A Model for Error Propagation in the Surface Profile for Solid Freeform Fabrication

Rajeev Dwivedi, Srdja Zekovic, Radovan Kovacevic

Research Center for Advanced Manufacturing
Southern Methodist University, 1500 International Parkway Suite 100
Richardson TX- 75081

Reviewed, accepted August 26, 2005

ABSTRACT

Inherent to Solid Freeform Fabrication (SFF) is the sequential addition of material in the form of layers along one direction or multiple directions. Addition of material in many techniques involves the phase transformation; which in turn, depends on a variety of process parameters. For most of the SFF processes, surface profile obtained during the material addition is not flat, and several methods such as overlapping are used to get a near flat surface profile. This paper describes a mechanism to model the profile of deposition in the Laser Based Direct Metal Deposition (LBDMD) for SFF. The method can be applied to other SFF methods used for metal deposition. The method also suggests a mechanism to model the error propagation.

1. INTRODUCTION

The computer model of layered deposition, in most of the solid freeform fabrication techniques is not exactly replicated in the process implementation. The process model assumes that the top surface generated by the deposition is flat. In the actual implementation of the process, parameters associated with the process pose various limitations. In the Solid Freeform Fabrication methods such as Selective Laser Sintering and Shape Deposition Manufacturing, the solidification of the pre-shaped raw material is done; therefore, the top surface generated by the deposition for a layer is nearly flat.

For methods such as the Laser Based Deposition, Gas Metal Arc Welding, and Gas Tungsten Arc Welding, the metal addition is done by generating a molten pool and adding the raw material in the form of wire or powder to the molten pool. The generated beads are not square. The amount of material along the bead profile changes. Fig. 1 shows the bead profiles obtained for a set of process parameters. The amount of material changes with respect to the distance from the center. Near uniformity of the top surface of the layer is obtained by overlapping the beads; however, the deviation from the desired profile cannot be completely eliminated. For the initial layers the deviation introduced is not pronounced; however, as the number of layers increases, the accumulation of deviations leads to a large difference between the desired and the actual build up height. The layer based deposition has certain issues inherent to it such as:

Further information: (Send correspondence to R. Kovacevic)
E-mail: kovacevi@engr.smu.edu

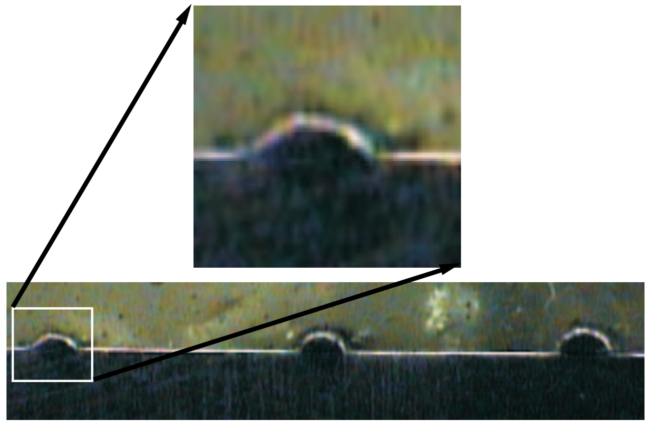

Fig. 1. Typical Bead Profile for different Process Parameteres (Power = 250,300, 400 W)

1. The deviation in the final dimension is a cumulation of the deviation along each layer.
2. The measurement of deviation for every layer is time consuming and requires lot of human intervention.
3. The process is comprised of discrete sub-tasks arranged in a sequential manner therefore is complex.

One of the widely used approaches to get a planar deposition profile includes machining at the intermediate level [1], [2], [3], [4] . The disadvantages that can be associated with the approach include :

1. Wastage of material
2. Slower process

The estimation, modeling, and optimization of the errors in rapid prototyping has been done by [5], [6], [7], [8]; however, the methods primarily address the variation due to the stair-case effect introduced in the process. Charney et. al. [9] suggest a quantitative model of tolerances and part geometry in relation to the process variables. The surface profile and the subsequent errors play a very important role in the geometry of the final part; however, the published results are very limited [10].

This paper suggests a method that models the geometry of deviation and the pattern of deviation accumulation. The model focuses on the Laser based deposition; however, it can be applied to other methods of metal deposition such as Welding and Plasma based metal deposition. The model suggests the geometry of deposition profile. The deposition process parameters can then be adjusted to obtain a smoother geometry and reduce the overall deviation in the part geometry.

