Functionally Gradient Material Representation by Volumetric Multi-Texturing for Solid Freeform Fabrication

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Abstract

In order to fabricate parts from Functionally Gradient Materials (FGM) by layered manufacturing methods, the SFF community needs a method to represent material distributions in solid models. Gradient material distribution requires accurate and systematic representation, and must be compatible with existing geometric data. This paper presents a method, called Volumetric Multi-Texturing, to represent a three dimensional density gradient by exploiting hypertexturing and volumetric density functions. This method utilizes procedural and implicit methods to design/acquire density information. The implicit procedural approach, as opposed to an input database, allows users to interactively create and modify the design patterns without explicitly changing the values stored in the database. Further, it promises convenience in process planning, and efficiency in data storage and computation time. The theoretical approach, design procedure, and tool path generation for fabrication of an example part are presented in the paper. The design procedure presented is based on specifying the material gradient by surface modification.

1 Introduction

One of the potential capabilities of Solid Freeform Fabrication (SFF) is production of parts that have not been feasible by other techniques. The fabrication of Functionally Gradient Materials (FGM) is imminently feasible in near future. Several researchers are endeavoring to fabricate FGM parts through creation of new methods or modification of existing techniques [3, 5, 7, 13, 16].

To support the fabrication of FGM parts, a new data exchange format must be created to transfer material information to the manufacturing process. Most commercial solid and surface modelers represent parts as homogeneous solids. Currently, these tools only provide support for attaching material information as a simple annotation to lumped regions for rendering and mass property calculations (e.g., mass and moment of inertia). Therefore, existing schemes are not sufficient to define volumetric properties of parts.

1.1 Related Work

Kumar and Dutta [10] first introduced the concept of a material subset, (r_m-set), where a material dimension is added to the spatial dimensions R^3. The complete material space is constructed by combining these r_m-sets through Boolean operations. Recently proposed models for specifying are based on two approaches: interpolating functions or discrete density data sets. Several trivariate function forms have been proposed to describe the spatially varying material gradient [11, 6]. These functions can be categorized as either parametric or implicit representations. Both types have advantages and disadvantages, depending on how they are to be utilized for material processing. As a three dimensional extension of surface discretization, voxel-based and volumetric mesh schemes to store material density data have been extensively researched. Pegna proposed a representation method of spatially varying data by extending volumetric meshes used for finite element analysis [12]. His suggestion was that the nodal point set in the FEA mesh to model the physical state of a part could be used to model a spatially varying material distribution as well. This approach not only provides a means to store material data but also provides an interface to material gradient design using FEA [9, 8].

2 Approach

The major requirement of this work is providing access to exact values at any given spatial point inside the solid. To meet this requirement, we chose to explore an implicit procedural scheme. In this approach, a global implicit material space defined in a geometry space G^3 is generally represented by a set of procedural functions F_m with the following conditions:

$$F_m^i(\vec{p}_{int}) = d_p^i, \quad \vec{p}_{int} \in G^3,$$
$$\sum_{i=1}^{n} F_m^i(\vec{p}_{int}) = \sum_{i=1}^{n} d_p^i = 1.$$

$\vec{p}_{int}$ = an interior point
i = material index
n = number of materials
d_p = density at $\vec{p}_{int}$

The function F_m^i does not explicitly or parametrically describe the pattern. However, the information does exist procedurally and implicitly, and is obtainable by $\vec{p}_{int}$ query of its density in the material space M^3. Since the material composition at any $\vec{p}_{int}$ must sum to 1, $n-1$ functions F_m's are necessary to fully constrain M^3. Each F_m consists of a number of sub-functions, f_m's that influence a query point $\vec{p}_{int}$. The number of required f_m's can vary throughout the part's geometry. A single material region may require no sub-functions. Interior regions near corners or edges may be composed of many contributing sub-functions. These sub-functions interact with each other as multipliers or by blending operations. The f_m's are thus wrapped together as a single procedure, and the wrapped procedure is only visible to users in the form of level sets of $F_m^i(\vec{p}_{int}) = d_p^i$. The information flow is graphically described in figure 1.

