A NOVEL METHOD FOR PART DECOMPOSITION BASED ON UNDERCUT EDGES FOR EFFICIENT HYBRID RAPID PROTOTYPING

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Abstract

Layered Manufacturing allows physical prototypes of 3D parts to be built directly from their computer models, as a stack of 2D layers. This article proposes to decompose the model to be built into minimum number of layers and then build each layer separately by performing machining operations on a given sheet of metal, using a 3-axis CNC milling machine and adhering the layers, allowing large models to be built in parallel. The decomposition of the model is carried out in such a fashion that it decomposes all the inaccessible regions in the object such as under-cut concave edges. For a given build up direction, the undercut edges, which cause a part to be inaccessible by a tool, are extracted and eliminated in two decomposition steps ensuring that the number of layers are as least as possible, thus minimizing the build time.

Keywords: Part decomposition, Hybrid Rapid Prototyping, Undercut edges

Introduction

In Hybrid Rapid Prototyping, a part is build by performing both machining and deposition. Thus, part decomposition is not only a necessary process before build operations but also determines the build time. The build time in this process consists of machining time and deposition time. The deposition time is the time spent for sheet transfer, setup, and deposition, which takes a major portion of the total build time and is proportional to the number of layers. Therefore, to reduce the build time, the part should be decomposed into as fewer layers as possible while ensuring that each layer has a shape that can be fabricated from the given sheet using 3-axis milling machine. In other words, a part should be divided into three-dimensional layers by considering the following requirements:

- *Tool accessibility*: a part should be decomposed into such layers that all the machining features in each layer are accessed by tool.
- *Sheet thickness*: the thickness of each layer should not be bigger than the maximum thickness of the available sheet.
- *Layer number*: it is essential to minimize the number of the decomposed layers in order to reduce the build time.

The existing researches on part decomposition consider these requirements and suggest a method based on Undercut Concave Edge (UCE) and Interference Concave Edge Pair (ICEP). Our proposed algorithm takes an integrated approach and treats the concave edges without distinction.

Hybrid Rapid Prototyping System: The process of the system is composed of four steps, i.e. back-face machining, sheet reverse and deposition, front-face machining, excess region removing. In this system, the shape of the layer is generated by performing two directional 3-axes machining in two setups, i.e. back-face machining and front-face machining. In order to make sure that the decomposed layers can be generated by back-face machining and front face machining, a part should be decomposed into layers such that surfaces of each layer can be visible from either the back or front at least. Fig. 1(a) shows the machineable layer, which has only two intersection points with any ray along the build direction.1 However, if a ray intersects a layer at more than two intersection points along the build direction, as shown in Fig. 1(b) and 1(c), then, this layer is not machineable and cannot be generated in the proposed hybrid rapid prototyping system.

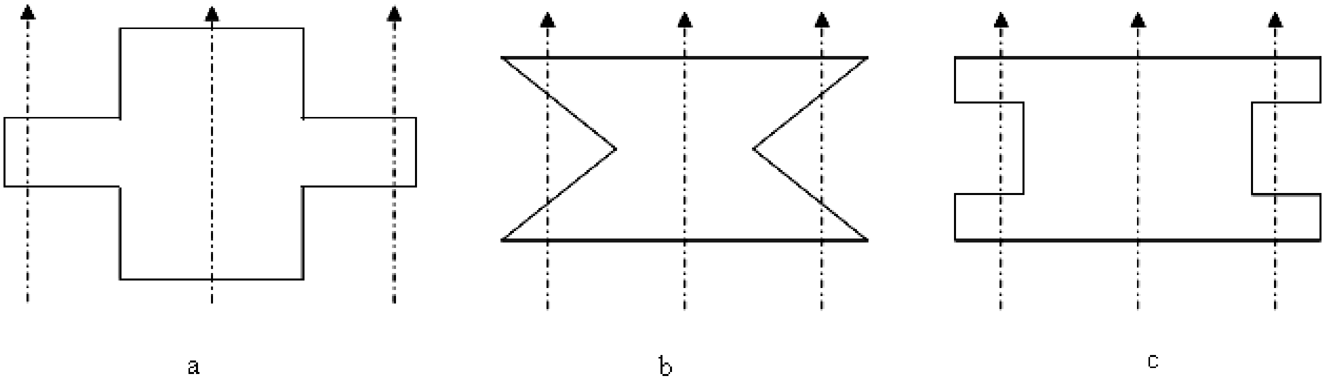


Figure 1. Tool accessibility condition of the layer (Courtesy, Hu & Lee [1])

