

Application of Topology Optimization in Modern Additive Manufacturing

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The current work examines the principle of topology optimization (TOP) and solving the problem of minimal compliance, and its applications in modern additive manufacturing (AM). The theory of TOP has been excessively investigated for the past few decades; however its practical use is hindered by manufacturing limitations of manifesting the designs into physical parts. TOP numerically determines the favorable topology configuration contained in a workspace that is constrained by a specific set of external supports and applied loads. It overcomes conventional design traditions, and solves for finding the path of least resistance, mimicking phenomenon found in natural structures, such as bone and wood. The present research focuses on the different methodologies invested in the field, and also inspects the possibilities and efficiency of using numerical design, which is powered by TOP, in modern AM processes such as Electron Beam Melting enabling the production of highly complex parts. The major interest is the production of functionally graded porosity (FGP) in bio-adaptable dental implants able to reduce stress shielding, promote faster osseointegration, and provide micromotion between the dental prosthesis and the root form of the implant. Future implications of the field include optimized porous structures aiming towards application-defined stiffness and pore size, enabling a digital design with advances analogous to modern AM.

Introduction

Topology optimization (TOP) is a relatively new field of study. It deals with the mathematical problem of finding the optimal distribution of material of a component based on specific criteria of performance or physical aspects. Although some of the earliest principles date back to 1904[1], the field have not received any remarkable improvement until the emergence of computational power and its maturity in the mid-1980's. Analogous to biomatter in nature, TOP is believed to portray a more goal oriented design routine when compared to conventional means where human intuition and experience plays the most crucial role in the preliminary design conceptualization of a structural component. It is the optimal solution that provides a more efficient use of a material by maximizing the performance of user-set resource parameters such as mass or cost, or by minimizing the needed resources to accomplish a user-set performance.

Two major approaches of representing topology prove mostly eminent: a voxel-based topology, and a truss-based topology. The current work will examine both approaches while exhibiting an example where TOP is used as a design tool which applies to the design of lattice structures and their consequent building using an AM process; i.e. electron beam melting (EBM®).

According to Li et al., TOP deals with identifying the areas of a domain that have inefficient levels of stress, and then removing them in order to increase the overall efficiency. By doing so an optimal material distribution would emerge [2]. Li elaborates two approaches to achieve the aforementioned goal:

the homogenization method [3], and the discrete variable method which he credits to the work of Shim and Manoochehri in 1997 and Xie and Steven [2]. The former requires complex and cumbersome mathematical techniques, and proving the existence of a solution was the main objective of the corresponding studies instead of practically finding the solution. The latter approach, which employs discrete methods, takes advantage of computational techniques where an evolutionary scheme is used in parallel with a finite element model to find the optimal distribution of material. The next few sections will portray and discuss the mathematical model of one of the most common TOP objectives (minimal compliance problem), and its corresponding implementation into a Matlab code by Sigmund et al [4], the expansion of the Matlab code into solving three dimensional problems, using ANSYS® to solve TOP problems to produce an optimal design for a lattice structure under uniaxial compressive stress, and finally implementing truss-based TOP to produce an optimal lattice structure designed to perform under a tri-axial strain field.

Minimal Compliance problem

The minimal compliance problem, also known as the maximum global stiffness problem, is perhaps the most common TOP approach. A domain in either \mathbb{R}^2 or \mathbb{R}^3 governed by a set of constraints, loads, and a user-defined volume reduction is evaluated for the optimal material distribution (See Figure 1).

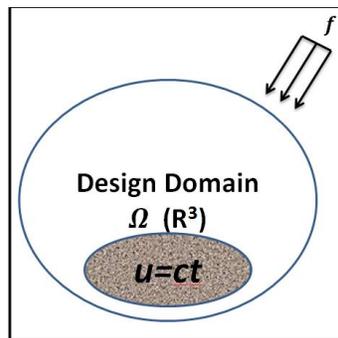


Figure 1: Schematic of a topology optimization environment

According to Sigmund and Bendsoe, from a discrete model point of view where the domain is meshed into discrete elements, the mathematical formulation of the problem at hand is as follows [5]:

<p>(1) Minimize: $c = f^T u$ Subject to: $K(E_e)u = f$</p>
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where c is the compliance tensor, f the force field, and u the displacement field. The minimization of the compliance is governed by a common finite element method relationships used in solid mechanics which includes the force and displacement fields, as well as the stiffness matrix which is affected by the change of density. The solution of Equation (1), results in a distribution of densities throughout the domain, which after several iterative steps will converge towards the minimal compliance. However, a binary solution is strongly desired, where a density of 1 would indicate the presence of material at a certain element, in contrast to a density of 0 or, in avoidance of singularities, a minimal density value, would indicate the presence of a void at the corresponding element. Consequently the binary solution outlays the optimal topology of a structure.

