

# **A DITHERING BASED METHOD TO GENERATE VARIABLE VOLUME LATTICE CELLS FOR ADDITIVE MANUFACTURING**

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## **Abstract**

This paper covers the principles of a novel method to efficiently spatially vary the size of tetrahedral cells of a lattice structure, based upon finite element analysis stress results. A dithering method, specifically error diffusion, is used to represent a grayscale stress fringe with variably spaced black dots. This enables linkage of the spacing between lattice cell vertices to stress level thereby providing a functional variation in cell density. This method is demonstrated with a simple test case in 2D and the steps involved for extension to 3D are described.

## **Introduction**

Lattice structures generally contain a large number of cells, each made up of several structural members. They can be used to control the stiffness of a structure or to provide tailored impact absorption capacity, usually through plastic deformation. Additive manufacturing (AM) is more suited to the manufacture of these complex structures than traditional manufacturing processes. The design of these structures, however, remains a challenge. With increased geometric complexity comes increased design complexity, and this is exacerbated when including computational analysis methods and mathematical optimization techniques in the design process. Some approaches focus on how to handle the geometric complexity problem while others focus on how to handle the analysis/optimization complexities. Both of these approaches should be unified to make a useful lattice design tool.

Different regions of a component generally require lattice support to different extents. With a fixed lattice, this variation can only be achieved by varying the structural member dimensions. This leads to a large number of design variables, depending on the resolution required. In these cases, the variation in support cannot be tuned by adjusting the sizes of the lattice cells and so the end result would be expected to be less than optimal due to the design freedom restrictions. Some approaches combine different cell structure designs together, which is difficult, and the cells are kept the same size [1]. Optimizing the size of each cell in a lattice is possible, but is computationally very expensive and awkward to implement, and so this approach is generally avoided. Some approaches have used different sized cells, not for structural performance reasons, but only as a by-product of requiring the lattice structure to conform to a shape with curved faces. Instead of trimming tessellated cells to this shape, they are either swept [2] or based upon an unstructured mesh [3]. Spatially varying the sizes of the lattice cells based upon structural analysis in an efficient manner is the topic of this paper.

The method in this paper uses a dithering or halftoning method to define spatially varying points which are subsequently connected with lattice members. The points determine the spatial

variation of the lattice cell volumes. This approach was inspired by previous work on linking dithering to meshing techniques by [6-9].

### Method

This section begins with a brief introduction to dithering before moving onto specific method details. Dithering is a procedure that converts continuous tone images into a binary representation. This is useful for bi-level printers and displays. When viewed from a certain distance, the binary representation appears similar to the continuous representation to the human eye. There are several dithering methods, which can be split into two categories: ordered dither and error diffusion [5]. Error diffusion was used for this work as there is better contrast performance and reduced ordered artifacts compared to ordered dithering. Error diffusion uses an adaptive algorithm based on a fixed threshold to produce a binary representation of the original input. Each pixel value is modified by minimizing errors caused by the thresholding at previous pixel locations. In this way the thresholding error is diffused to adjacent pixels, hence the name of the method. The method of error diffusion is outlined in the following steps for each pixel:

1. Threshold the value of the pixel of the original or modified image using a fixed threshold value.
2. Calculate the absolute error between this thresholded value and the original value.
3. Diffuse this error by modifying the value of adjacent pixels in the original image using a filter.
4. Repeat from step 1 until all pixels have been processed.

The actual proportions of the error diffused to adjacent pixels is determined heuristically and a typical filter for this in 2D is that proposed by Floyd and Steinberg [4] which is shown in Figure 1. This filter is passed over the image during step 3 listed above where  $x$  is the current pixel. The fractions of the error specified in the filter boxes are added to the original image pixel in step 4.

	$x$	7/16
3/16	5/16	1/16

Figure 1 – 2D filter proposed by Floyd and Steinberg for diffusing the binary thresholding error to adjacent pixels, where  $x$  is the current pixel.

3D error diffusion is less common due to fewer potential applications. This is a simple extension of the 2D process where the error is diffused in 3D space to voxels around the current voxel. Again, the proportions of the original error to diffuse are determined heuristically and the filter used in this case was that proposed by Lou and Stucki [5], which is shown in Figure 2.