Related Work: Hu and Lee in 2005 proposed new part decomposition algorithm for the hybrid rapid prototyping process considering the total number of layers and the maximum allowable sheet thickness as well as the tool accessibility. For the given build-up direction, the undercut edges were extracted and classified into undercut concave edge and interference concave edge pair and then the decompositions for eliminating the two kinds of undercut edges were performed in two steps [1]. Hur et al proposed a new methodology for the development of a hybrid-RP system in 2001 [2]. This method combines the advantages of the one-setup process of RP and the high accuracy offered by CNC. They presented an algorithm to decompose a part into layers by grouping the neighboring faces whose normal vectors have the same sign of z coordinate value and took into consideration the sheet thickness as well. Such a case is illustrated in Fig. 2, in which the model is decomposed into six layers by Hur's method.

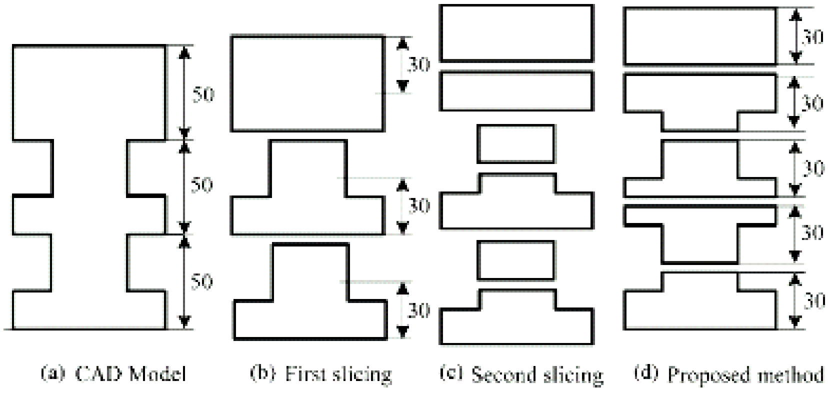


Figure 2. The layers generated by Hur's method (*Courtesy Hur et al [2])*

Dhaliwal et al [3] in 2001 described a decomposition method for the design of multi-piece mold, which consists of three steps viz. feature based decomposition along planar faces of primitives constituting the part, decomposition of components having inaccessible concave edges and recombination of accessible components. Ki and Lee in 2002 proposed a method which allows various machining directions to deduce the number of decomposed components [4]. In addition to the feasibility of two-setup machining, it also considers the machining easiness and the machining efficiency during the decomposition. Chang et al. [5] presented a method of part decomposition for Shape Deposition Manufacturing (SDM). In this method, silhouette curves, which serve as the boundary between undercut and non-undercut portions of the surface, were used to split the surfaces into sub-surfaces that serve as the basic build units and to generate the build sequence. Taylor et al [6] presented the SWIFT (Solvent Welding Freeform Fabrication Technique) rapid prototyping process which is feasible and has advantages over most commercially available rapid prototyping processes with regard to cost, accuracy and speed. Figure 3 shows the proposed SWIFT machine configuration.

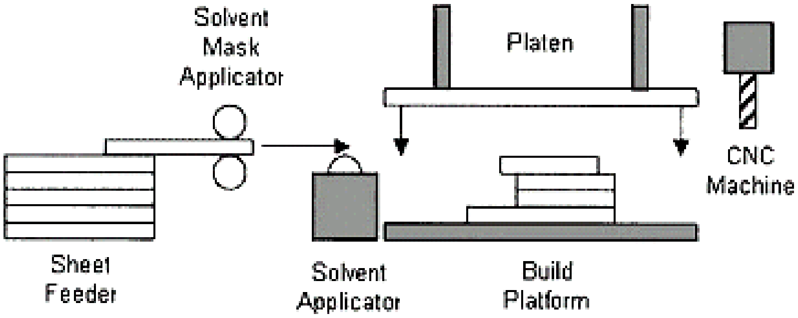


Figure 3. The proposed SWIFT machine configuration.