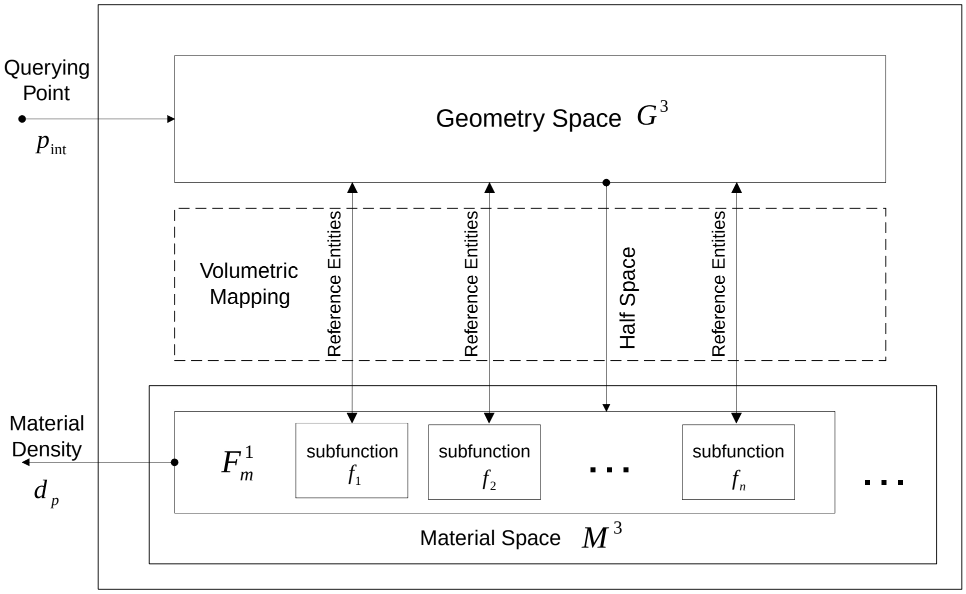

Figure 1: Information Flow of FGM Data

2.1 Volumetric Texture Mapping

Our approach is based on volumetric rendering schemes by texturing in computer graphics [2]. Such schemes are very effective at representing fuzzy objects such as cloud and smoke. By analogy, FGM design is envisaged as the creation of material clouds in a confined geometric space in structured and controllable manner. Another motivation for pursuing this approach is that, based on our research into expected applications, material gradients will be emphasized near the surfaces of a part. This is because many FGM applications are meant to replace or improve the surface coatings on components designed for severe thermal and mechanical stress environments. Hence, from the user's point of view, it is logical to optimize the design scheme for surface-to-interior material gradients.

The material spaces are related to the geometry only by reference entities. The reference entities can be any topological entities such as, vertices, edges, or faces. These topological members can be either structural or non-structural members. Non-structural members include non-manifold and multi-dimensional geometries as shown in figure 2. Most modern boundary representation (B-rep) kernels allow the coexistence of these members in their data structures. In case a mapping needs to be defined apart from the B-Rep structure, non-manifold members can be created anywhere inside the geometry and volumetric mappings applied to these entities.

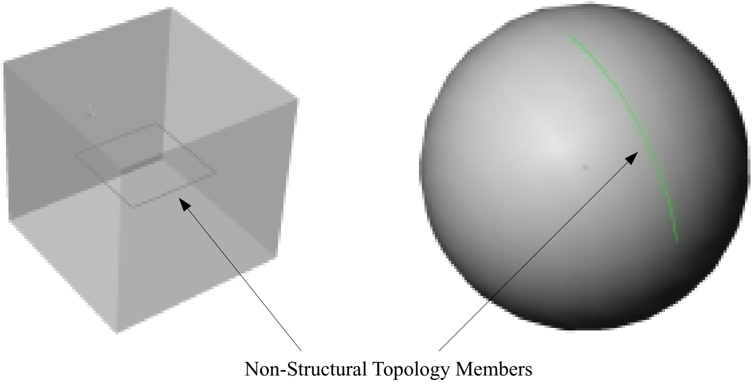

Figure 2: B-Rep Models with Non-Structural Topologies

The material gradient in the part is procedurally designed and stored. The term ‘procedural’ was adopted to emphasize that the patterns are described by program code rather than by data structures. The major defining characteristic of procedural data is that it is synthetic, generated from a program or model, not from a discrete data set and its interpolation. Structured data sets can be included among the gradient representation in a procedural design by incorporating the gradient interpolated from the data as one of its primitive operations.