The initial part of the paper suggests the sources of deviation. Next, a quantitative model of the process and the model of geometric reconstruction by depositing metal is suggested.

The model then is extended to incorporate the deviation. The cumulative effect of the error propagation is then modeled. The paper concludes with a case study.

2. A STATE SPACE MODEL OF DEVIATION PROPAGATION

The process planning in all of the SFF techniques start with the CAD model of the object. Next, the model is sliced by a set of parallel planes. The slicing in essence generates a 2D contour geometry. A suitable path is generated to fill the 2D contour area. The metal deposition head sweeps along the path to fill the material in the area. The 2D contour area generation and the sweeping is performed for all the layers in a suitable sequence to fabricate the part.

The proposed model primarily focuses on the accuracy along the direction of growth of the part. In order to model the deposition process, the deviation along the x and y coordinates is ignored. The path planning used in the deposition introduces one bead along the boundary in order to account for the material deficit along the turning points of the path 8. The extra material is removed by machining in the post processing.

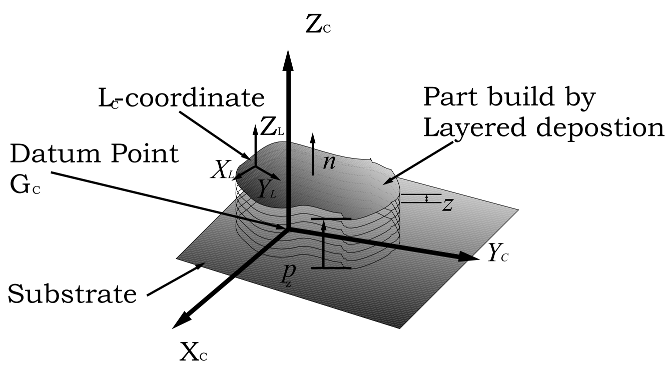

Fig. 2. The Coordinate Systems used in the Error Propagation Estimation

The modeling uses two coordinate systems (Fig. 2). The first coordinate system is the global coordinate system (G_c) and is attached to the substrate; whereas, second coordinate system is attributed to the immediate surface used for deposition (L_c). The change in dimensions due to temperature rise is ignored and, therefore, the location and orientation of the G_c is ignored. L_c, however, changes as each layer is deposited.

The desired shape of the object is characterized by the final z-coordinates of various points along the top layer. The deviation of the z-coordinate in the k^{th} layer, thus can be expressed by the following relationship:

$$Z_k(x, y) = \sum_{i=1}^{k} z_i(x, y) \tag{1}$$

where z_i is the contribution to thickness in the i^{th} layer. The deposition, therefore, can be characterized by the set of thicknesses for each layer expressed as:

$$Z(x, y) = [z_1,\ z_2,\ \ldots,\ z_n] \tag{2}$$

The behavior of the error propagation is assumed to be monotone; however, depending upon the geometry and the process parameters, the machining in an intermediate stage becomes inevitable. For such cases, the method suggested in this paper can be introduced between two stages of machining.

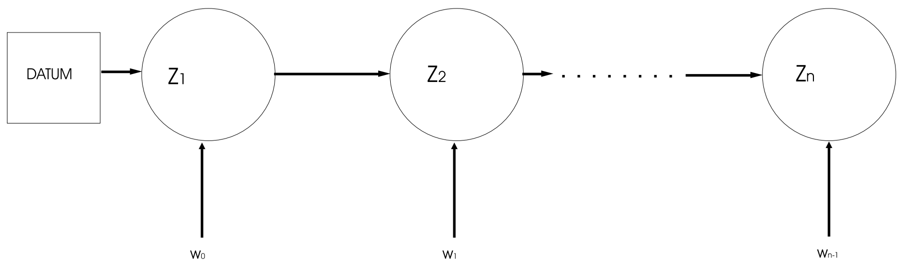

Fig. 3. The Model of Error Accumulation

Any arbitrary surface is characterized by an infinite number of points; therefore, we focus on a representative set of points for the diagnosis and the measurements. The total number of points may vary for different geometries. The formulation of the deviation propagation is based on a state space model. As described in fig. 3. the deviation for a given layer is the result of :

1. The error introduced during the deposition for the layer.
2. The accumulation of errors due to deposition along the previous layers.