In 2001 Qian and Dutta presented a feature-based fabrication methodology in LM domain to resolve the dilemma between improving the surface quality and decreasing the build time [7]. Experimental results and quantitative analysis presented in this paper demonstrate that significant build time can be saved by using feature-based fabrication method. Feature-based fabrication was also found to be an effective way to localize curvature effects. Majhi et al in their work gave efficient geometric algorithms for certain optimization problems arising in layered manufacturing [8]. Ye et al in 2001 described a new approach to recognize undercut features from a B-rep model [9]. The proposed method combined the strength of graph-based methods and hint-based methods to recognize undercut features effectively. In 2003 Pandey et al proposed an approach for adaptive slicing which was based on the realistic build edge profile and was implemented using two approaches, namely direct slicing and tessellated model (STL) [10]. Ma and He [11] developed algorithms for adaptively slicing an object represented in NURBS surfaces. A selective hatching strategy was proposed to shorten the build time. Ilinkin et al [12] presented a new decomposition-based approach to LM, which reduces substantially the support requirements of the process, while also realizing other benefits such as, it allows the construction of large models that cannot be accommodated in the workspace as a single piece. Ilinkin et al again in 2002 [13] presented algorithms to decide whether a simple polygon can be decomposed into by a line into two terrains.

Part Decomposition

The undercut edges shown in Fig. 4 cause a part to be inaccessible by a tool under the given machining direction. If a part contains any undercut edges, decompositions are needed to eliminate them. The key issue is how to detect those undercut edges and decompose a part into layers such that the part can be built in as short time as possible by the hybrid rapid prototyping system.

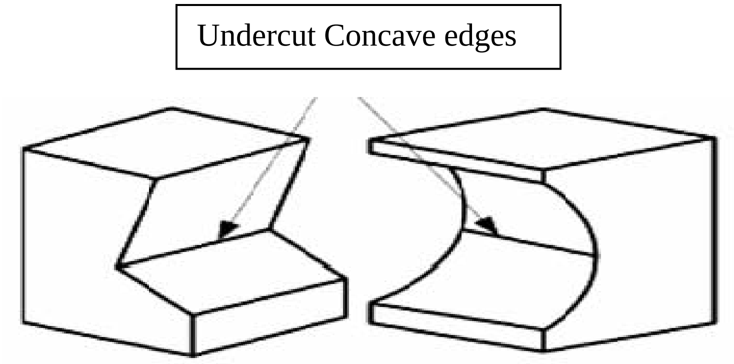


Figure 4. Objects with undercut concave edges

Concave Edge/ Silhouette Curve Detection: The algorithm proposed for concave edge detection uses the fact that the sign of the component of the normal vector (in the build direction) to a surface changes whenever a concave edge is encountered. The same fact has been used in case of ruled and freeform surfaces where instead of an edge a silhouette curve might be generated. Specific algorithm for various cases has been discussed in detail below.

2D Concave Point Analysis: Starting from the origin i.e. (0, 0) while traversing in an anticlockwise direction, if the slope $m = \tan\theta$ of an edge of the polygon changes its sign from negative to positive then the point common to both the lines is the concave point otherwise it is a convex point. For instance, in Fig. 5, starting from (0, 0), line-1 has the value $m > 0$ and line-2 has $m < 0$, therefore, point-1 is a concave point. Similarly point-2 can also be classified as a concave point. Then, in the proposed method we go on to join the concave points on the opposite sides to decompose the object into separate layers.

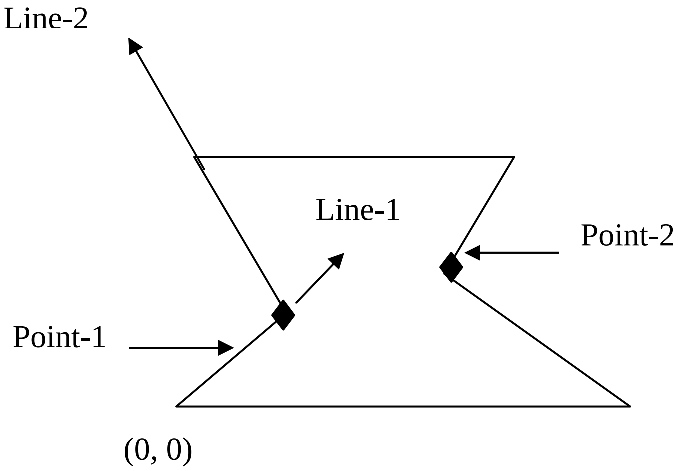


Figure 5. Description of 2D Concave point Algorithm

Concave Edge Analysis in Planes: Suppose a body is designed using planes only and let the normal vector to each plane be $n_{ix}\hat{i} + n_{iy}\hat{j} + n_{iz}\hat{k}$, where i denotes the plane number and n_x, n_y and n_z denotes the component of the normal vector in x, y and z directions respectively. The proposed algorithm states that if $n_x > 0$ and n_z changes its sign from negative to positive or if $n_x < 0$ and n_z changes its sign from positive to negative, then the common edge between the two planes can be classified as a concave edge. Similar concept holds good for concave edges on $x-y$ plane and in that case the sign of n_y is considered as well.