In order to enforce a binary solution, a solid isotropic material with penalization model (SIMP) has been suggested; where a penalization factor is added to relay the relationship between a material's density and its stiffness, in order to lead towards a binary solution of the minimal compliance problem [6, 7, 8]. Equation (2) portrays the mathematical formulation of SIMP [5]:

$$E_{ijkl}(x) = \rho(x)^p E_{ijkl}^0(x), \quad p > 1$$

$$(2): \quad \int_{\Omega} \rho(x) d\Omega \leq V, \quad 0 \leq \rho(x) \leq 1, \quad x \in \Omega$$

$$p \geq \max \left\{ 15 \frac{1-\nu^0}{7-\nu^0}, \frac{3}{2} \frac{1-\nu^0}{1-2\nu^0} \right\} \text{ in } (3-D)$$

where $E_{ijkl}(x)$ corresponds to the stiffness at a certain indexed element, $\rho(x)$ is the density of the element, $E_{ijkl}^0(x)$ is the stiffness of the base material, and p is the penalization factor. This relationship has been proven to be physically permissible corresponding to a minimal value of penalization which depends on the material's Poisson's ratio [9]. By implementing the SIMP on the minimization equation of Equation (1), the modified objective function to minimize becomes Equation (3):

$$(3): \quad c = x^p f^T u = x^p u^T K u$$

where x , corresponds to the density field, which after convergence corresponds to the solution of the mathematical problem.

Three main approaches for solving the SIMP minimization problem can be cited

- The method of **m**oving **a**symptotes (MMA) [10]
- **S**equential **l**inear **p**rogramming (SLP)
- The method of **o**ptimality **c**riteria (OC) [11]

Sigmund presented a paper in 2001 [4], where a 99 line Matlab code is used to solve two-dimensional static TOP problems, with the options of using multiple load scenarios. The code employs the SIMP model, as well as the OC method for solving (See Figure 2). In addition it uses a mesh independency filter to prevent check board areas in the solution.

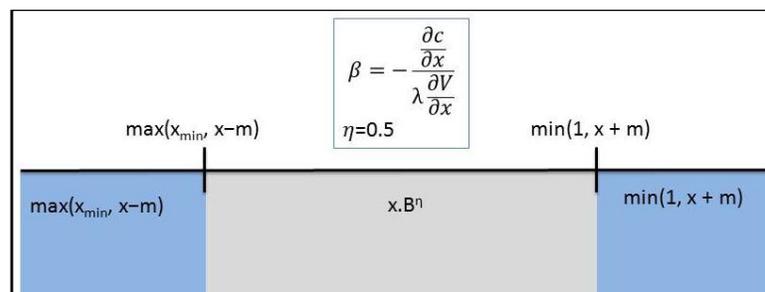


Figure 2: OC method employed by Sigmund [4]

Figure 3 and Figure 4 displays three examples of solutions found using Sigmund's code. The former portrays a cantilever beam with a load at the middle. By using a symmetry approach the number of elements can be reduced, hence improving computational efficiency. As portrayed in the figure,

computing time was 25 seconds and the result is a familiar topology found in the design of suspended bridges.

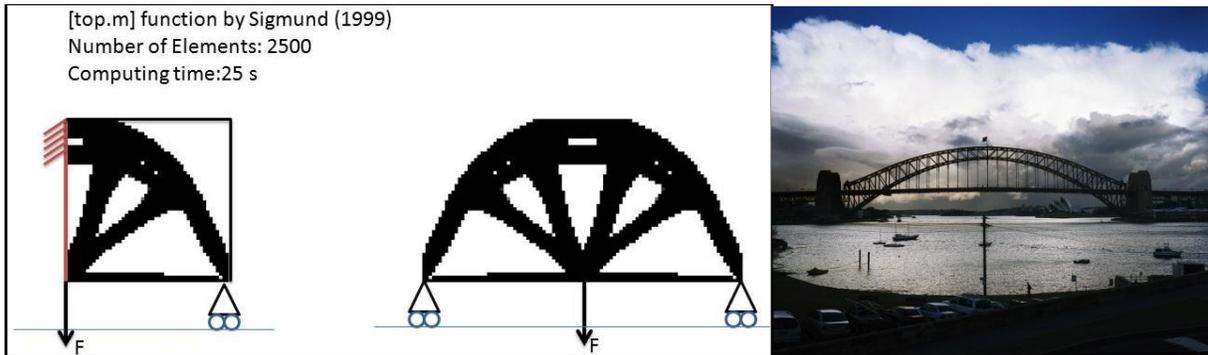


Figure 3: An example of a topology optimization using the code provided by Sigmund et al. [4], and a modern suspended bridge (courtesy of ©Trent Parke/Magnum Photo)

Solutions generated by TOP depend on different factors such as mesh resolution, methods used for solving, and iterative/evolutionary parameters. Effects of such factors include checkboards and mesh dependent designs. Some literature surveys the occurrence of such problems and the available techniques to resolve them [12]. Bourdin presents an example of a filter used to deal with these problems to ensure a sharp and valid solution [13].