The deviation in the top profile of the deposition is therefore expressed by the following relationship:

$$\tilde{Z}_k(x, y) = \tilde{Z}_{k-1}(x, y) + \tilde{F}_{k-1}(x, y)\tilde{w}_{k-1}(x, y) \tag{3}$$

where $\tilde{Z}_k(x, y)$ is the deviation in the height of the k^{th} layer at the location (x, y). $\tilde{w}_{k-1}(x, y)$ is the variation associated with the deposition of the layer $k - 1$, and the $\tilde{F}_{k-1}(x, y)$ matrix transforms the variation in the layer $k-1$ with respect to the substrate. By introducing another equation:

$$S_k(x, y) = \tilde{Z}_k(x, y) \tag{4}$$

the deposition process can be represented to be in a state space form [11] with Eq. 3 and Eq. 4 representing the state and the output equations, respectively.

One of the assumptions that can be made towards the suitable treatment of the errors is that, the variation $\tilde{w}_k(x, y)$ is a normally distributed random variable. Further, a suitable distribution function could be attributed to the random variable.

3. A MODEL FOR MATERIAL ADDITION IN SFF

As described earlier, the deposition for most of the SFF techniques does not allow a uniform profile and the amount of material varies in space. The near planar surface profile can be obtained by overlapping two beads.

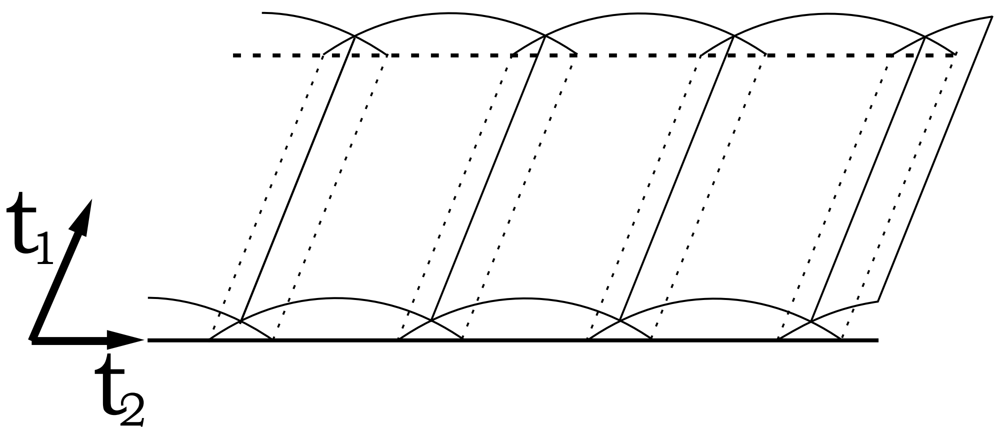

Fig. 4. Material deposition defined by the directions t_1 and t_2

One of the important factors in the overlap is the selection of the phase difference between the two adjacent beads. Let the function $f(t_1, t_2)$ represent the cross-section profile of the bead (Fig. 4); t_1 and t_2 define the datum plane of the layer onto which the deposition is done; then for two beads separated by the vector $\triangle t_1 \hat{e_1} + \triangle t_2 \hat{e_2}$, the cumulative profile is expressed by:

$$F(t_1, t_2) = f(t_1, t_2) + f(t_1 + \triangle t_1, t_2 + \triangle t_2) \tag{5}$$

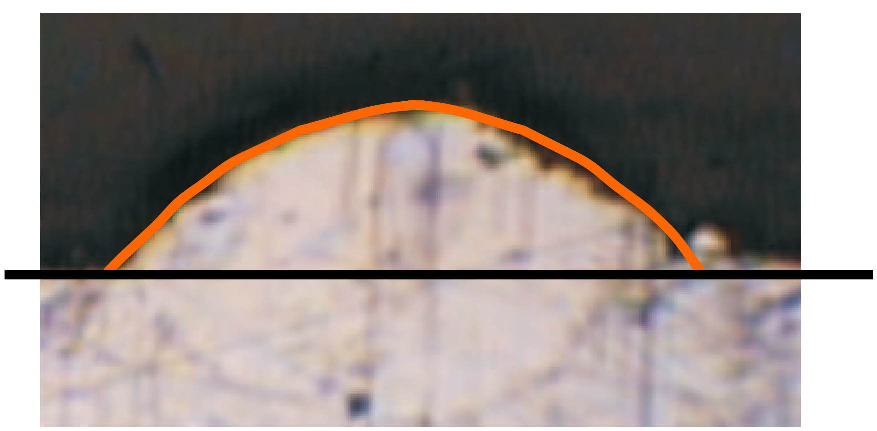

Fig. 5. Sinusoidal Approximation of the Bead profile

Observations (Fig. 5 and 1) suggest that, for a wide range of process parameters, the profile of the deposition bead for the laser based deposition can be approximated as a sinusoid.

