

# A Study of Static and Dynamic Mechanical Behavior of the Substrate in Ultrasonic Consolidation

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**ABSTRACT:** A new 2-D FEM model is developed for a fundamental study of the time dependent mechanical behavior of the substrate in ultrasonic consolidation. The simulation shows that for a given vibration condition, the amplitude of contact friction stress and displacement stabilize to a saturated state after certain number of ultrasonic cycles. With the increased substrate height, the amplitude of contact frictional stress decreases, while that of contact interface displacement increases. The energy density and transfer coefficient at the contact interface with different substrate heights can be used as parameters to predict the potential for ultrasonic bonding. The reason for the decrease in the frictional stress and displacement at the contact interface for certain substrate height seems to be caused by the complicated wave interference occurring in the substrate. A specific substrate geometry generates a minimum strain state at the interface as a result of wave superposition. Such minimum strain state is believed to have produced the “lack of bonding” defect.

## Introduction

Ultrasonic consolidation (UC) of thin foil is a promising rapid prototyping technology for fabrication of complex shaped solid objects. The UC process is designed to continuously weld layers of metal foil to previously deposited material, during which the profile to each layer is created by contour milling, to build-up a 3D structure. As with ultrasonic welding, most commercially available metal foils, such as aluminium, titanium, magnesium, copper and steel can be used in the UC process [1, 2]. In general, The UC process combines ultrasonic welding of metals with layered manufacturing techniques to produce solid components. The UC process possesses fast bonding times, multiple material combinations and low process temperatures (at typically 20% of melting temperature of the base metal)[3].

The primary mechanisms for bond formation are thought to be plastic deformation and diffusion. Although many experimental data, including SEM microstructure and mechanical property's tests, are used to study and validate the UC bonding theory, a good modeling of ultrasonic consolidation is still not available to understand the essence of this complicated manufacture process. The modeling of ultrasonic consolidation of metals and alloys began only recently, with the development of emerging new technology that employ ultrasonic welding in ultrasonic consolidation. The weld strength of UC is modeled and the calculation method is derived on the basis of the theory of surface and volume effects [4]. A 2-D spot-

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welding model in mechanical field is also constructed and the variation of friction coefficient in whole process is measured and studied in detail [5].

A recent study shows that there is a maximum height-to-width ratio of freestanding structures that can be built using ultrasonic consolidation. If the height-to-width ratio is higher than 1:1, no ultrasonic bonding can be achieved [6]. None of the current ultrasonic welding theory or understanding is able to explain such experimental observation. This paper attempts to develop the mechanics for ultrasonic consolidation that can explain the experimental results and point to the direction for ways to overcome the problem.

## The Model

Previous research on ultrasonic bonding has focused on the interface area where the bonding occurs [5, 3]. However, we found out the wave interference within the substrate has significant effect on joint formation. Therefore, a dynamic FEM model is developed to include the sonotrode and the entire substrate for a fundamental study of the time-dependent mechanical behavior of the substrate in ultrasonic consolidation.

Figure 1 shows the 2-D dynamic model involving the sonotrode that is pressed to contact with and vibrates against the substrate. The baseplate is not shown in the model and its influence is simulated by adding a fixed boundary condition at the bottom face of the substrate. The titanium sonotrode surface has been roughened so that there is no slipping between the top surface of the aluminum foil and the sonotrode during vibration. Therefore a very large friction coefficient is assumed between the top aluminum foil and the titanium sonotrode. Any relative sliding is between the bottom surface of the top foil and the substrate. The substrate is assumed to be homogeneous.

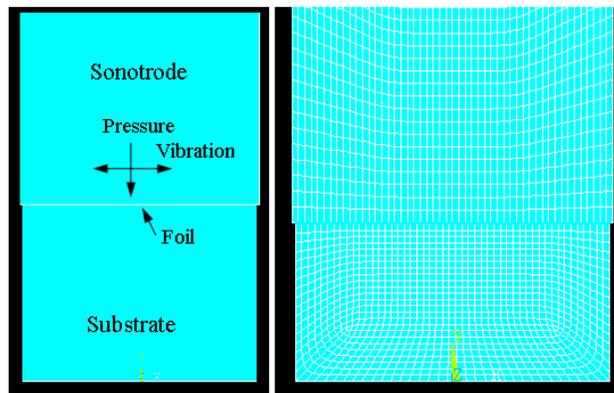


Figure 1: 2-D dynamic model for ultrasonic consolidation

In this analysis, the stress-strain relationship is defined as follows:

$$\{\sigma\} = [D]\{\varepsilon_e\}, \quad (1)$$

where,  $\{\sigma\}$  is stress vector,  $[D]$  is stiffness matrix, and  $\{\varepsilon_e\}$  is elastic strain vector. For the deformation of metals the von Mises yield criterion is employed and the elastic strain is given by

$$\varepsilon_e = \varepsilon - \varepsilon_p - \varepsilon_t, \quad (2)$$

where,  $\varepsilon_e$  is the elastic strain,  $\varepsilon$  is total strain,  $\varepsilon_t$  is thermal strain. The stress-strain relationship now becomes

$$\{\sigma\} = [D]\langle\{\varepsilon\} - \{\varepsilon_p\} - \{\varepsilon_t\}\rangle, \quad (3)$$