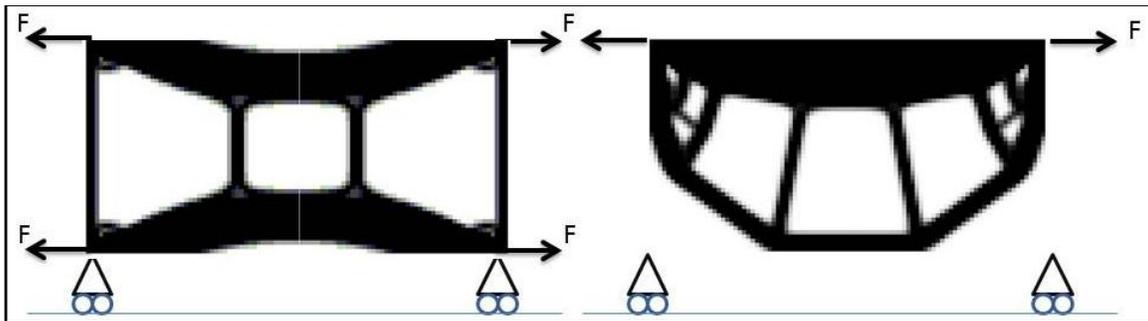


Figure 4: Additional Examples of a topology optimization using the code provided by Sigmund et al [4]

An extension to conventional TOP which deals with static loads, is dynamic TOP, where instead of just maximizing compliance, it becomes of interest to maximize fundamental eigenvalues, where resonance of the structure is to be avoided by making sure that its natural frequency is higher than any loading frequency imposed any kind of dynamic loading device. After examining the code and methods employed by Sigmund, the current paper portrays the implementation of a Matlab code and algorithm analogous to the former in order to solve three-dimensional TOP problems.

Implementation of a voxel-based three-dimensional TOP algorithm

The first step for implementing the three-dimensional analysis is to define the unit element that is used to mesh the domain of interest. A structured brick element similar to elements used in commercial finite element software is established, constituting of the 8 nodes and 24 DOFs (See Figure 5).

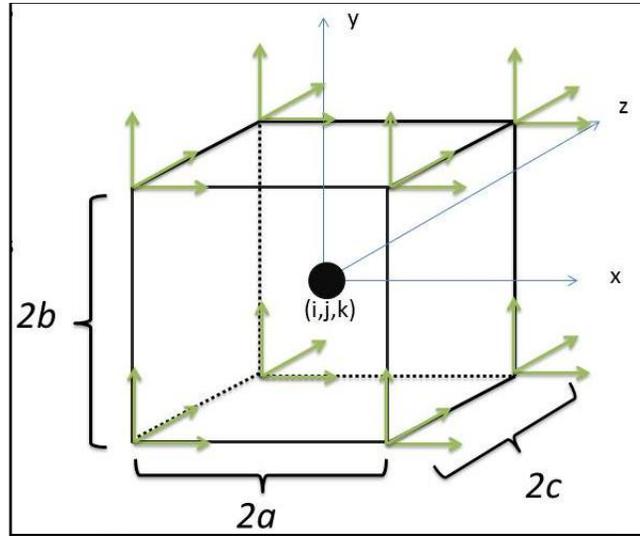


Figure 5: Schematic of the 8 node, 24 DOF, brick element used in the algorithm

The next step is to generate the stiffness matrix associated with the brick element. After examining the existing literature, no resources providing the matrix are found. However, employing principles of modern solid mechanics [14], and using Mathematica® to perform some parametric calculations the matrix is generated following the set of Equations in (4).

$$(4): \quad [E] = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ \frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ \frac{\nu}{E} & \frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \quad [k]_{24 \times 24} = \int_{-c}^c \int_{-b}^b \int_{-a}^a [B]^T [E] [B] dx dy dz$$

$$[B] = [\partial][N]$$

$$[N] = \begin{bmatrix} N1 & 0 & 0 & N2 & 0 & 0 & \dots \\ 0 & N1 & 0 & 0 & N2 & 0 & \dots \\ 0 & 0 & N1 & 0 & 0 & N2 & \dots \end{bmatrix}$$

$$N_i = \frac{(a \pm x)(b \pm y)(c \pm z)}{8abc}$$

where $[E]$ is the 6x6 stiffness matrix of the material. $[B]$ is a 6x24 matrix found by the differentiation of $[N]$ which depends on the geometrical aspects of the element. The result of the generation is $[k]$ the 24x24 stiffness matrix of an 8 node brick element. The algorithm used for the three-dimensional TOP is analogous to the one used in Sigmund's code. A TOP function solves the minimal compliance problem employing an OC method. The condition for optimization, which follows an FEM relationship, is implemented using the $[k]$ matrix generated earlier.