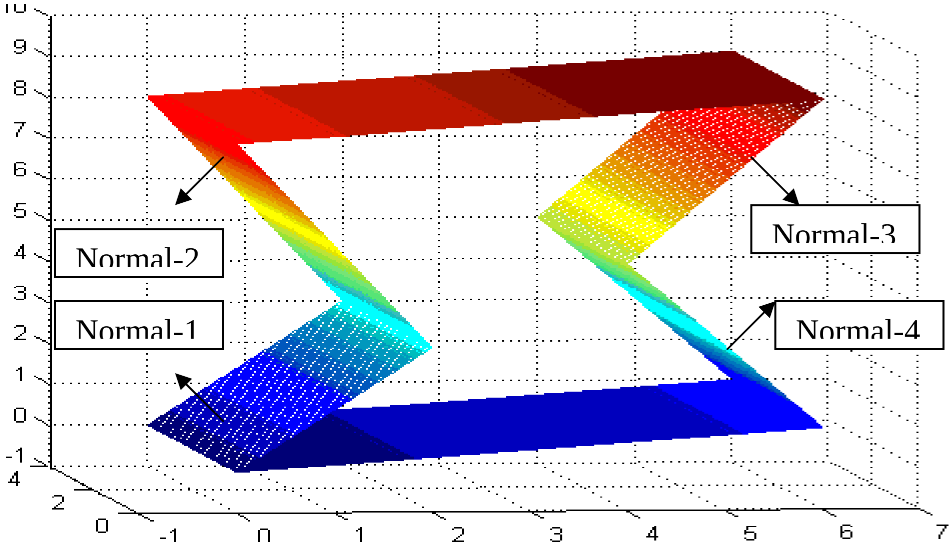


Figure 6. Description of 3D Concave Edge Algorithm in Case of planar body

As shown in Fig. 6, for Normal-1 (normal to plane-1, starting from left bottom and moving clockwise), $n_x < 0$ & $n_z > 0$, and for Normal-2 (normal to plane-2) we have $n_z < 0$. Therefore in this case n_z changes sign from positive to negative and n_x is negative; hence the edge common between planes 1 & 2 can be classified as a concave edge. Similarly it can be shown for planes 3 & 4 where n_z changes sign from positive to negative and $n_x > 0$.

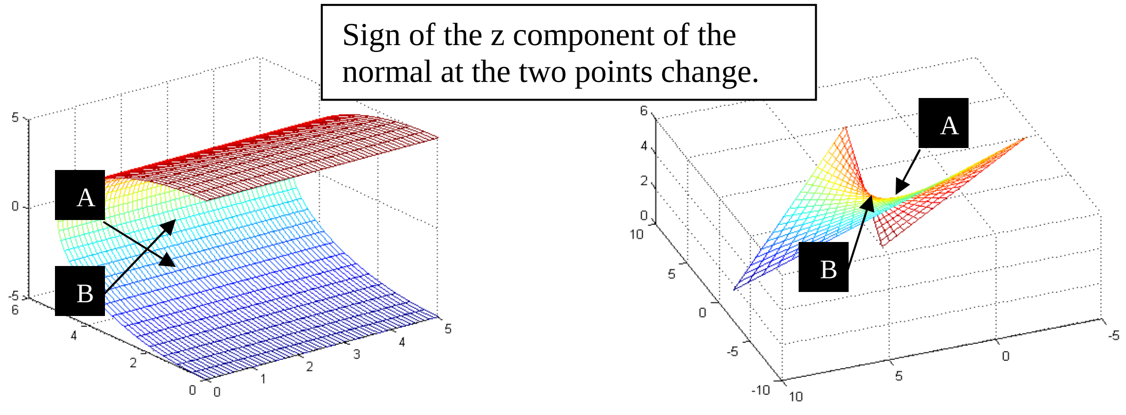
Concave Edge/Silhouette Curve Analysis in Ruled Surfaces: Most of the ruled surfaces can be treated mathematically as follows:

$$f(u,v) = u * G(v) + (1-u) * D(v)$$

Or alternatively as:

$$f(u,v) = G(v) + u * D(v)$$

Where $u \in [0,1]$ and $G(v)$ and $D(v)$ are two independent functions known as the directrix and the generatrix functions. The ruled surfaces are plotted by plotting the curves for different values of u and v. As a result a grid of intersecting curves is formed. For finding the concave edge/silhouette curve, the value of normal are calculated at the grid points. A point will lie on the silhouette curve if the sign of the components of the normal at its adjacent points (before and after) change in the same manner as in the case of planes. In Fig. 7a and 7b, the point lying in between points A and B will lie on the concave edge and silhouette curve respectively.