The cumulative profile representation of the deposition can be represented by a specific form of equation 5 expressed as:

$$Z(x) = C \sin(\frac{\pi x}{Bw}) \;\; such \;\; that \;\; 0 \leq x \leq Bw \tag{6}$$

where Bw is the total width of the bead. C is a suitable constant corresponding to the geometry of the bead. The overlap of two beads can be expressed by the following:

$$Z_{overlapped}(x) = C(\sin(\frac{\pi x}{Bw}) + \sin(\frac{\pi x}{Bw} + \frac{\pi l}{Bw})) \tag{7}$$

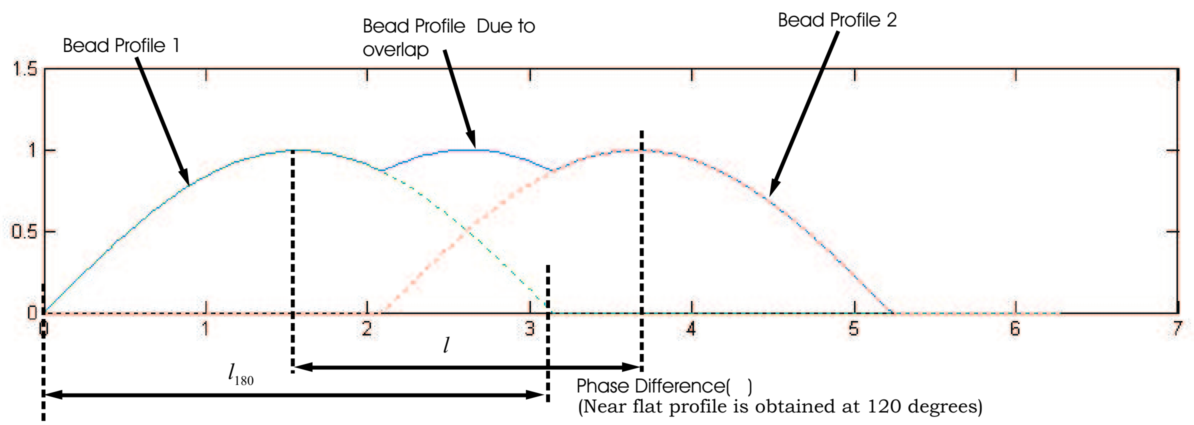

Fig. 6. The Optimal Profile Obtained by Overlapping Two Beads

where l represents the separation between two overlapping beads. A suitable value of the l can be determined to get a smooth top profile (Fig. 6). In the actual process implementation, the overlapping of the beads allows the remelting of previously deposited layer and hence a smoother surface; however, it is difficult to get a flat top surface. The cumulative overlap of multiple beads allows a near smooth deposition.

Fig. 7 shows the cross-sectional profile of two overlapping beads. The overlap allows the extension of the molten pool beyond one bead and, therefore, remelting of the previously deposited bead. The remelting and the overlap allow a smoother top surface profile.

Though the overlap provides a smoother surface profile, the deposition pattern shows that the region between two adjacent beads has material deficit. A suitable shift in the pattern of the paths for the adjacent layers should allow a smoother profile of deposition. An experimental investigation for the shift in profile is performed and reported in the later sections.

3.1. Modeling of layer profile and variation

For most of the metal deposition techniques, the deposition follows a zigzag pattern or variants of the zigzag pattern.