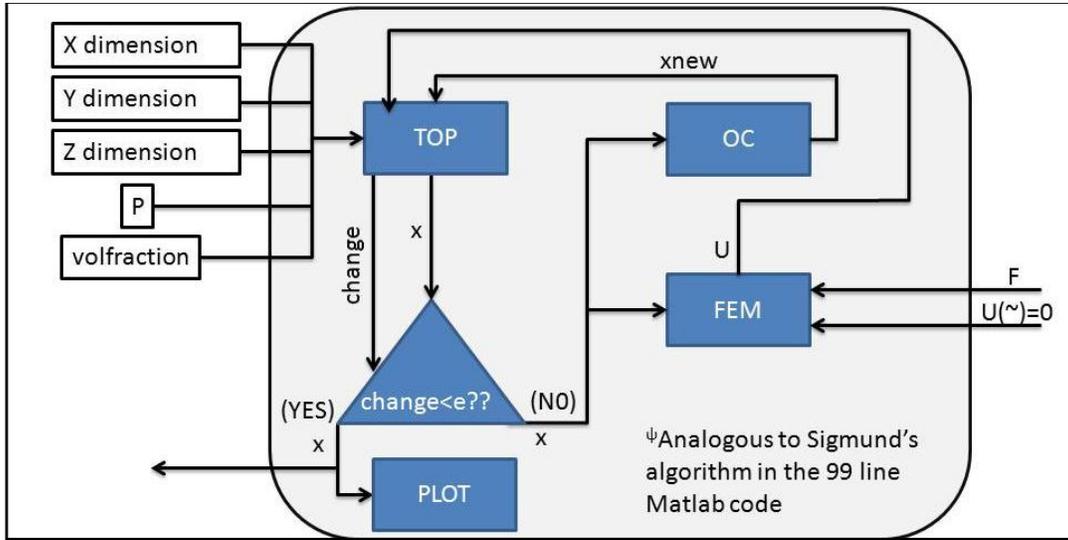


Figure 6: Algorithm used in the three-dimensional topology optimization

The user inputs the dimensions of the domain, a penalization factor value, and a volume fraction to the algorithm, along with loading conditions and constraints. The result is a binary distribution of material, where a “minimal value” corresponds to void, and a maximum value of “1” corresponds to material. Figure 7 and Figure 8 display two examples of TOP where a domain containing 1000 brick elements is loaded under an axial and shear force respectively.

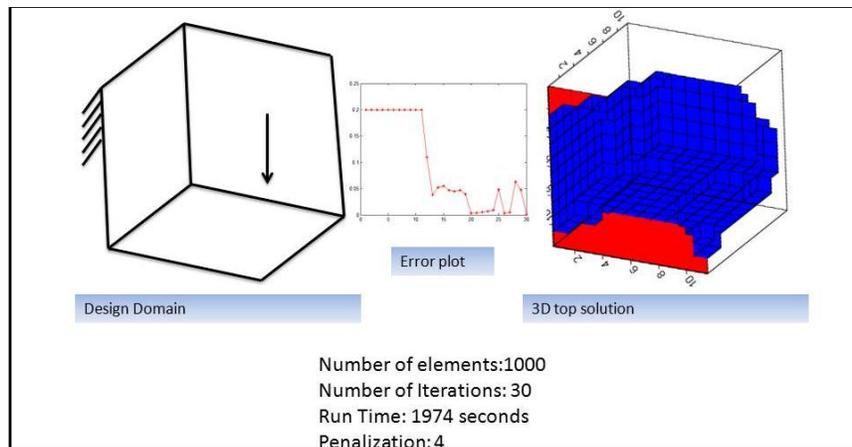


Figure 7: Topology Optimization of a beam under a shear force

The time of convergence is in the order 10^3 seconds, and it requires over 20 iterations for the solution to converge. Increasing the definition of the mesh, producing more elements, leads to a severe increase of computing time, hindering the practicality of employing the developed code to produce three dimensional topology with decent resolution. Nonetheless, the code proved very helpful to explore an application aspect of TOP, similar to Sigmund’s code which as mentioned before deals with two-dimensional topologies [4].