a **b**

Figure 7. Description of 3D Concave Edge/Silhouette Curve Analysis in Case of ruled surface

Concave Edge/Silhouette Curve Analysis in Freeform Surfaces: The freeform surface can be expressed mathematically as S given by $S(u, v) = (x(u, v), y(u, v), z(u, v))$. The normal to the surface can be expressed as $N = S_u \text{ X } S_v$, where S_u and S_v are the tangent vectors along the $u \& v$ directions respectively. The similar procedure to that of a ruled surface is followed in finding the silhouette curve in case of freeform surfaces. Fig.8 shows the silhouette curve detection in case of a Bezier surface. The red line shows the silhouette curve.

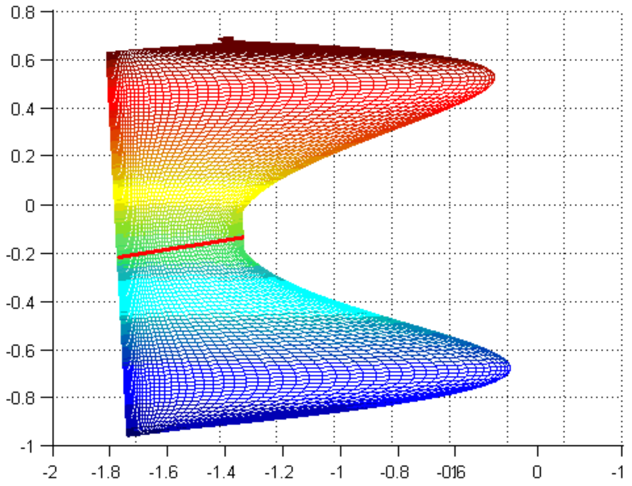


Figure 8. Description of Silhouette Curve in Case of freeform (Bezier) surface

Layer Decomposition: For decomposition of the object into layers to eliminate the concave undercut edges, the layers are formed by decomposing the object in the fashion as shown in Figure 11. The combinations of vertical and horizontal cuts are used to form the layers in this case because this would help in easy stacking and thus assembly of the layers. This is in stark contrast with previous works where it was proposed to slice the object along the concave edge in the horizontal direction. This subtle change in the algorithm produces a good difference in the number of layers generated thereby reducing the build time.

Machining and Deposition Process: The hybrid-RP process consists of two cycles. One is the main process cycle, which repeats periodically until the whole building process is completed, and the other is the additional machining process, which can be carried out at any optimal instant to machine the machining feature segment. Fig. 9 shows the whole hybrid-RP process. The left part of the figure shows the main process cycle, which is composed of 4 steps, i.e. Stock material placement, front face machining, back face machining and excess material removal, and the right part does the additional machining process. Though stacking thick layers accompanies many hidden geometries that reside within a sheet material, hybrid-RP effectively solves this problem by inverting and machining the backside of the sheet.