2.2 Blending Functions

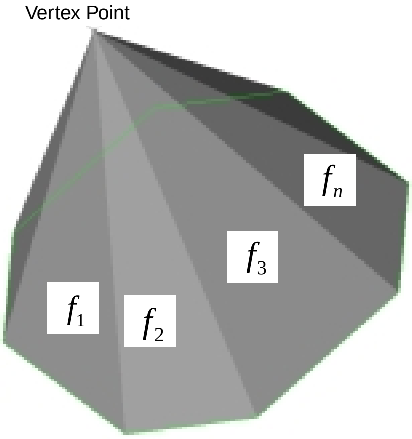

Figure 3: A Part with Many Faces Joined at One Vertex

Blending functions are used to union multiple f’s defined on the surfaces of a part. Th number of the f’s for blending can be from 2 to n depending on the location of joining. For example in figure 3, n faces are joined at one vertex point, while only two f’s may be required at the joining edges for unioning the neighboring faces. At the vertex point at which n faces meet, the corresponding f's need to be joined simultaneously. A blending function, based on the Ricci's smoothing approximation of union operations [15], has been formulated to accommodate these situations. The function can be generalized into the following form:

$$f_{blend}(\vec{p}_{int}, a) = \left(1 - \left| \begin{array}{cccccc} f_1^a & f_2^a & \cdots & f_{n-1}^a & f_n^a & 1 \\ 1 & f_2^a & \cdots & f_{n-1}^a & f_n^a & 1 \\ 1 & 1 & \cdots & f_{n-1}^a & f_n^a & 1 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 1 & 1 & \cdots & 1 & f_n^a & 1 \\ 1 & 1 & \cdots & 1 & 1 & 1 \end{array} \right| \right)^{\frac{1}{a}}$$

where a is a positive real number.

The basic idea is to blend the overlapping portions linearly with respect to each other. The order of blending is then controlled by a single parameter a. The smoothness is determined as $\lim_{a \to \infty} f_{blend}(\vec{p}_{int}, a) = min(f_1, \cdots, f_n)$. Figure 4 demonstrates the change of smoothness in blending of f_1 and f_2 as a varies.

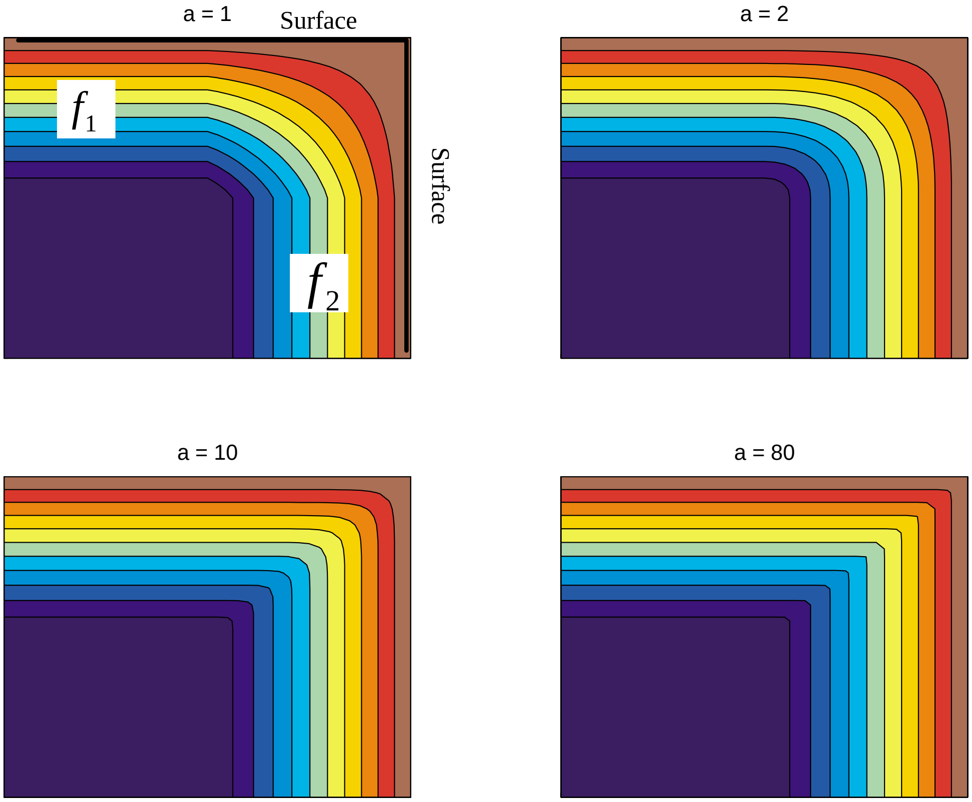

Figure 4: Blending of f_1 and f_2 with $a = 1, 2, 10, 80$

2.3 Advantages

The procedural representation is very compact in storage because it needs to store only procedural instructions and their parameter values. The size of a procedural gradient can be measured in kilobytes, while the memory size of voxel-based gradient data may require on the order of megabytes. Efficiency in memory becomes particularly significant when the material pattern is repetitive or uniform through the geometry, because the data for the mapping can be reused.