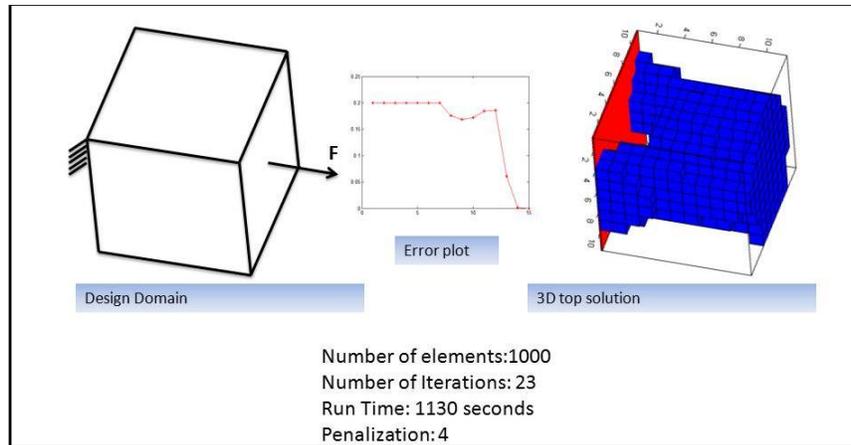


Figure 8: Topology Optimization of a beam under axial force

Topological Optimization using ANSYS®

Due to the educational aspect of the code produced by Sigmund, and the consequent three-dimensional implementation, the code lacks the required computational efficiency to handle higher resolution and a larger number of elements. In order to produce topologies with decent resolution that can be built using AM techniques, a commercial TOP software is used in the current work; i.e. the topological optimization module in ANSYS®. Figure 9 displays the model used for the TOP. The main objective is to produce a structure that can be periodically replicated in order to span over a volume. In order to do so, the resulting topology has to exhibit symmetry in the 3 axial directions. The mentioned model represents 1/8th of the total volume of the topology. Material models are established with a Young's Modulus of 110 GPa, and a Poisson's ratio of 0.33 which corresponds to the mechanical properties of Ti6Al4V. The cube is meshed into 24 389 SOLID95 10 node elements. A zero-displacement constraint is imposed at one of the corners of the cube, while a force distribution in the Z direction is imposed to the diagonally opposing corner.

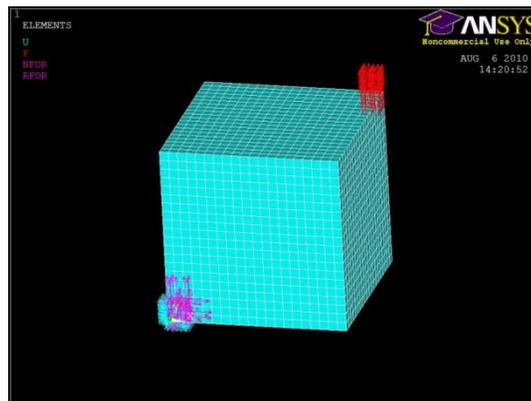


Figure 9: Model, mesh, loads, and constraints for the domain used for the ANSYS® topological optimization

The model is solved for a static structural solution, to make sure that all the parameters in the model are properly defined. A topological optimization is executed, with a converging error <0.001 , using the OC

method to find the optimal topology for a 50% volume reduction. The solution takes 4425 seconds to converge, after 4 iterations.

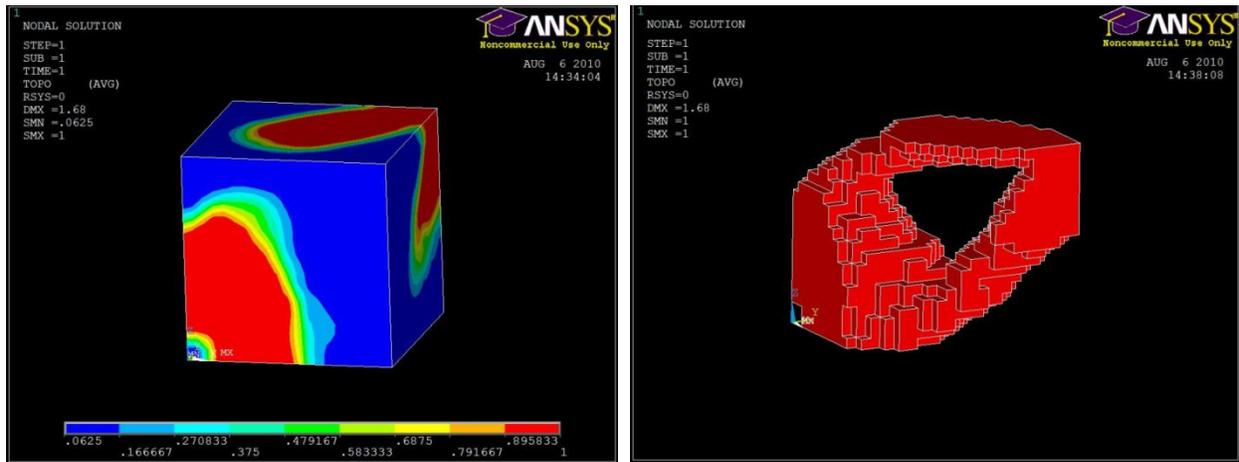


Figure 10: Density plot after topological optimization (left) and the isolated elements with density over 0.4 (right)

Figure 10 displays the resulting density plot of the optimization. By identifying the nodes with a density exceeding 0.4, and the attached elements, the optimal topology of the structure emerges. The next step is to export the nodes into Geomagic©; a scanning data processing software which specializes in reverse engineering and pre-CAD preparation.