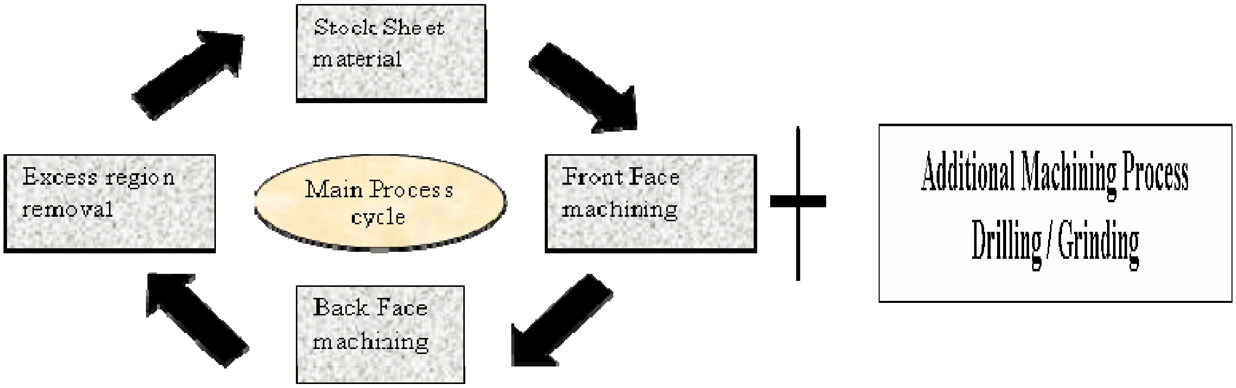


Figure 9. Process cycle of hybrid-RP system

The part should be sub-divided into deposition feature segments such that every such segment is visible either from the top or from the bottom, i.e. should be manufacturable in two setups at most. In other words, no interference should occur in the deposition or machining process of the main process cycle in the build up direction. Fig. 10 shows the faces of a deposition feature segment to be machined in each setup. The 3-axes milling capability of the machining center is also considered to determine the shape of these deposition feature segments. The shape accuracy of the machining features can be guaranteed by machining each machining feature segment in a single machining operation and the strength of a part can be improved because the number of part segments is reduced.

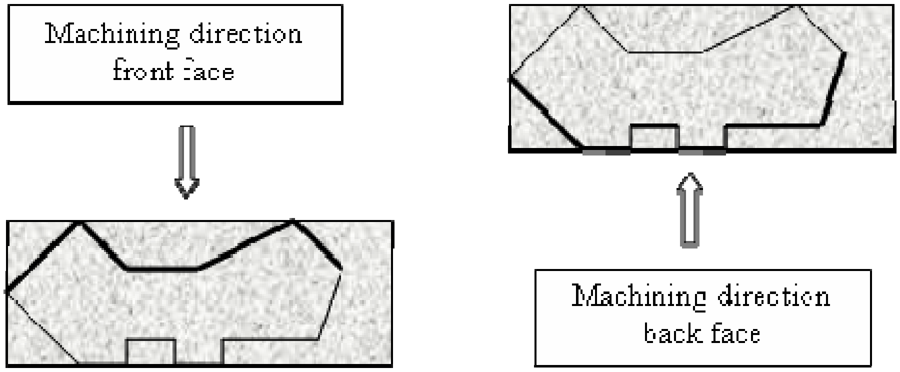


Figure 10. Deposition feature segment generation by front and back face machining process

Results

The decomposition algorithms for both 2D and 3D cases as mentioned above were implemented in *Matlab* environment and the following results were obtained which give lesser number of layers as compared to the earlier research works. The number of layers were reduced drastically thus reducing the cost and time for the fabrication of the object. Fig. 11 shows the example and hence compares the result obtained by our proposed algorithm and the algorithm proposed by Hu and Lee [1].