To find out the effect of sonotrode's cyclic motion, a transient dynamic analysis is conducted on the model. The governing dynamic equation is as follows for a linear structure [7]:

$$[M] \left\{ \frac{\partial^2 u}{\partial t^2} \right\} + [C] \left\{ \frac{\partial u}{\partial t} \right\} + [K]\{u\} = \{F^a\}, \quad (4)$$

where,  $[M]$  is the structural mass matrix,  $[C]$  is the structural damping matrix,  $[K]$  is the structural stiffness matrix,  $\left\{ \frac{\partial^2 u}{\partial t^2} \right\}$  is the nodal acceleration vector,  $\left\{ \frac{\partial u}{\partial t} \right\}$  is the nodal velocity vector,  $\{u\}$  is the nodal displacement vector, and  $\{F^a\}$  is the applied load vector.

This 2-D model is solved using the ANSYS package through an iteration algorithm. A custom-designed program is developed for the execution of the algorithm. During the simulation, the following conditions are used. These conditions represent the actual ultrasonic consolidation process parameters:

1. Load on the sonotrode is 310 lbs
2. Amplitude and frequency of the ultrasonic vibration are  $1.6 \times 10^{-5}$  inch and 20 kHz respectively
3. Moving speed of the sonotrode is 1.12 inch/sec
4. The substrate remains at 300°F
5. The substrate has a width of 0.94 inch with various height from 0.5 to 2.0 inches
6. The friction coefficient between aluminum foils at 300°F is 0.408
7. Other mechanical and physical properties of Aluminum 3003-H18 at 300°F are used as inputs

## Simulation Results

### Static Analysis

Upon the initial application of the pressure on the substrate, and before the first vibration cycle is applied, the substrate exhibits a static distribution of stress and strain. For the

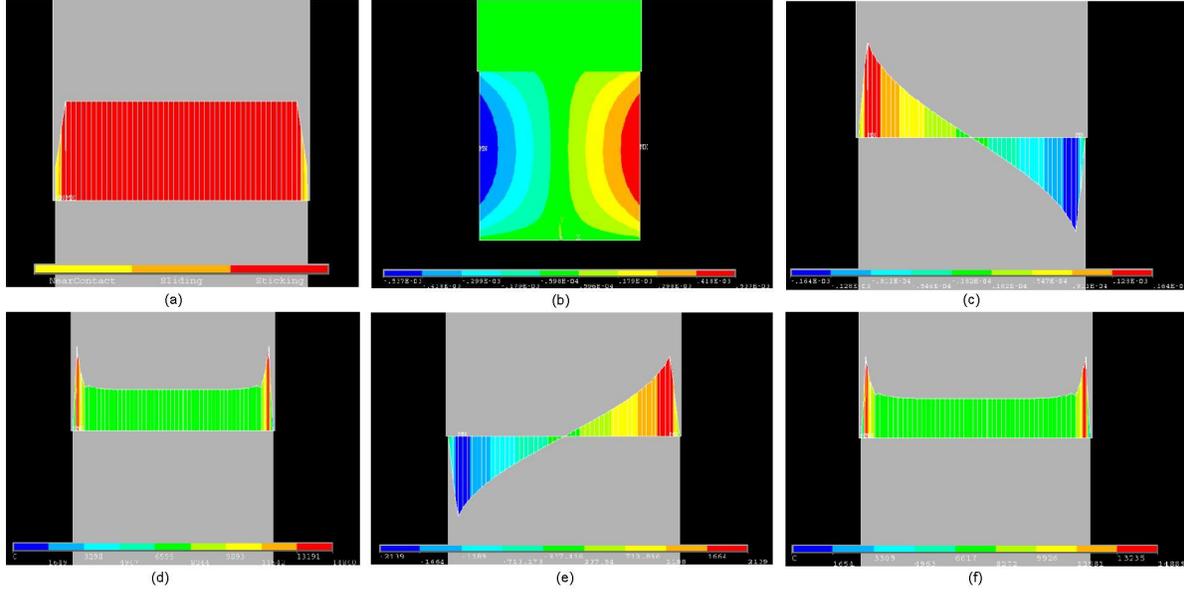


Figure 2: Static distribution of mechanical status for substrate with an 1:1 height-to-width ratio: (a) contact interface friction status, (b) substrate’s horizontal displacement, (c) contact interface horizontal displacement, (d) contact interface pressure stress, (e) contact interface frictional stress, and (f) contact interface total stress.

substrate with an 1:1 height-to-width ratio, the distribution of mechanical status is shown in Figure 2.