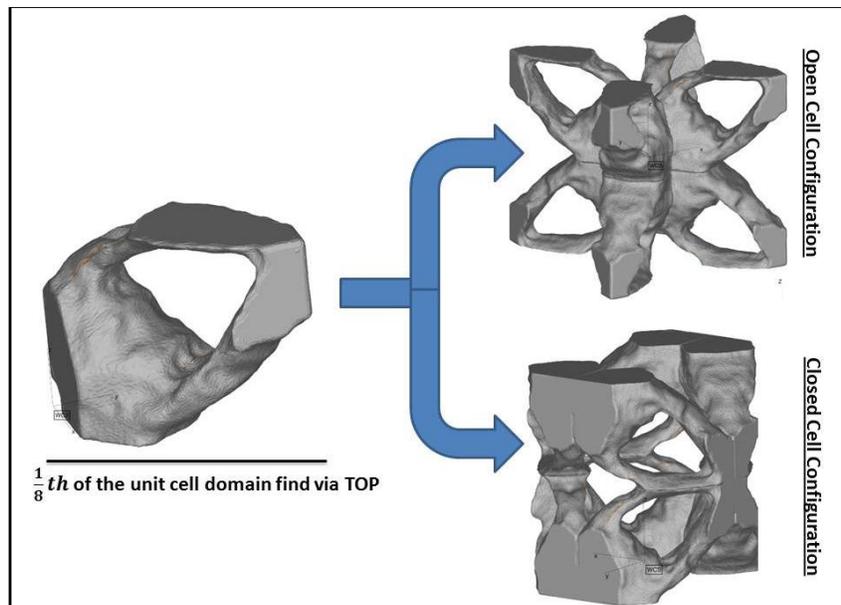


Figure 11: Extrapolation of the result of TOP which represents 1/8th of the domain into the full domain constituting the unit cell of a lattice structure

After a series of volume wrapping, surface smoothing, and triangle count reduction operations, the 1/8th of the unit cell is replicated and joint to form a unit lattice cell, with either an open cell or closed cell configuration. The resulting cell is a voxel-based topology optimized unit, which can be extrapolated to produce porous material, and lattice structures that exhibits maximum stiffness in the uniaxial direction of

the predefined load. By changing the volume fraction reduction, altering the level of porosity, the optimal stiffness of the structure will vary, hence providing the user control over the mechanical performance of the material.

Truss-based TOP

An alternative approach to the aforementioned voxel-based TOP is a truss-based TOP. The basic method for solving the problem is referred to as the ground structure method [5]. The technique deals with setting a fixed number of nodes, and then determining every possible truss connecting the nodes to each other. The areas of the trusses are then varied according to the loads and constraints, with a possibility of the areas being zero, or a minimal value to prevent singularities. A minimal area means that the truss is unnecessary and is consequently removed. Figure 12 display the possible 2016 trusses according to a 4*4*4 node matrix.

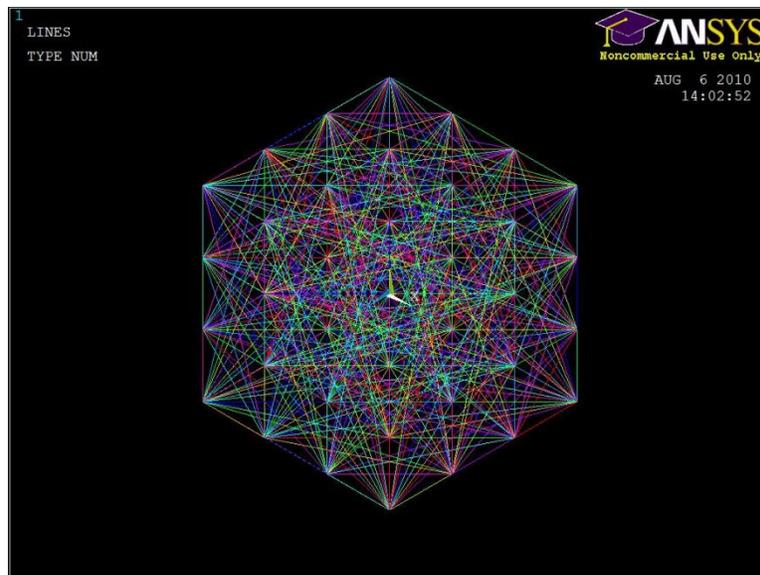


Figure 12: All possible 2016 connections of a 4x4x4 unit cell

Work done by Sigmund et al. in 1995, deals with finding the optimal truss topologies according to a strain field $\varepsilon = [1 \ 1 \ 1 \ 0 \ 0 \ 0]$ [15]. The truss topologies displayed in Figure 13 are analogous to the one found by Sigmund; however, keeping the cross-sectional areas constant throughout the unit cell for simplicity. The topologies are designed using netfabb®, software that specialized in the design of lattice structures and their extrapolation into solid designs for AM preparation. The four cell structures are different, even though they follow the same strain field. However, some added criteria drove towards the various designs. For example cell A has no trusses on the outer surface of the domain cube. In contrast cell B only has truss connecting nodes on the outer surface. Cell A and cell D have an isotropic geometrical distribution, while cell C and cell B manifest anisotropy. In addition, cell D is a merger of three cells of type C oriented in three different orthogonal directions.