Another advantage of the procedural method is that it has no fixed resolution. In most cases, it can provide fully detailed information for any resolution required. Since a set of instructions procedurally defines the gradient information, the affected region can be edited simply by modifying the instructions or definition without visiting individual nodes or voxels. Dimensional changes of a part with procedurally described material information do not require redefinition of the material space. Uniform or non-uniform scaling of the material space, therefore, becomes trivial.

Compared to other representation schemes such as voxels, parametric formulations, and volumetric meshes, implicit procedural models appear to be more suitable for generating machine instructions for fabrication. For material delivery tool path generation, for example, material gradient samples are usually evaluated in an order determined by the process planning algorithm, not by the gradient design procedure. The implicit procedure fits perfectly in such an environment because it is designed to answer a query about any point in the part at any time. Parametric or discrete models, however, lend themselves to path generation based on a fixed sequence or discretization, which may not match the needs of the process planning algorithm. In most process process planning programs, using an explicit routine or discrete data requires running the gradient design procedure as a pre-process to generate the material gradien, which must be stored in a buffer for retrieval as necessary for rearrangement or interpolation during process planning. This reduces the efficiency of the process.

3 Design Procedure

In this section we demonstrate a possible design procedure to incorporate material gradient information in a given geometry. A FGM drill bit that we currently explorating for fabrication was chosen for demonstration. For this application, the customer wants more erosion resistance near the tip of the drill bit where wear is most likely to occur. Away from the tip toward the shank of the bit, fracture resistance becomes more important. Tungsten carbide provides abrasion and erosion resistance, while cobalt maintains ductility and fracture resistance. The design procedure for this gradient pattern is divided into the following step: importing geometry, global gradient design, surface gradient design, and integration of all information.

3.1 Geometry

The material gradient modeler is exercised on a boundary representation of the part. Commercially B-Reps are used almost exclusively in geometric modelers. Therefore, this scheme can be easily integrated with accessible formats such as ACIS [1], STEP, and IGES.

Certainly, a B-Rep is not the only way of describing the shape of an object to a computer. In academia, research continues into other methods, such as polyhedral models (e.g., the STL format popular in SFF), octrees, and implicit boundary models.

Although there exist several other methods of defining the shape of solid objects, a B-Rep provides several advantageous features to this project. Every geometric entity displayed on the screen is explicitly held in the model's data structure, making interactive editing, interrogation, and annotation of objects straightforward. For example, the attachment of attributes to entities such as faces, edges, and vertices are very efficient. Subdivision of geometric entities in a B-Rep is very meaningful in defining the material space. The regions that are emphasized for FGM design match well with B-rep entities.

3.1.1 Importing Geometric Data

the ACIS 3D Toolkit^{TM}, developed and commercialized by Spatial Technology Inc. was used extensively in this work. ACIS integrates wireframe, surface, and solid modeling by allowing alternative representations to coexist in an unified data structure. ACIS bodies can have any of these forms or any combination of them. Linear and quadratic geometries are represented analytically, and NURBS surfaces are used to represent free-form geometry [1]. Figure 5 shows the relationship between the topological elements that define the boundary representation of an ACIS model.

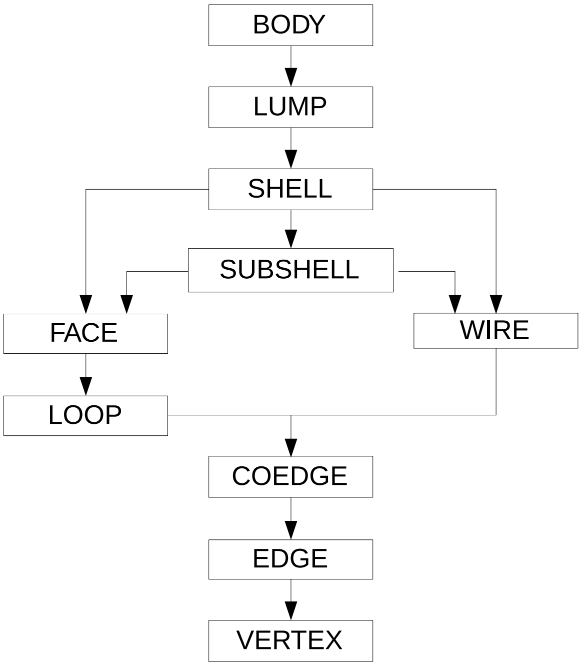

Figure 5: ACIS Topology Tree for B-Rep [1]

ACIS saves, or stores, model information in an ACIS save file called the '*SAT*' format. This format is widely accepted as a geometric data exchange format. Currently, most commercial solid modelers support the conversion from their own internal formats to the *SAT* format. Therefore, existing solid modelers, such as ProEngineer, SolidWorks, and CATIA ^{TM}, can be used for defining part geometry, which can then be saved as a *SAT* file and imported in to ACIS for adding material gradient information.