As can be seen in Figure 2a, the friction status at the contact surface is mostly “sticking”, i.e., there is no relative motion in the horizontal direction between the sonotrode and the substrate. Near the two edges, however, there are small areas where “sliding” occurs. Right at the edges, the status of the interface is “near contact”. In Figure 2b, the horizontal displacement of the substrate shows a typical barreled shape: the mid-height of the substrate flows out undisturbed, while near the contact interface the frictional force opposes the outward flow of the material. This effect is also seen in Figure 2c, in which the horizontal displacement at the contact interface reaches its maximum at the boundary between the “sticking” and “sliding” areas and decreases to zero at the center and the edges of the contact interface.

Figures 2d, 2e, 2f show the pressure, frictional stress, and total stress distributions at the contact interface respectively. The pressure and total stress at the contact interface are mostly uniform, with peaks near the two edges. The frictional stress points to the opposite directions for the two sides of the substrate, with a gradual decrease of intensity to the center of the interface, where the zero-friction point is located. A common feature for these stress distributions is that the maximum values are located at the boundary between the “sticking” and “sliding” areas.

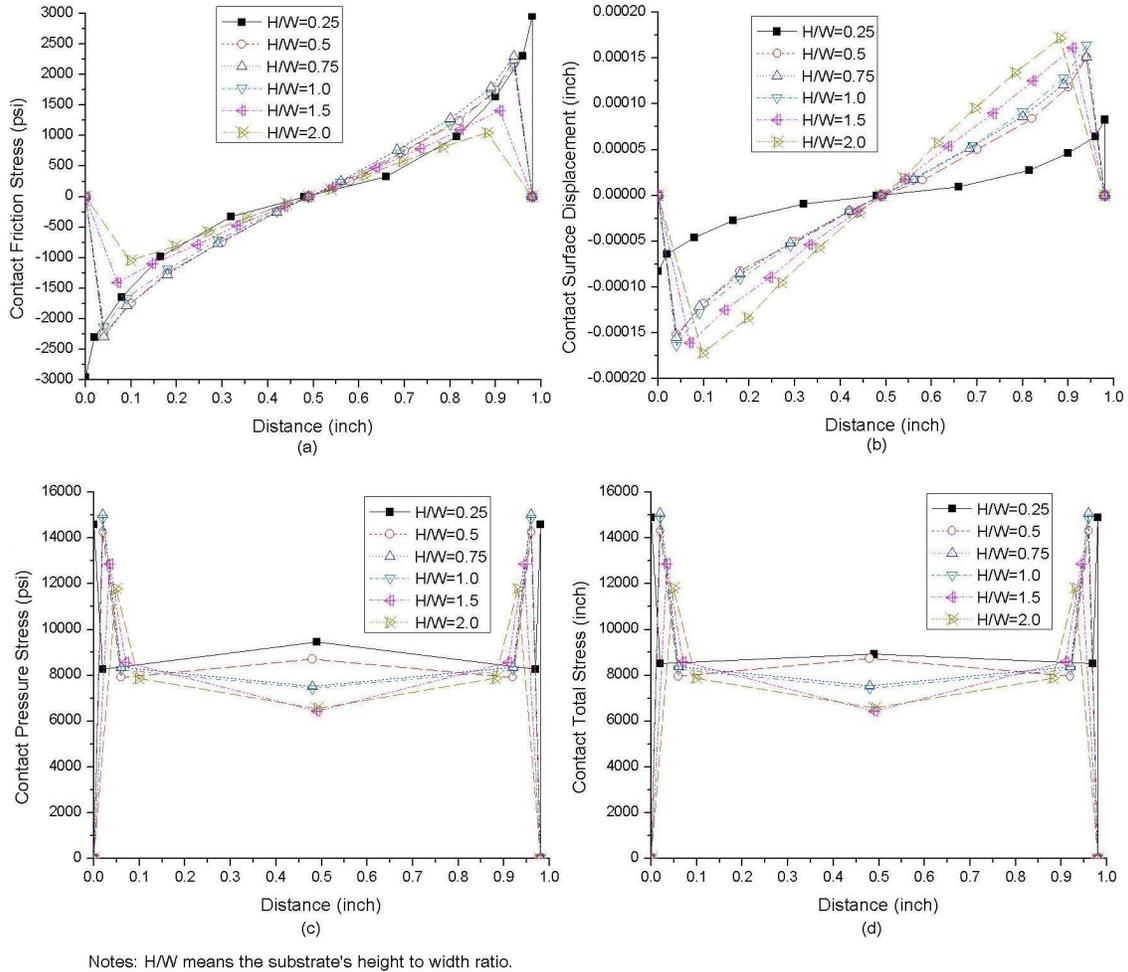


Figure 3: Static distributions of stresses and displacement as a function of the H/W ratio: (a) friction stress, (b) horizontal displacement, (c) pressure, and (d) total stress.