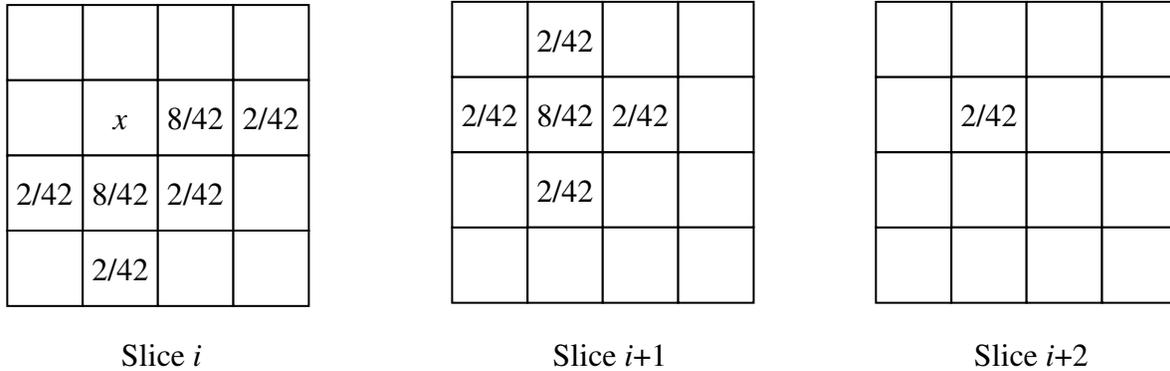


Figure 2 – 3D filter proposed by Lou and Stucki for diffusing the binary thresholding error to adjacent voxels in the current and adjacent slices, where  $x$  is the current voxel.

For this work, the above error diffusion methods were used to define lattice cells constructed from cylindrical members. This method is demonstrated first with the simple 2D cantilever plate problem shown in Figure 3a using the Floyd-Steinberg filter and is then extended to a 3D problem using the Lou-Stucki filter. The first stage is to conduct a finite element analysis (FEA) of the problem. This is used to calculate the stress or some other measure of the part's performance and the results can be plotted as an image as shown in Figure 3b. The handling of multiple problem load cases can be achieved by combining stress results from the individual analyses by taking the maximum for each pixel across the load cases. This correlates to taking the minimum grayscale level value for each pixel. This grayscale image is then subjected to an error diffusion algorithm as was explained earlier which generates the binary representation of variably spaced black dots on a white background shown in Figure 4. There are several issues that have to be addressed to make this method useful:

1. How to map the stress values to the grayscale fringe image?
2. How to reduce the effect of local boundary condition stress concentrations?
3. How to provide some local control over generated node spacing, i.e. min/max spacing?
4. How to control the quantity of generated nodes?

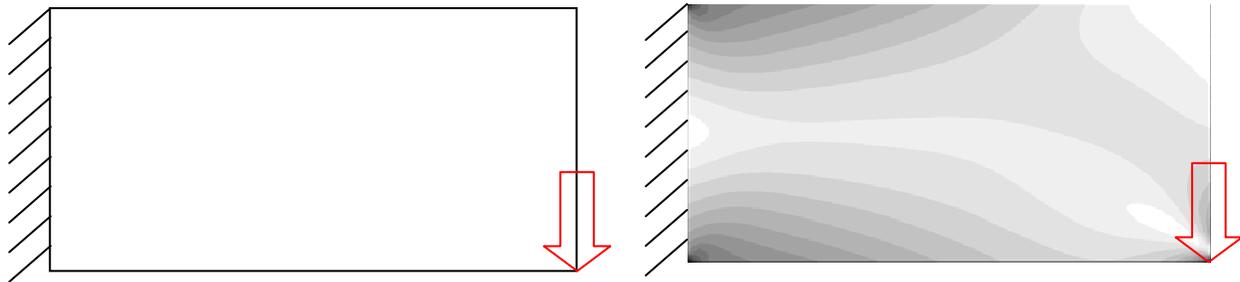


Figure 3 – a) Cantilever plate 2D test case problem and b) grayscale stress fringe result from FEA where darker grey = higher stress.