3.2 Design Global Gradient

A global gradient is used to impose an overall trend or pattern of material distribution in the part independent of any geometric entities. This information is stored at the BODY level of of the ACIS topology. If the gradient pattern and the geometry are relatively simple and the density distribution propagates in a single direction either linearly or radially, a global gradient in the material space is all that is needed to satisfy the design intent.

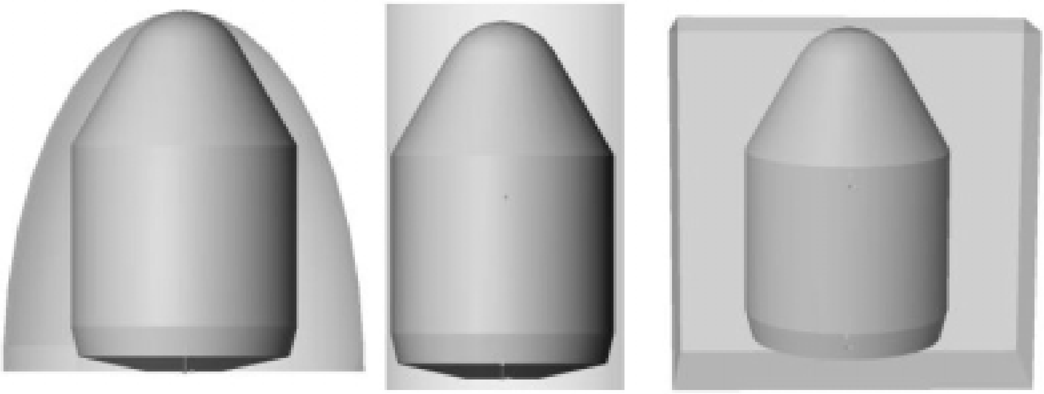

Figure 6: Different Types of Global Gradients on Geometry

Global gradient design basically exploits a three dimensional geometric mapping of implicitly designed material space to the part geometry. The mapping is constrained by defining skeletal points and the orientation of the global material space in the part geometry. Since the B-rep geometry already defines the half space of the part, the global gradient space is not necessarily required to conform to the shape of the part. Figure 6 gives simple examples of global gradient material space mapping to s geometry. The first example in figure 6 creates an ellipsoidal or spherical material gradient, while the second represents radial gradient in cylindrical coordinates. A simple one dimensional gradient can be designed by mapping an implicit cube that entirely covers the part, as shown in the third example in figure 6.

3.2.1 Implicit Surfaces with Gradient Texturing

Algebraic implicit formulation is used as a representation scheme for the global material space. Implicit surfaces are two dimensional, geometric shapes that exist in three dimensional space; they are defined according to a particular mathematical form. Intuitively, an implicit surface consists of those points in three dimensional space that satisfy some particular constraints. The constraints are represented mathematically by a function, generally denoted f, whose argument is a point $\vec{p}$. In implicit methods, the gradient pattern consists of a level set of f, that is, a set of all points at which the function has a particular value. Since implicit models tend to be continuous throughout a region of the modeling space, they are appropriate for this application for their ease of specification and their smoothly blending density distributions. The implicit density functions are best defined by summed, weighted, primitive implicit surfaces.

3.2.2 Gradient Profile

A gradient profile that affects the entire gradient material space should be defined in a one-dimensional parametric form, where the parameter ranges from 0 to 1. The curve can be formulated from spline curve fitting of data points, or from a predefined exponential sketching function, called the Density Modulation Function (DMF) (See [14]). The DMF consists of several procedural functions, such as bias, gain, and noise functions, that are the base level functions that higher order DMFs are built upon. The function is designed to roughly fit a generic monotonic material gradient profile. The shape of the profile can be interactively modulated by more meaningful parameters than the coefficients in polynomials. If the material gradient needs to designed by sketching a rough outline, a DMF is useful to initiate the design process.

3.2.3 More Complex Global Gradients

Global material space is not necessarily bound to a primitive shape. It can be shaped into more complex forms by performing Boolean operations. Further, constructive implicit modeling schemes, such as skeletal models, can be imported to generate more complex material spaces independent of the geometry space. An example that uses a complex global material space is shown in Figure 7.

3.3 Designing Surface Gradients

Once the global material space has been defined, detailed surface gradients can be specified. This process may be considered as an extension of volumetric texture or hypertexture mapping used in computer graphics. The software will give a variety of choices in selecting the surfaces for material gradients. Users can choose surfaces from the set of sub-surfaces (FACE) of part, or the entire surface (SHELL) can be selected. Surface patches for the gradient can be created on the part if the topological entities do not provide appropriate surface area for grading.