Substrate with different height-to-width ratios have been simulated and the results are summarized in Figure 3. The frictional stress at the contact interface is shown in Figure 3a. It can be clearly seen that as the height-to-width ratio is increased, the peak frictional stress decreases. For the H/W ratio of 1:1, the peak friction stress is less than 1500 psi, while a minimum of 2000 psi friction stress is seen for H/W ratios less than 1:1.

The horizontal displacement at the interface has an inverse trends relative to the frictional stress. In Figure 3b, the substrate with the lowest H/W ratio, has the lowest interface displacement. As the H/W ratio is increased, the interface displacement increases.

The pressure and total stress distributions for substrates with different H/W ratios are shown in Figures 3c and 3c. As the H/W ratio increases, the pressure and stress distributions exhibit a decreasing trend. There will be less pressure at the contact interface for a higher substrate. In Figure 3, all curves share a common trend: the maximum (or minimum) values in the

Table 1: The proportion of the sticking area at contact surface as a function of the H/W ratio

H/W ratio	0.25	0.5	0.75	1.0	1.5	2.0
$\frac{A_{sticking}}{A_{total}}$ (%)	100.0	95.5	94.6	93.9	91.1	87.0

curve move inwards towards the center of the contact interface as the H/W ratio increases. Since the maximum values occur at the location where the “sticking” and “sliding” boundary is, this feature means that as the H/W ratio increases, more portions of the contact interface becomes sliding. The way in which sticking and sliding areas change as a function of H/W ratio is shown in Table 1. Other conditions the same, a higher substrate has a greater sliding area at contact interface.

### Vibration Analysis

When the sonotrode moves in opposite directions, the distributions of the contact frictional stress and interface displacement retain the same general features as the static case. However, the zero-value point vibrates with the sonotrode (Figure 4). The stress value at each point of contact exhibits a fluctuating variation with the vibration of the sonotrode. As the sonotrode moves to the left, the zero-value point moves to the left and vice versa.

The magnitude of fluctuation in the stress and displacement levels for a given point at the contact interface is defined as the “amplitude”. The distribution of contact frictional stress amplitude and interface displacement amplitude with different substrate H/W ratios and with different cycles is shown in Figure 5. With the H/W ratio increasing, the amplitude of contact friction stress decreases (Figure 5a), but the displacement amplitude increases (Figure 5b). This result agrees with the static analysis discussed earlier. The maximum variation of both friction and displacement occurs at the edge areas of the substrate, and the central areas have less variations.

Figures 5c and 5d show the amplitude of frictional stress and displacement as a function of number of vibration cycles for a substrate with an H/W ratio of 0.5. For the same substrate height, as the number of cycles increases the frictional stress increases initially, but stabilizes to a “saturated” distribution after about 500 cycles. This result indicates the sonotrode velocity should be adjusted to ensure enough vibration cycles for the contact interface, but too slow a velocity will not increase the friction action beyond the saturated level. A similar trend exists for the interface displacement as a function of vibration cycles: after 500 cycles, the displacement amplitude stabilizes to a “saturated” distribution.

Away from the contact interface, the stress distribution within the substrate will influence the bond formation. Figure 6 shows the shear stress distributions for the substrate with a H/W ratio of 0.5 at different vibration cycles. In the substrate, there are four stress concentration regions that are located at four corners. With the increasing number of cycles, the originally concentrated stress regions start to spread and branch toward the inner regions