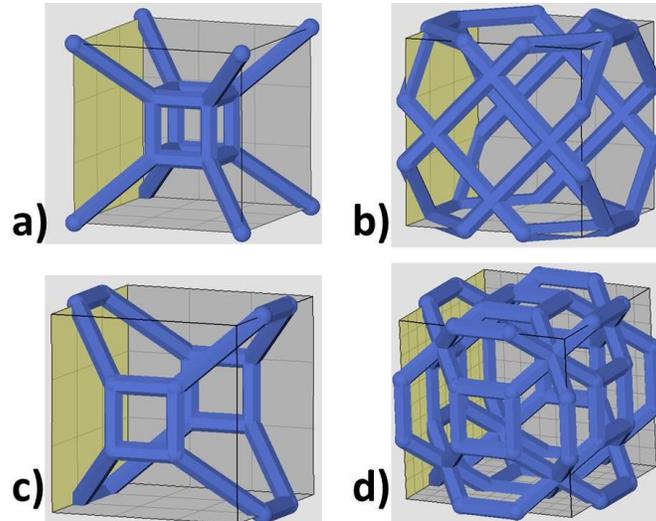


Figure 13: 4 different optimal truss topology solutions corresponding to a strain field $e = [1 \ 1 \ 1 \ 0 \ 0 \ 0]$, found in Sigmund's work in reverse homogenization

Lattice Structures Designed by TOP by EBM®

One of the main objectives of the current work is to find the application of TOP in AM. Due to the abilities of modern technologies of rapid manufacturing to produce parts with highly complex geometry, the use of TOP to produce optimal geometries according to a wanted performance proves to be intuitive. In the case of the design and production of lattice structures, the few previous steps displayed in this paper can be used as design inputs to produce the corresponding lattice structures (See Figure 14).

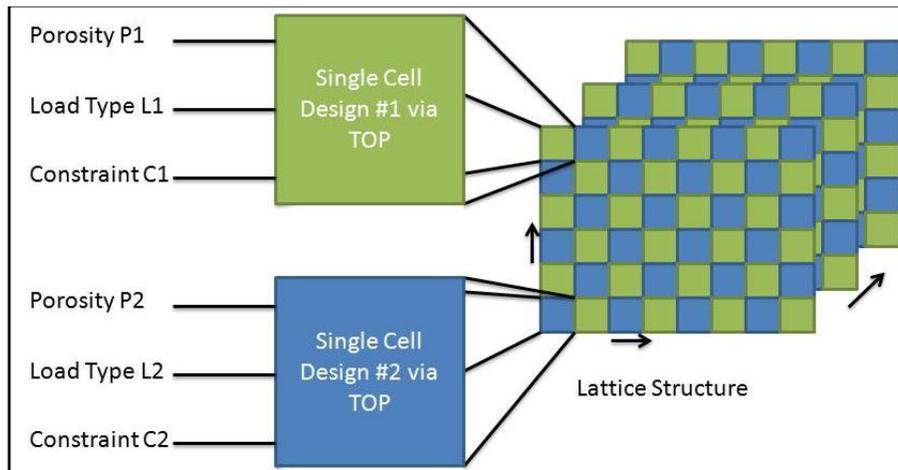


Figure 14: TOP, a design input to lattice structure

Functionally graded porosity (FGP), a concept introduced by previous work [16] deals with locally tailoring the microstructure of a material in order to convey a locally tailored mechanical performance. FGP is one of the major design features found in the concept of bio-adaptable dental implants, where the variant mechanical performance profile plays an important role of matching the mechanical properties of the surrounding bone; hence, preventing stress shielding and promoting healthier osseointegration.