3.3.1 Gradient Profile

The schemes used for the global gradient profiles can be applied to the design of profiles for surface gradients. Preferably, the profile starts from the surface of the geometry and propagates into the interior of the part.

3.3.2 Uniform Surface Gradient

In a case where a surface gradient with the same profile needs to be applied to all surfaces of the part, the profile can be added to be the SHELL in the topological hierarchy. This formulation becomes very concise and provides

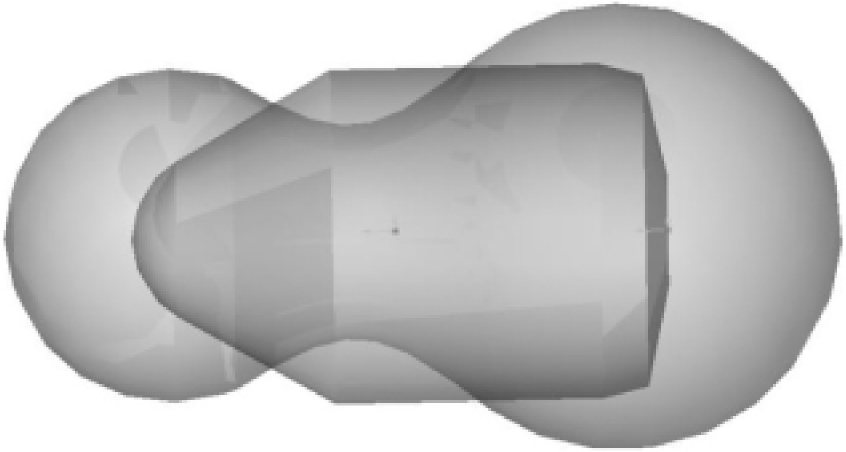

Figure 7: Use of Implicit Skeletal Geometry for Global Gradient

uniform control of the surface gradient.

3.3.3 Face Specific Gradient

If the surface gradient is applied on specific regions and matches subdivided FACEs in topology, the program supports this procedure in a very efficient way. Each FACE entity can store surface profile information. For example, the part shown in figure 8 consists of 11 FACEs. A gradient profile can be designed on any combination of these FACEs. Each FACE may a contain different profile or no surface gradient profile at all.

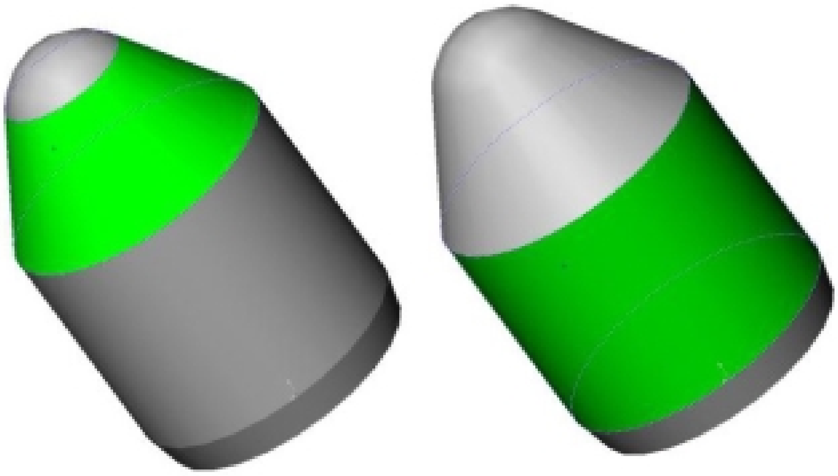

Figure 8: Selecting FACEs for Face Specific Gradient Design

3.4 Integration

Once the global gradient and the surface gradients are defined, interaction and blending properties need to be determined between the global gradient and the surface gradients, and among the surface gradients. Unless the surfaces are coplanar and have identical profiles, the instructions for blending the material gradient from each surface must be defined at the joining EDGEs or VERTEX.

3.4.1 Surface Trimming

The surface gradient profile acts as a multiplier of the global gradient to ensure the primary material density reaches zero at the part surface. The global gradient, which is not bound by the geometry when originally defined, is now trimmed to the surface of the geometry by incorporating the surface gradient profiles. The overall material gradient is formed by the product of the global gradient profile and the surface profiles as plotted in figure 9.