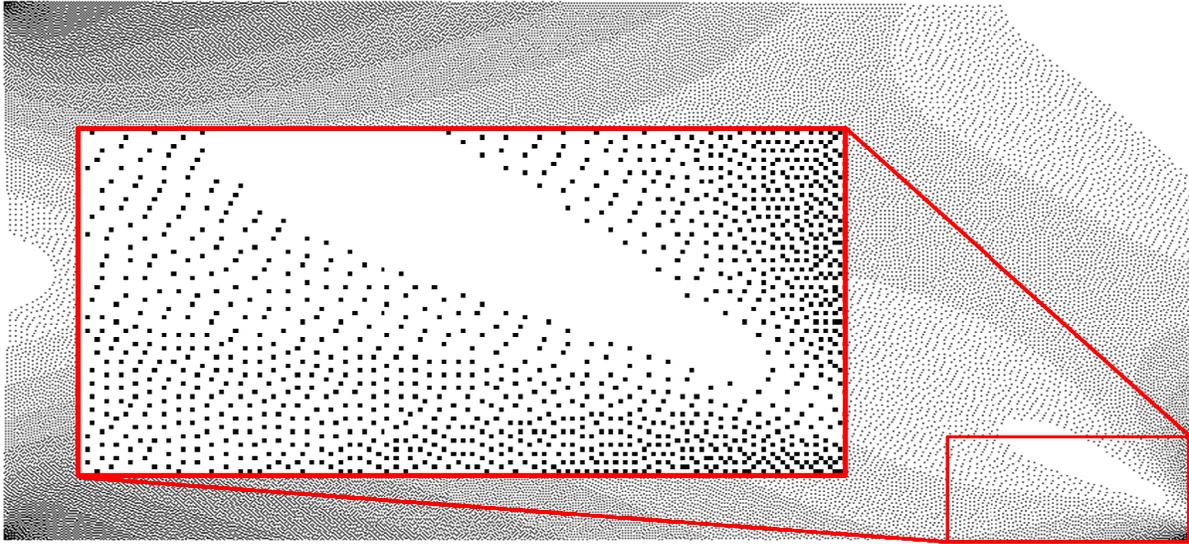


Figure 4 – Variably spaced black dots from error diffusion.

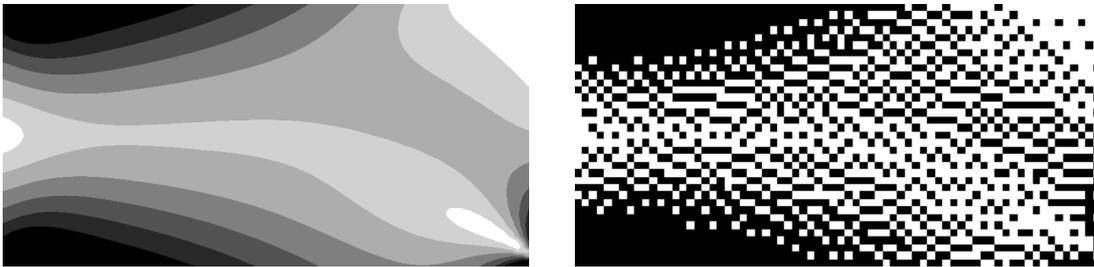


Figure 5 – a) Contrast modification to reduce skewing effect of stress concentrations at boundary condition application nodes, and b) Error diffused result for resized image.

The second and third of these issues are related to the first issue. Regarding issue two, due to the nature of the FE method, it is usual for the degrees of freedom of individual nodes to be constrained and for loads to be applied to individual nodes. This results in unrealistic values being calculated for these nodes which skews a linear mapping of stress values to grayscale values. This can be accommodated by modifying the mapping of the stress to grayscale values by using a simple image processing technique as shown in Figure 5a. In this stage, the contrast of the image is adjusted to reduce the upper limit of the stress value that is mapped to a grayscale value of zero (black).

Issue three is more involved and requires a quantitative link between the user requirements and the dithering method. The maximum and minimum sizes of the lattice cells can be affected by application requirement or because of manufacturing constraints. For example, in the case of the selective laser melting (SLM), the process will only currently self-support up to a certain maximum overhang horizontal distance. It would also not be desirable for the lattice cell sizes to be too small as there are minimum feature size limitations due to the powder size and laser beam diameter, and issues regarding powder removal from dense lattice regions. The resulting sizes of the lattice cells are dependent on the dithering method, specifically the size of

the input image and the grayscale values themselves within this image. The effect of these parameters on the resulting lattice cell sizes was investigated. Both issue two and three relate to the first issue in that all are modifications to the mapping between the analysis results and the dithered results. This paper focuses on how to adjust the mapping to ensure certain user requirements are satisfied.

Regarding issue four, the overall quantity of the generated dithered points can be controlled by simply modifying the size of the original image as this resolution determines the resolution of the resulting dithered image. A reduced size dithered result is shown in Figure 5b.