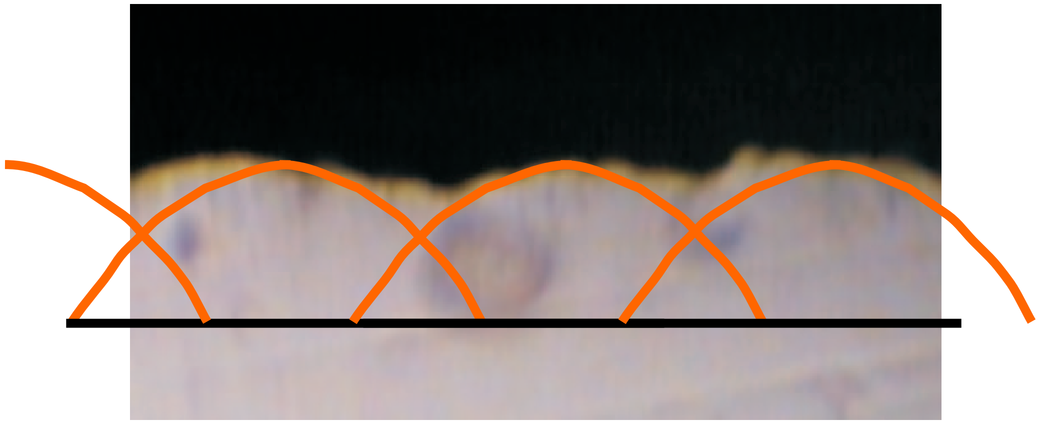

Fig. 7. The Cross-sectional profile of the deposited overlapping beads

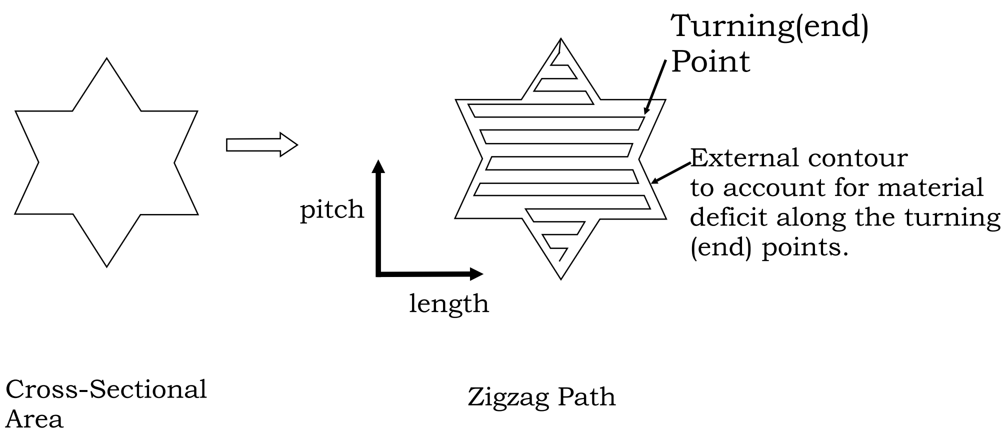

Fig. 8. A Cross-sectional area and the corresponding zigzag path

The zigzag pattern is characterized by a set of interconnected parallel line segments as shown in the Fig. 8. In order to simplify the modeling of deposition the turning effects along the end of the path segments is ignored. The distance between the parallel lines is characterized by the extent of overlap between two path segments that in turn depends on various process parameters. A 2D coordinate system is attributed to characterize the path pattern. The first coordinate axis is directed along the length of the path; whereas, the other coordinate axis is directed along the pitch of the path. The profile of the deposition about a given length of the path is therefore expressed by:

$$Z_n(x) = \sum_{i=1}^{n} C_i(\sin(\frac{\pi\ modulo(\frac{x}{2l_i})}{Bw_i}) + \sin(\frac{\pi\ modulo(\frac{x}{2l_i})}{Bw_i} + \frac{\pi l_i}{Bw_i})) \tag{8}$$

where n is the maximum number of layers, x is the location of the path segment, l_i is the distance between two consecutive beads, and $modulo()$ is the 'modulo' or the 'remainder' function. The variation therefore, in the deposition can be modeled as:

$$\tilde{Z}_n(x) = f_1\Big(\sum_{i=1}^{n-1} C_i\big(\sin\big(\frac{\pi\ modulo(\frac{x}{2l_i})}{Bw_i}\big) + \sin\big(\frac{\pi\ modulo(\frac{x}{2l_i})}{Bw_i} + \frac{\pi l_i}{Bw_i}\big)\big)\Big) + F_{n-1}(x) f_2\Big(C_n\big(\sin\big(\frac{\pi x}{Bw_n}\big) + \sin\big(\frac{\pi x}{Bw_n} + \frac{\pi l_n}{Bw_n}\big)\big)\Big) + err(n) \tag{9}$$