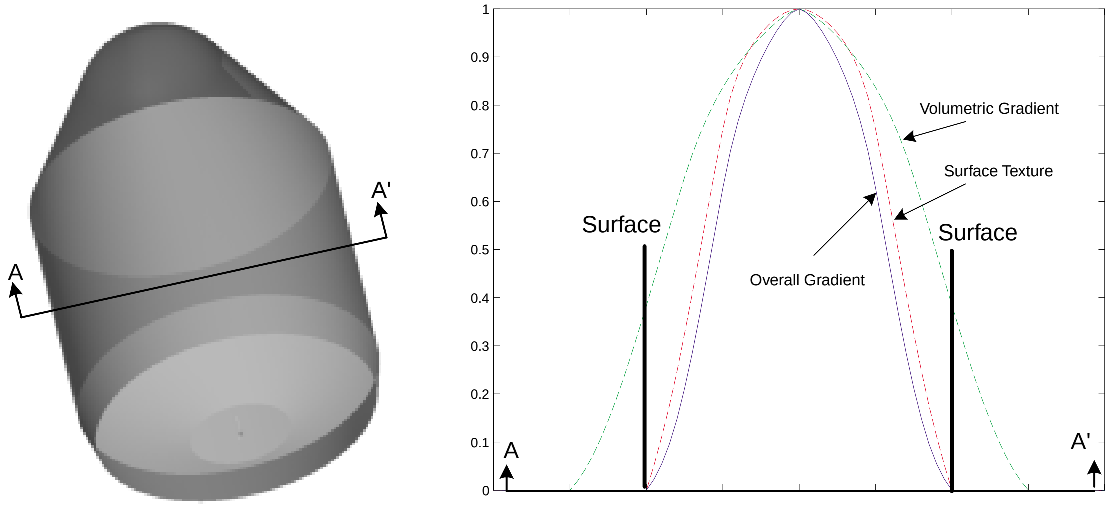

Figure 9: Surface Trimming by Surface Texture as a Multiplier

The final gradient can be adjusted to the designer's intent by modifying either the global gradient or each surface gradient.

3.4.2 Edge Blending

Just as chamfer, fillet, and blend operations create new surfaces for joining two tangent discontinuous surfaces in solid modeling, a material chamfer or fillet can be used to generate intermediate regions for blending the material properties of two different regions. At every joining edge and vertex where more than two surfaces meet, the designer should specify the blending properties of the material spaces from each surface. The joining contour is simply shaped by changing the parameter a described previously. The value of a determines the tightness or the radius of curvature of the corner at the joining edges. A sharper corner is formed as a increases. The value of a can be as low as 1 and increased to infinity for the desired sharpness.

3.4.3 Vertex Blending

Multiple surface gradients can intersect at a vertex, while only two surfaces are allowed to meet at an edge. The blending of the gradients near a vertex also can be controlled by the parameter a. The same approach as the edge blending is used for this case. A higher value of a will form a gradient shape more tightly bound to the reference geometry near the vertex.

3.5 Design Change and Modification

The procedure for designing material gradients is intended to be analogous to that of feature based modeling. In this way, the design of the material composition can be seen as an extension of geometric design. Therefore, modifying the design should be intuitive and user friendly.

Dimensional changes of the solid generally do not require redefinition of the material space because the procedurally designed material gradient adapts to the modified geometry based the mapping and profiling instructions. When new entities are introduced to the geometry space by adding more features, new mapping, profile, and blending information should be inserted in the material space.

4 Results

A FGM drill bit was designed using the Volumetric Multi-Texturing scheme. First, an ellipsoidal implicit material gradient was specified for the global gradient. Surface gradient profiles defined by DMFs were then added to all 11 faces. Material fillets or chamfers were applied at all the joining edges and vertices. The ATTRIB class, which is derived from the ENTITY class on every ACIS topological entity, was exploited for storing material design information. Therefore, no auxiliary data structures were necessary to store material gradient data. Figure 10 shows a cross section of the FGM drill bit. The tungsten carbide rich region is located near the tip and the shell of the part. The composition gradually changes from the tip to the shank.

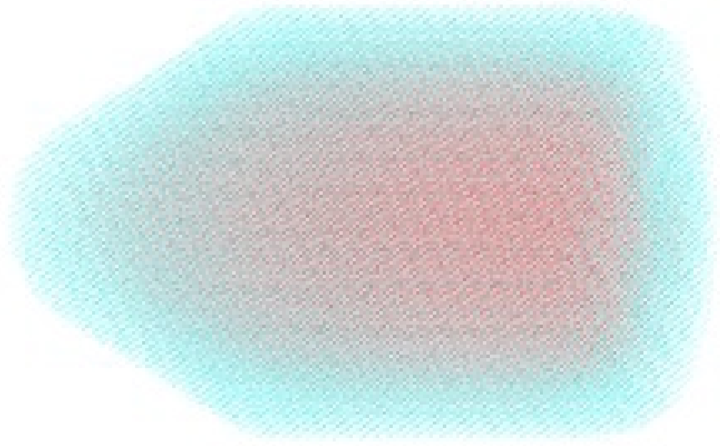

Figure 10: A FGM Drill Bit, (Blue: Tungsten Carbide, Red: Cobalt)

