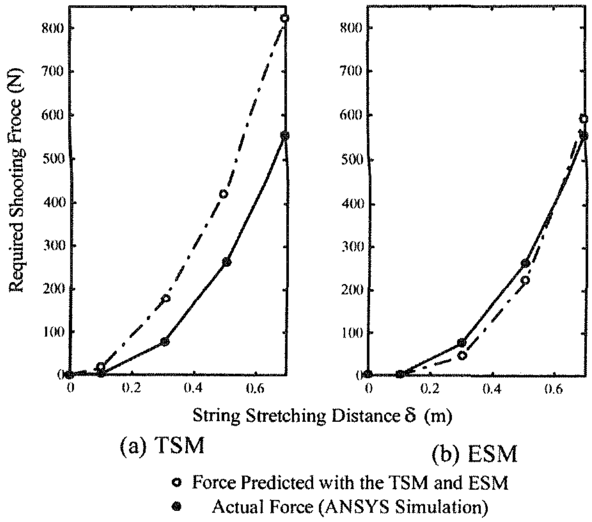

Figure 5: Prediction of the Shooting Force with the TSM and ESM

One way to perform scale testing with the TSM is to assume that the influence of the bow string to the reaction force is relatively small in comparison to the bow frame. In that case, the ratio of the reaction force should be $\frac{F_p}{F_m} = \frac{E_p^b}{E_m^b}$, and it should be equal to $16/7.46 = 2.14$. Figure 5(a) shows the force predicted through the TSM ($F_p = 2.14F_m$), neglecting the effect of the bow strings. As we expected, there exists large discrepancy between the actual and predicted reaction forces of the product bow (Figure 5(a)). In contrast, the bow force predicted through our new lumped ESM (Equation (7) is plotted in Figure 5(b), and one can notice the remarkably improved prediction accuracy. The ESM results are comparable to the full-scale product testing, and one may save significant effort to fabricate full-scale product bow for testing, especially fabricating scaled models from rapid prototyping processes.

4 Conclusions

Through a simple bow example, the capability of the ESM to correlate systems with distorted configurations has been demonstrated. In general, specimens are used to estimate specific material properties or determine form factors. In contrast, the ESM explicitly estimate neither system parameters or form factors. Instead, the ESM implicitly abstracts the relative influence of system information (e.g., material properties, form factors, boundary conditions) through a specimen pair. By adopting this approach, one can correlate two distorted systems even without knowledge of system parameters.

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