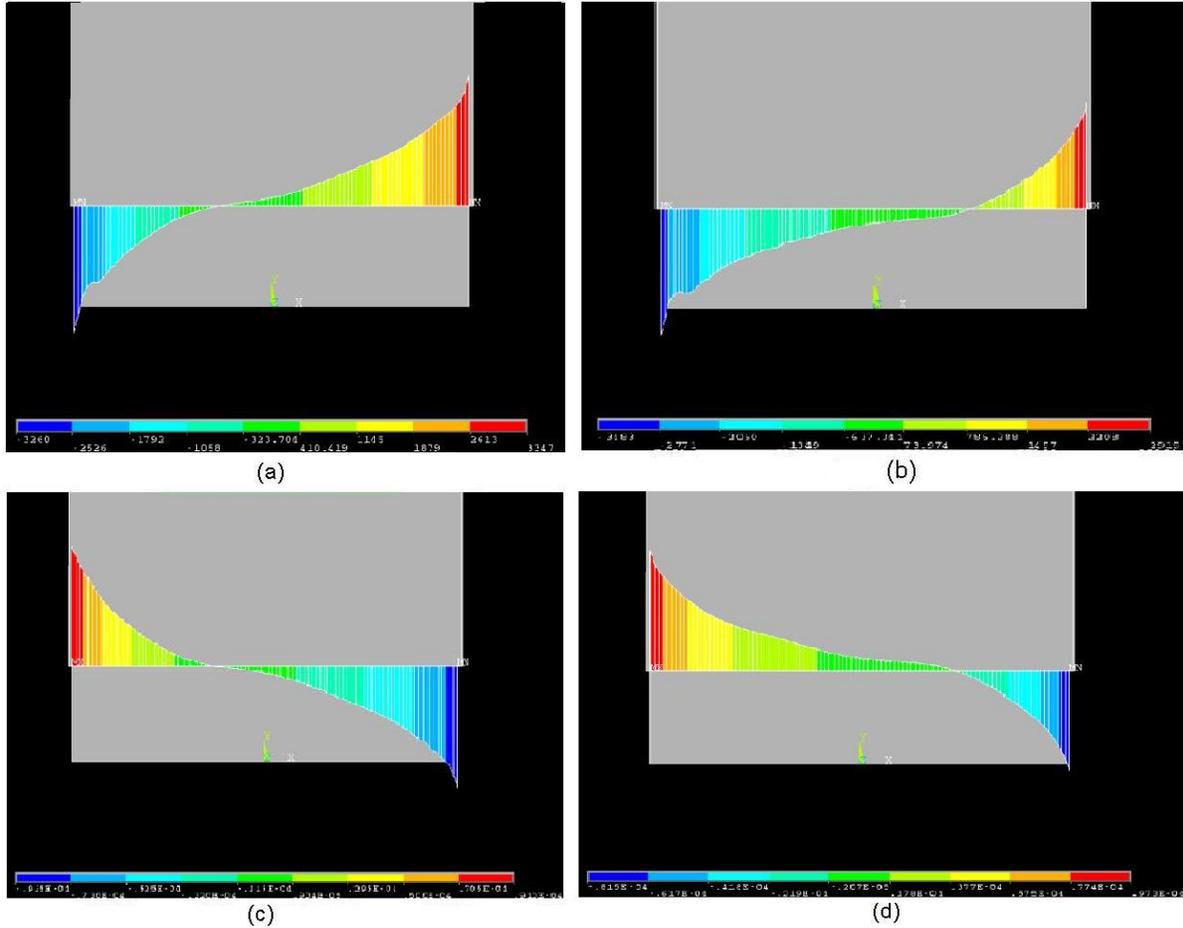
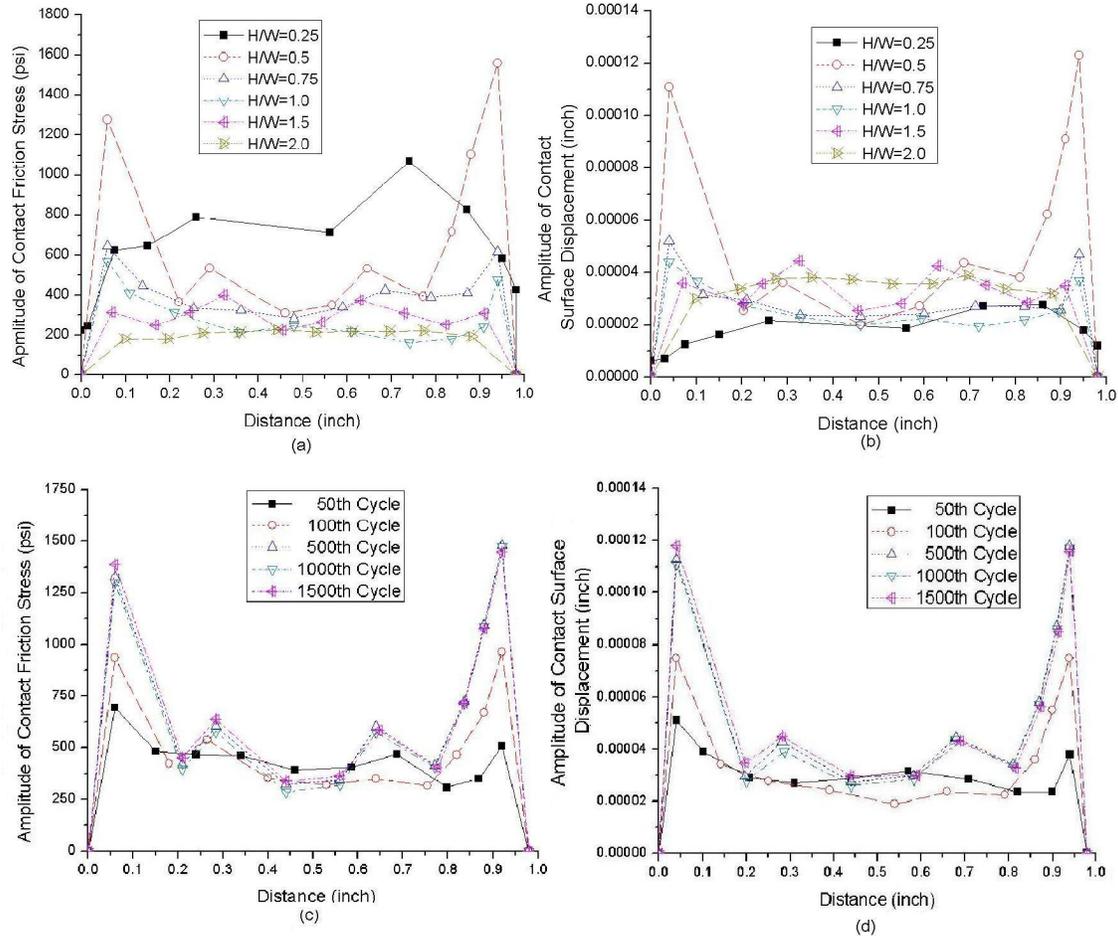