The functions $f_1()$ and $f_2()$ described in equation 9 capture the influence of the remelting during the deposition and $err()$ captures the random errors. However, modeling the functions $f_1()$, $f_2()$ and $err()$ is not trivial due to the involvement of a wide range of process parameters and other variables such as the influence of the underlying substrate geometry. A set of experiments and observations are performed to arrive at a model. The pattern of the growth of the layer surface geometry is observed and compared. Ten experiments are performed in order to establish the model of error propagation. The allowable limit of the deviation is of the order of 0.2mm. Once the limit for the deviation exceeds the suggested limit, the top surface is faced off and prepared for further deposition.

4. EXPERIMENTS AND THE OBSERVATION

Parameter	**Value**
Power	200W
Torch Speed	5 mm/s
Phase Difference	$\frac{2\pi}{3}$
Gas Flow	12 cu-ft/hr
Layer thickness	0.4 mm

Table 1. The process parameters used for the experiments

The calculations suggest that the value corresponding to the phase difference of $\frac{2\pi}{3}$ between the adjacent beads gives a near planar profile (Fig. 6). The deposition geometry depends on a number of process parameters. The experimental results used for the analysis are based on the set of process parameters described in the Table 1. The measurements for the top layer profile are performed. The model of the bead overlap suggest that a shift in the bead pattern such that the maxima of the deposition profile overlap with that of the minima of the adjacent layer provides a very smooth surface. A comparison of the measured surface profile and the profile in the CAD model is done. A comparative study includes following models:

1. Continuous deposition
2. Machining and deposition pattern based on the suggested offsetting

The measurements are based on the deposition for a 10 $mm \times 10$ mm area by the zigzag path. Of primary concern is the deficit of material due to deviation. The excess material can be removed by machining in the post processing; however, filling material for the deviations leading to deficit is extremely complex, therefore is avoided. The experiment is performed and the measurements are done for every layer. The total number of layers is recorded (NL_{max}) before the deficit is observed . Also, the number of layers is stored before the positive deviation exceeds 0.2 mm (ND_{max}) .

Pattern	NL_{max}	ND_{max}	**max-deviation**
Continuous	3	2	+0.25
Offset	9	not determined for the experiments performed	-0.23

Table 2. The experimental results

The table 2 suggests a comparison of the total number of layers and respective deviations. The desired thickness of the layer is 0.4 mm.