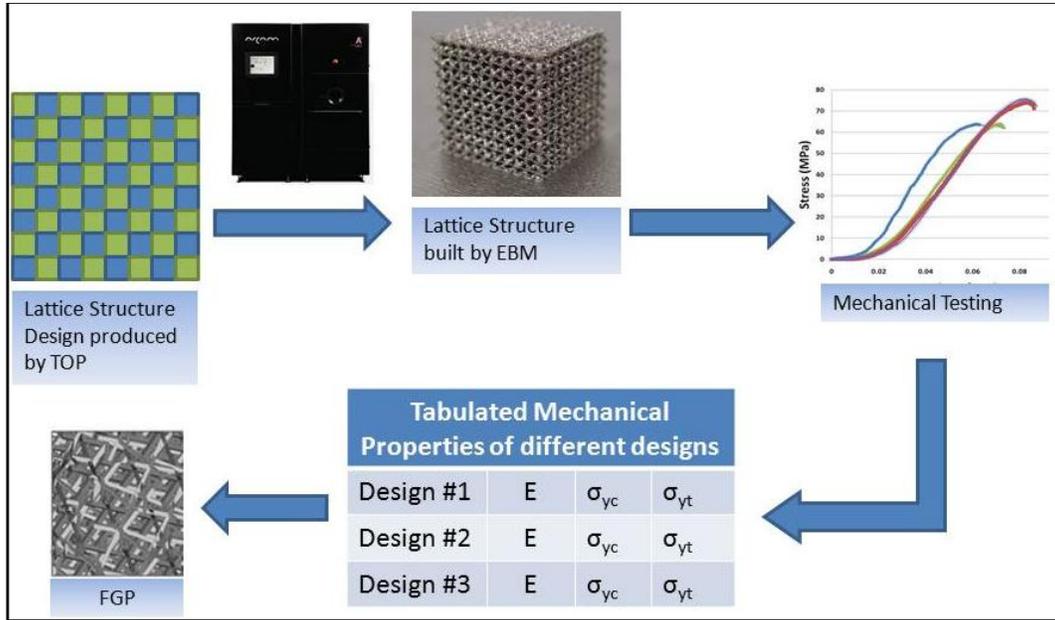


Figure 15: Process track to implementing TOP for FGP design

Lattice structures produced by EBM following TOP design can be implemented to FGP following the process depicted in Figure 15. The lattice structures designed using TOP, are built using EBM and then mechanically tested for validation. Design variables such as truss aspect ratio, or porosity are altered in order to see their effect on mechanical performance. The results of the mechanical testing are then statistically tabulated and analysed. The tabulated data can then be used to design FGP, where a user defined mechanical performance distribution is matched with the corresponding geometrical features. For example, in the case of bio-adaptable dental implants, one design scenario suggests that the surface stiffness profile of the implant matches that of the surrounding bone, which similarly to any biomatter has anisotropic mechanical properties. The stiffness profile is detected using micro-computed tomography (μ CT) scans, where the gray scale values of the images are directly dependent on bone's density and stiffness.

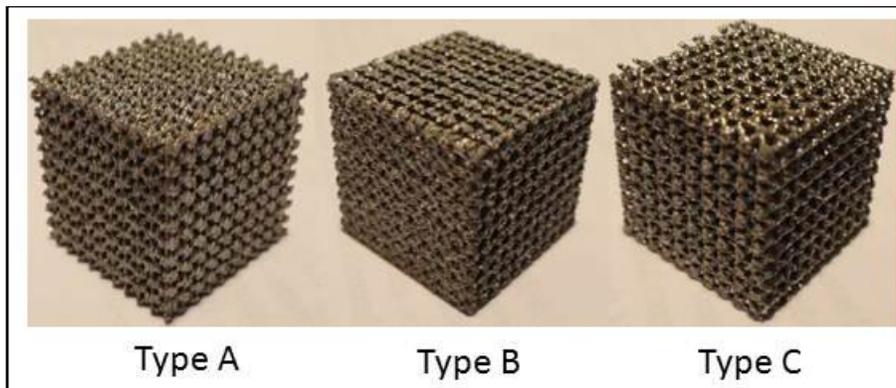


Figure 16: Three different optimal structural designs, extrapolated into 20 mm lattice cube, and built using an EBM®

The cell types A, B, and C of Figure 13 are scaled to 2 mm unit cells and then extrapolated to cover a volume of 800 mm³ using the netfabb® software. The designs are then built using and ARCAM EBM®

A2 machine, using 300 mesh size Ti6Al4V powder, with a layer thickness of 100 μm (See Figure 16). The corresponding cubes will go through mechanical testing to characterize the mechanical performance of the three different cell types. The biggest pore size in type C cells measured 1.42 mm, as displayed in Figure 17, which is slightly smaller than the design pore size of 1.6mm.

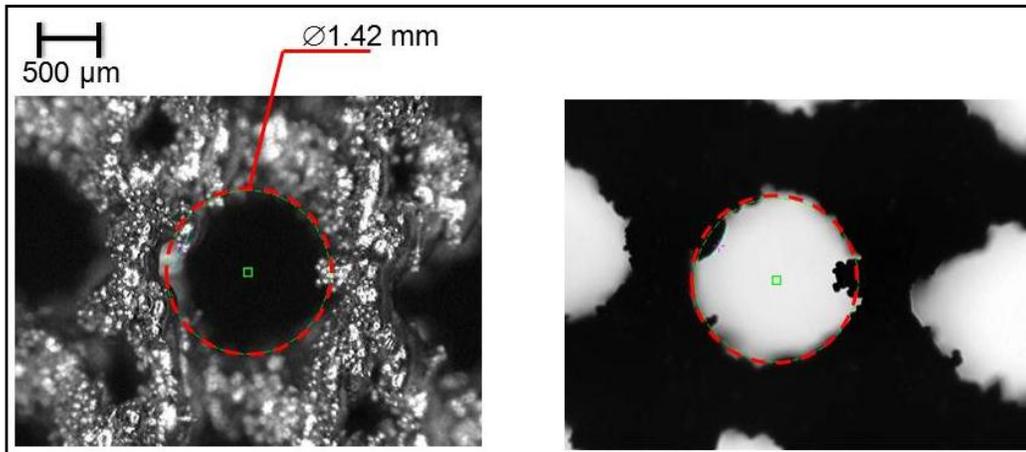


Figure 17: Micoview® snapshot of the lattice structure with axial front lighting (left), and back lighting (right)

Summary

The current work presents a brief overview of TOP that covers voxel-based and truss-based optimization and their potential application towards the production of lattice structures and FGP. Contemplating TOP as a numerical design tool completes the loop of digitizing the modern track of design and production of components where the potential of material performance reaches a more efficient position. TOP enables the design of features with highly complex geometrical aspects of porosities, where AM enables the production of such complex parts. These features play an essential role in improving the performance of biomedical implants, where the customization of implants matches the singularities found in every patient's case, contributing toward longer implantation life and the improvement of patient satisfaction and well-being.

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