Figure 4: Distribution of contact friction stress and displacement at the 1500th cycle with a H/W ratio of 0.25 when the sonotrode moves to left or right directions: (a) friction stress (left), (b) friction stress (right), (c) displacement (left), and (d) displacement (right).

of the substrate. After about 500 cycles, the shear stress distribution approaches a steady state. This confirms the earlier observed contact friction and displacement saturation at 500th cycles. When the sonotrode moves in the opposite direction, the pattern of the stress distribution does not change, except in an inversed mirror image (compare Figure 6e with Figure 6f).

Figure 7 shows the shear strain distributions at the 1500th vibration cycle in substrates with various H/W ratios. The shear strain measures the degree of elastic/plastic deformation, which measures the potential for bond formation. For build height-to-width ratios less than 1:1, greater levels of shear strain exist near the bond interface. In addition, significant shear strain exists inside the builds. Such internal shear strain appears to distribute in horizontal bands, apparently due to the interference of the traveling vibration waves. For build height-to-width ratios higher than 1:1, the shear strain has a much lower level at the bond interface. At these build heights, the internal shear strain bands are no longer seen.



Notes: H/W means the substrate's height to width ratio.

Figure 5: Distribution of the frictional stress amplitude and displacement amplitude with different substrate height-to-width ratio (a)(b) at the 1500th cycle, and at different cycles (c)(d) for the 0.5:1 height-to-width ratio.

## Discussion

The simulated friction and pressure distributions have confirmed and explained the experimental observation that ultrasonic consolidation cannot be achieved for free standing substrates with a height-to-width ratio higher than 1.0. However, the simulation has not answered why the friction and pressure drop to below the critical levels for bond formation.

To understand the wave interference during ultrasonic consolidation, a vibration analysis of the 2-D model is conducted here. The surface wave source caused by the vibration of sonotrode travels through the substrate and reflects when arriving at the bottom surface, meanwhile the ultrasound wave amplitude decreases in this process because of damping. A model that shows how the ultrasound wave travels in the substrate is given in Figure 8a. The

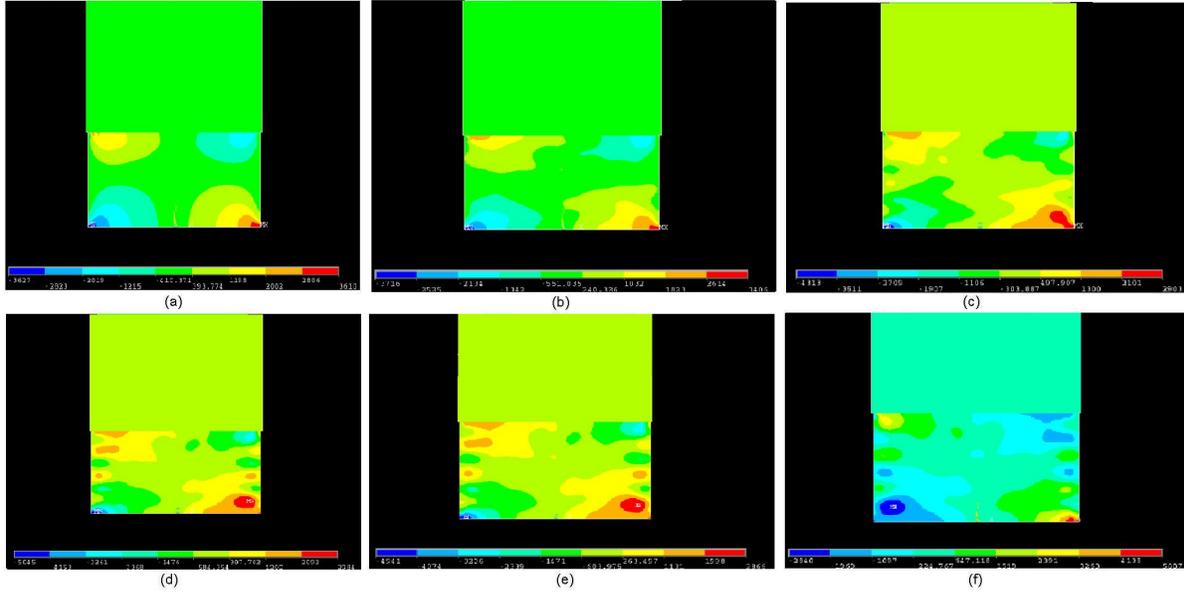


Figure 6: Distribution of shear stress with a 0.5:1 height-to-width ratio: (a) 1st cycle (right), (b) 50th cycle (right), (c) 100th cycle (right), (d) 500th cycle (right), (e) 1500th cycle (right), and (f) 1500th cycle (left).