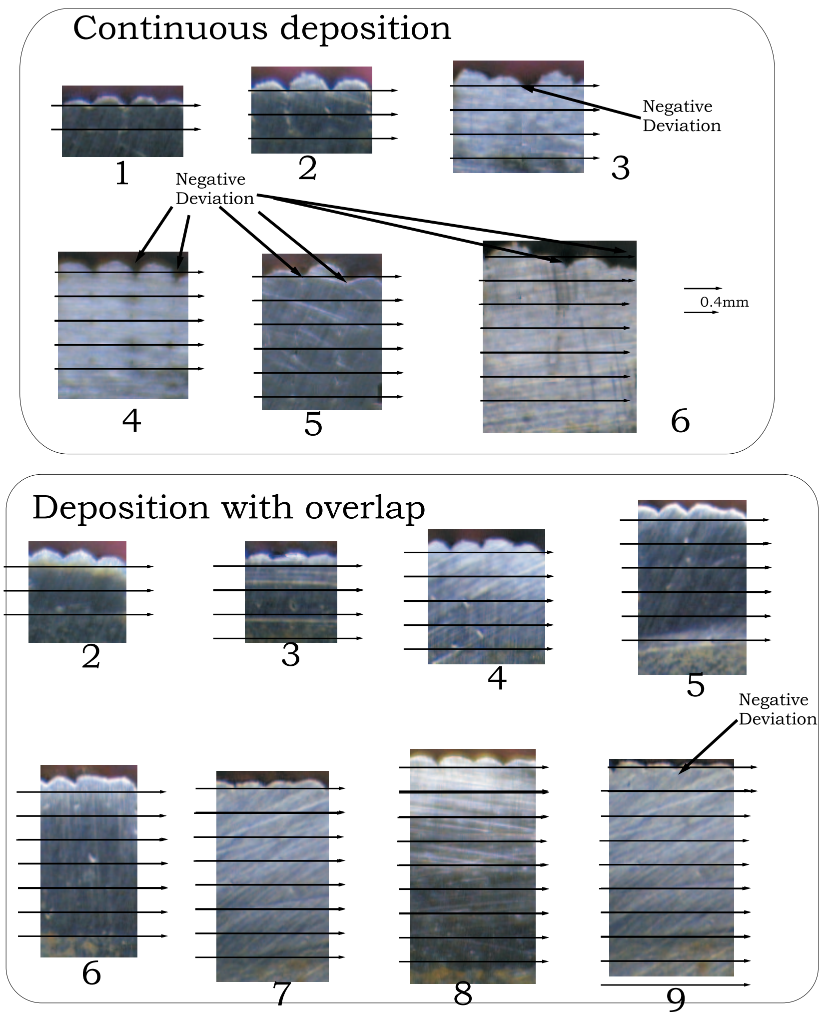

Fig. 9. A Cross-sectional view of the beads for different number of layers

Fig. 9 shows the cross-sectional view for the deposition for different number of layers. The deviations are small for fewer number of layers; however, the deviations exceed with an increase in the number of layers. A set of measurements, for the experiments performed, provide the following set of functions for the model:

$$f_1(n) = f_2(n) = 1.4nh(1 - 0.20\lfloor \frac{n}{2} \rfloor) \quad 2 \leq n \leq 8$$

(10)

where h is the maximum bead height. The functions $f_1()$ and $f_2()$ may vary for different set of process parameters. The sequential deposition for more than 8 layers introduces deficit and hence deviation in the surface profile.

5. CONCLUSIONS

A model of the bead geometry and the profile obtained by bead overlap was developed. A state-space model for the error propagation was derived. Observations suggest that a phase difference of $\frac{2\pi}{3}$ between the paths for adjacent layers reduces the deviation. Experiments are performed towards the estimation of errors and the pattern of error propagation.

ACKNOWLEDGMENTS

This work was financially supported by the grant DMI-0320663. Authors would like to acknowledge the help from Mr. Michael Valant Research Engineer, Research Centre for Advanced Manufacturing.

References

[1] F. W. Liou, J. Choi, R. G. Landers, V. Janardhan, S. N. Balakrishnan, and S. Agarwal. Research and development of a hybrid rapid manufacturing process. In *Proceedings of* 12^{th} *Solid Freeform Fabrication Symposium*, pages 138–145, University of Texas, Austin, August 2001.

[2] Zoran Jandric. *Optimization of Hybrid Rapid Manufacturing/Repair Process*. PhD thesis, Southern Methodist University, Department of Mechanical Engineering, May 2003.

[3] Radovan Kovacevic. Will we use welding technology to make rapid prototypes. *SME's Manufacturing Engineering Magazine*, (4):26–27, 2001.

[4] K. Hartman, R. Krishman, R. Merz, Gennady Neplotnik, Fritz B Prinz, Lawrence Schultz, M. Terk, and Lee Weiss. Robot-assisted shape deposition manufacturing. In *Proceedings of the 1994 IEEE International Conference on Robotics and Automation (ICRA '94)*, volume 4, pages 2890 – 2895, May 1994.

[5] Georges M Fadel and Chuck Kirschman. Accuracy issues in cad to rp translations. *Rapid Prototyping Journal*, 2(2):4–17, 1996.

[6] S. H. Choi and S. Samavedam. Modelling and optimisation of rapid prototyping. *Computers in Industry*, 47(1):39–53, 2002.

[7] F. Lin, W. Sun, and Y. Yan. A decomposition-accumulation model for layered manufacturing fabrication. *Rapid Prototyping Journal*, 7(1):24–31, 2001.

[8] Ramakrishna Arni and S. K. Gupta. Manufacturability analysis of flatness tolerances in solid freeform fabrication. *Journal of Mechanical Design*.

[9] Charity Lynn-Charney and David W Rosen. Usage of accuracy models in stereolithography process planning. *Rapid Prototyping Journal*, 6(2):77–87, 2000.

[10] R. Kovacevic, R. Mohan, M. Murugesan, and A. Seybert. On-line monitoring of the electric arc spraying process based on acoustic signatures. *Journal of Engineering Manufacture*, 209:369–379, 1995.

[11] Katsuhiko Ogata. *Modern control engineering (3rd ed.)*. Prentice-Hall, Inc., Upper Saddle River, NJ, USA, 1997.