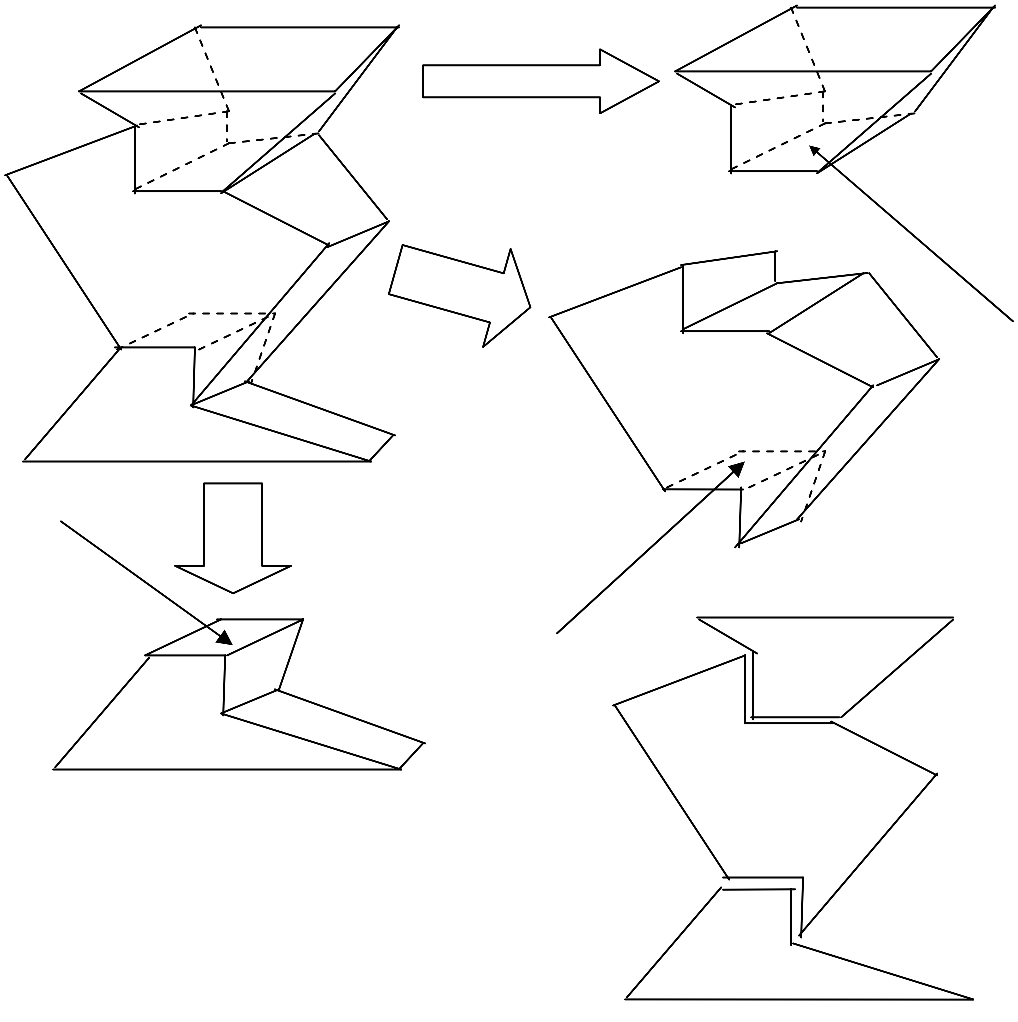


Figure 11. The proposed layer decomposition algorithm in which number of layers have been significantly reduced as compared to method presented by Hu and Lee [1].

It is evident from Fig. 11 that the number of layers is significantly reduced from 5 to 3 by applying the new algorithm for part decomposition. The assembly is done by depositing the layers on top of each other while machining. This ensures that the layers get easily stacked on top of each other.

In a similar way, the algorithm was also applied for a 3D body designed only using planes. The concave edges were determined using the algorithm described above the layers were created. Fig. 12 shows the case when the algorithm was applied to a closed 3D body comprising of planar surfaces. In that case the algorithm was applied and the blue lines show the concave edge.