Further, tool path generation for multiple materials was implemented to demonstrate the feasibility of the scheme for SFF manufacturing. Closed loop contours of given material ratios were iteratively generated until they fill up the entire cross sectional area of the drill bit. The path planning algorithm used the marching square algorithm to find material isocontours on a 1000 by 1000 grid. We assumed that the SFF process would use 10 different material compositions from 0% to 100% of the primary material density. The result is shown in figure 11.

It was successfully produced without running pre-processing for rearrangement or creating a buffering data storage.

5 Conclusion and Future Work

As a part of our research to produce FGM components by Selective Laser Sintering, a theoretical approach and its implementation for material design was introduced in this paper. A formulation responding to the initial specification, $F^i_m(\vec{p}_{int}) = d^i_p, \quad \vec{p}_{int} \in G^3$, was achieved using procedural implicit schemes in conjunction with the Volumetric Multi-Texturing.

We have designed a volumetric material gradient inside a drill bit as a demonstration. The material gradient was combined with geometry created by a commercial solid modeler. The gradient was composed of one global

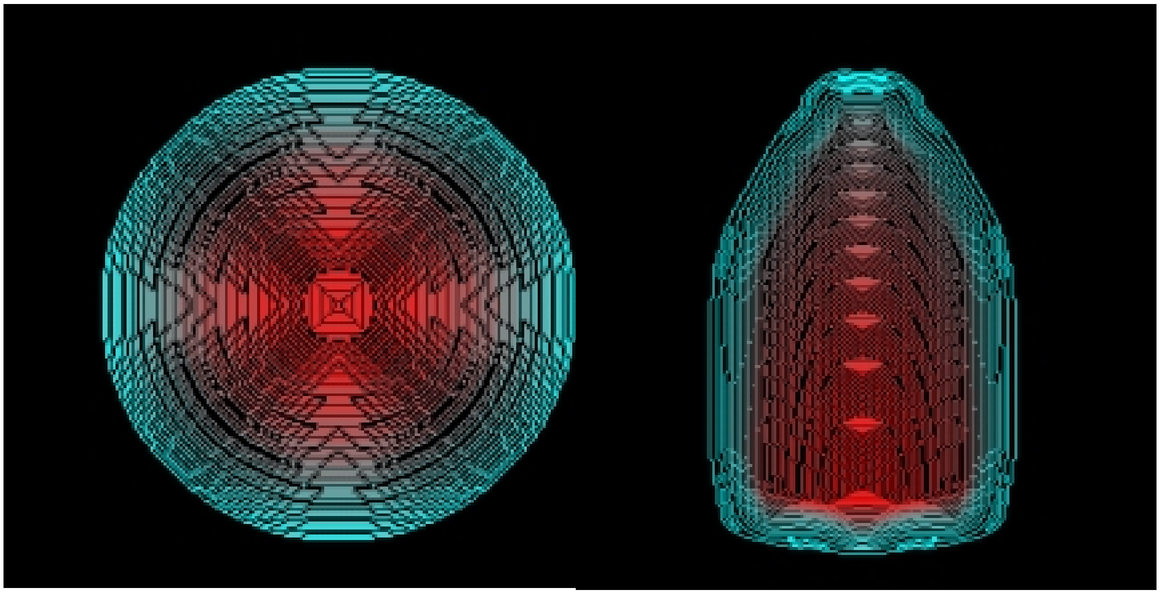

Figure 11: Multiple Loop Iso-Contour Tool Path for FGM Drill Bit

gradient and 11 surface gradients with joining and trimming operations. Both creation and modification of the material gradient were very systematic and concise. Tool path generation from the formulation showed its feasibility for SFF manufacturing.

We believe that our approach is a very efficient tool for designing material gradients in a part from a given geometry. To ensure the robustness of the approach, we suggest several areas of further research. Designers may have tools to numerically simulate and optimize the material distribution for analytical material gradient data. Many researchers are exploring this aspect of FGM design [8, 9]. However, even when the material data is numerically optimized for the intended purpose, the designer may wish to adjust the results for actual fabrication. Our approach will become a more attractive tool if discretized volumetric data can be converted to the Volumetric Multi-Texturing representation for convenience in modification and process.

Acknowledgment

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