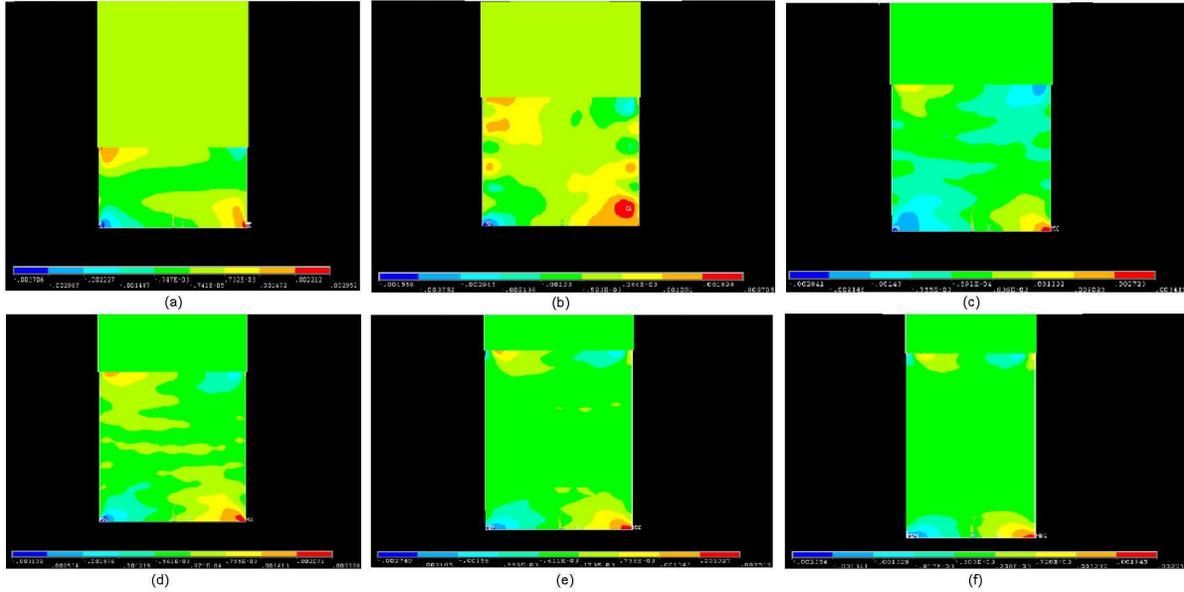


Figure 7: Distribution of shear strain at the 1500th cycle with different height-to-width ratios ( $H/W$ ): (a)  $H/W=0.25$ , (b)  $H/W=0.5$ , (c)  $H/W=0.75$ , (d)  $H/W=1.0$ , (e)  $H/W=1.5$ , and (f)  $H/W=2.0$ .

displacement amplitude at the contact interface is determined by the superposition of the source wave displacement ( $Y_1$ ) and reflection wave displacement ( $Y_2$ ). Their wave equations

are as follows:

$$Y_1 = A \cos \left( \omega t - \frac{2\pi y}{\lambda} + \psi \right), \quad (5)$$

$$Y_2 = \alpha A \cos \left( \omega t + \frac{2\pi(y - 2d)}{\lambda} + \pi + \psi \right), \quad (6)$$

where  $A$  is the wave amplitude,  $\omega$  is the angular frequency ( $\omega = 2\pi f$ ,  $f$  is the ultrasound frequency),  $\alpha$  is the damping coefficient of ultrasound in aluminum,  $\lambda$  is the wave length, and  $d$  is the height of the substrate.