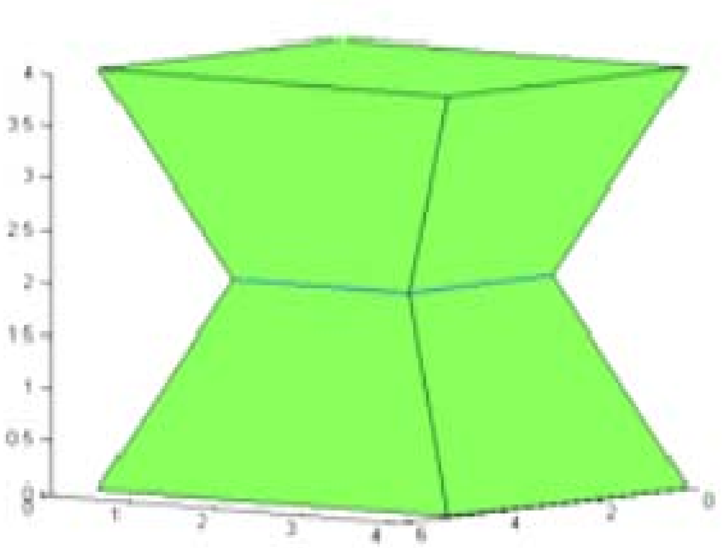


Figure 12. . Proposed algorithm applied to a closed 3D object

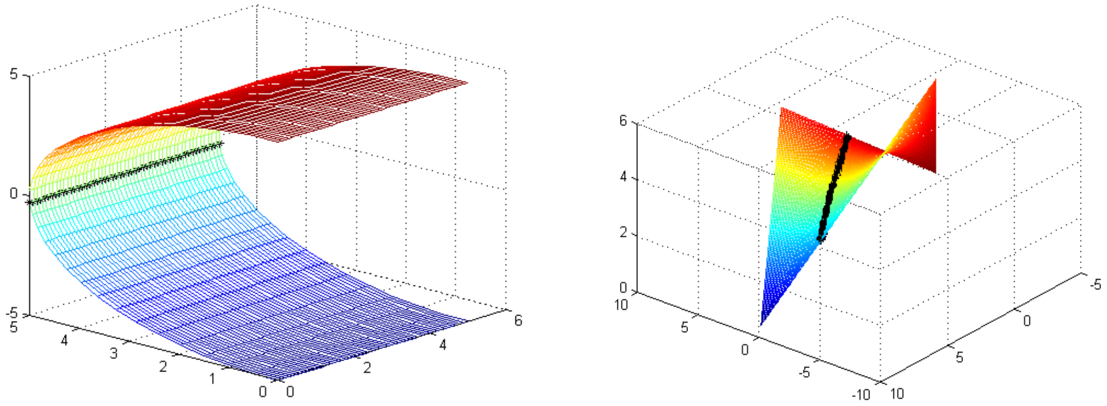


Figure 13. Silhouette curves used for concave edge decomposition in case of ruled surfaces

Fig. 13 shows the concave edge/silhouette curves determined in case of complex ruled and freeform surfaces. Fig. 14 shows a body comprising of both ruled surface as well as planar surface. The proposed algorithm was applied on this body to decompose it in layers along the undercut edge.

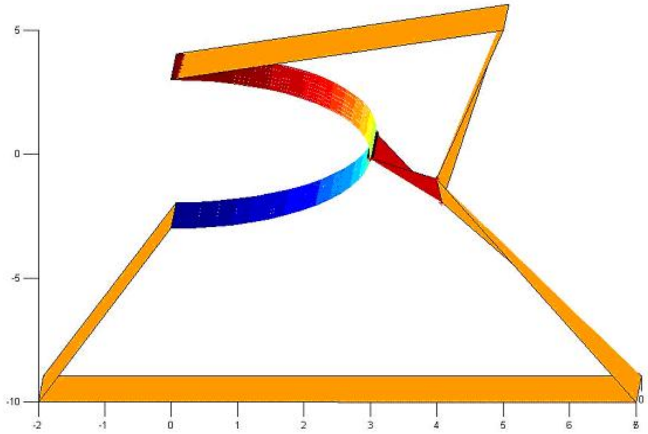


Figure 14. Ruled parting surface generated for a body consisting of both ruled and planar surfaces

These results show a remarkable improvement in the number of three-dimensional layers which needs to be machined out and then deposited on top of each other to manufacture the job by the process of hybrid-RP. Another important factor is the *sheet thickness*. It is very important to take into account that the thickness of each layer should not be bigger than the maximum thickness of the available sheet. Now to resolve this problem, the maximum sheet thickness required by the job can be calculated based on the information of the maximum and minimum z values of each sliced part. This may sometimes also lead to waste of material particularly in cases of thinner layers. Again this problem can be solved by adopting two different approaches. One is to use sheets of different thicknesses and provide sheets according to the requirement of the layer and Other approach that can be adopted is to supply a sheet having the mean thickness of all the layers and then decomposing the thicker layers in a way such that the criteria of minimum sheet thickness is satisfied. If the layer decomposition gives rise to complex parting surfaces then these ruled surfaces can be decomposed by combined vertical and flat sides which would simplify the machining process.

Conclusion

The proposed method of concave edge decomposition is noticeably superior to existing methods such as stratoconecption, of decomposition as it drastically reduces the number of layers and thus the build time. The effectiveness of the proposed algorithm becomes more visible as the number of concave edges and the asymmetry in the model increases. The algorithm uses a simple yet effective method to identify all the concave edges present in the object which should be decomposed to ensure better tool accessibility. The top surface of the sheets are anyway proposed to be machined by 3-axis, the proposed method is not asking for any extra-ordinarily complex machining procedure in machining the top and back portions in an inclined manner. It has also been suggested that the layers can be joined together by means of screws or bolts and dowel and pin like structures which would give more rigidity to the build prototype. It provides a new robust and simple procedure for part decomposition to be used in hybrid RP.

The future scope of the work involves applying of this algorithm to real life objects designed on work stations and check how effective it proves to be. Another major task would be to find a way of connecting more than two silhouette curves to form a parting surface in case of complex 3D objects. This problem can most probably be solved by scaling of the two curves at a time with respect to each other and taking relevant points to find a ruled surface which can then be machined using a 3-axis CNC machine.

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