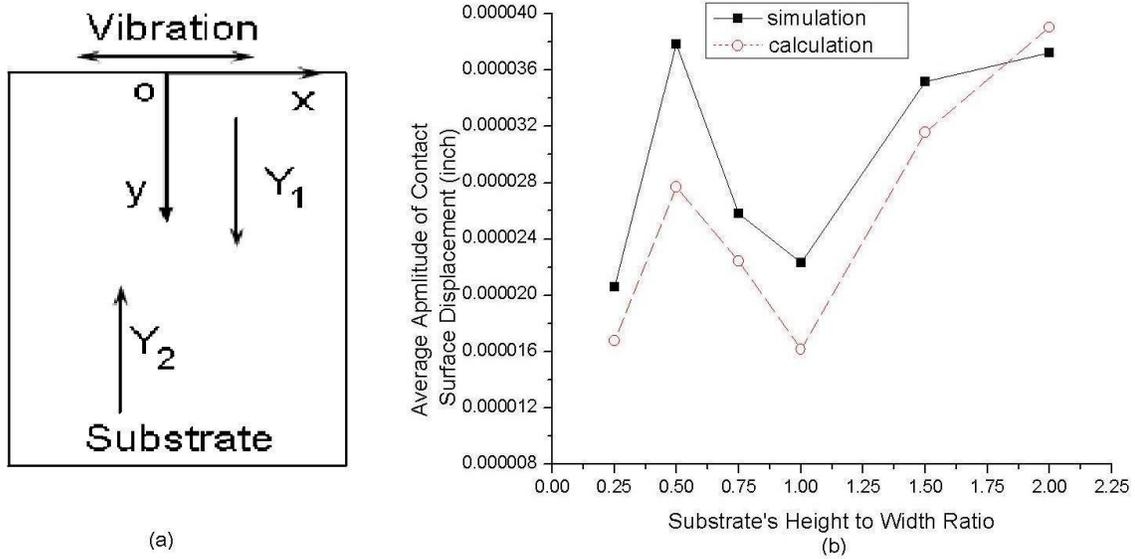


Figure 8: Sketch for the analysis of ultrasound wave traveling in the substrate (a); comparison of the average amplitude of contact surface displacement at the 1500th cycle between simulation and calculation based on Equation 7.

Substituting the corresponding parameters to Equations 5 and 6, the resultant wave equation for the contact interface ( $y = 0$ ) is shown in Equation 7.

$$Y = Y_1 + Y_2 = (0.92 + 1.49d)10^{-5} \left[ \cos(1.26 \times 10^5 t) - \alpha \cos(1.26 \times 10^5 t - 6.28d) \right], \quad (7)$$

where  $\alpha = 1 - 0.67d$  (for  $d \leq 1.5$  inches), and  $\alpha = 0$  (for  $d > 1.5$  inches).

The average displacement amplitude at the contact interface as a function of the H/W ratio has been calculated using Equation 7, and is plotted in Figure 8b. Compared with results from the ANSYS dynamic simulation (also plotted in the figure), Equation 7 clearly predicts the correct trend. Therefore, the effect of height on the bond formation is related with the geometrical conditions that cause persistent wave shapes through the traveling wave superposition. This explains the formation of bands for the shear strain shown in Figure 7. The wave superposition also helps to understand the reason for the critical H/W=1.0 ratio: at this particular substrate height, the wave superposition produces a minimum interface

displacement. A predication can be made here that if the vibration conditions are altered, a different critical height should be resulted.

From the energetic point of view, ultrasonic consolidation is a process that transforms the ultrasound energy into deformation energy and heat to facilitate bond formation at the interface. If we define the energy transfer coefficient as

$$K_e = E_m/E_u, \quad (8)$$

where  $E_m$  is the deformation energy density per cycle at the contact interface, defined as  $E_u = \sigma_{fric} \times Y$ , and the average ultrasonic energy density per cycle is defined as  $E_u = \frac{1}{2}\rho A^2\omega^2$ , where  $\sigma_{fric}$  is the average contact friction stress,  $Y$  is the average contact surface displacement, we can study the effect of the substrate geometry (H/W ratio) on the efficiency of ultrasonic energy transfer.

Based on the simulation results, the energy density and transfer coefficient at the contact interface for substrates with different height-to-width ratios are listed in Table 2. The maximum energy density and transfer coefficient occur at a H/W ratio of 0.5. The H/W ratio of 1.0 has the minimum values.

Table 2: The energy density ( $E_m$ ) and transfer coefficient ( $K_e$ ) at the contact surface at 1500th cycle with different height-to-width ratios (H/W).

H/W ratio	0.25	0.5	0.75	1.0	1.5	2.0
$E_m$ (psi×inch)	2.56E-2	4.58E-2	1.95E-2	1.18E-2	1.99E-2	1.55E-2
$K_e$ (%)	12.86	23.00	9.79	5.93	9.99	7.79

## Conclusions

1. A new FEM model is built for analysis of the dynamic mechanical behavior of the substrate in ultrasonic consolidation process. The simulation is accomplished through a custom-designed algorithm executed using the ANSYS package.
2. The pressure is found to determine the basic status of contact interface and substrate. The dynamic friction stress caused by vibration generates the cyclic variation in the distribution of interface displacement and strain state in the substrate. For a given vibration condition, the amplitude of contact friction stress and displacement stabilize to a saturated state after certain number of ultrasonic cycles. With the increased substrate height, the amplitude of contact frictional stress decreases, while that of contact interface displacement increases.
3. The reason for the decrease in the frictional stress and displacement at the contact interface for certain substrate height seems to be caused by the complicated wave

interference occurring in the substrate. A specific substrate geometry generates a minimum strain state at the interface as a result of wave superposition. The height of the substrate can be increased if a different set of ultrasonic parameters are used.

4. The simulation results show the contact interface with a H/W ratio of 1.0 has the lowest ultrasonic energy density and energy transfer coefficient. In this H/W condition, defects will be likely to occur at the edges of the contact interface. The experiment observation is therefore fully explained.

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