### Experiments

Some simple test image samples were generated of varying resolution and grayscale level as shown in Figure 6. The grayscale ranges from 0 to 255, but an upper limit of 250 was used as in a pure white region, no dithered points would be present. In total there were 28 samples and each of these constant grayscale level images was subjected to the 2D error diffusion method using the Floyd-Steinberg filter. The resulting dithered points were connected using a constrained Delaunay triangulation meshing technique and the areas of these elements calculated. Figure 7 shows the average element size for each sample plotted against the sample edge length, with fitted curves and their respective coefficients, calculated using MATLAB.

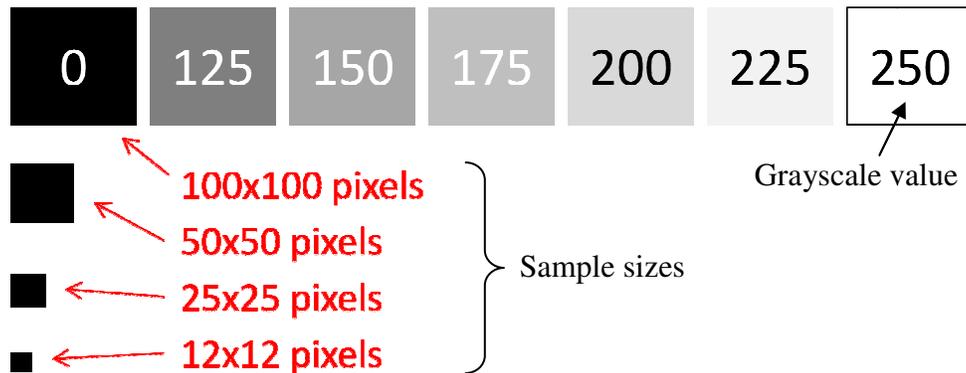
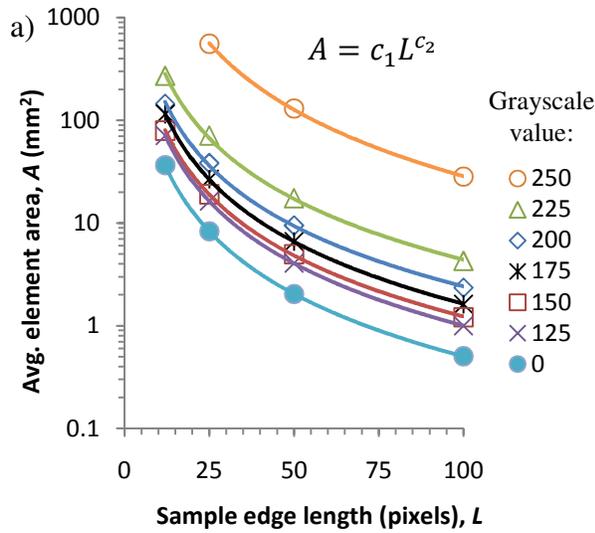
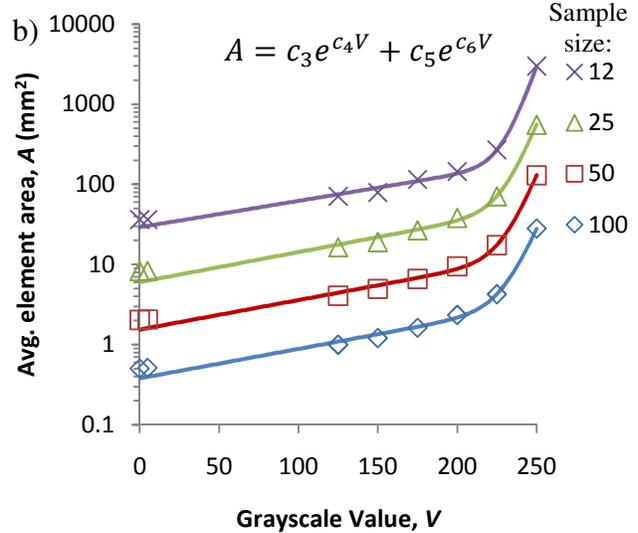


Figure 6 – Test samples used to investigate effect of image resolution and grayscale value on the dithered result.



Grayscale value, V	Coefficients	
	$c_1$	$c_2$
250	568605	-2.149
225	37080	-1.963
200	19257	-1.951
175	17102	-2.01
150	10768	-1.971
125	10550	-2.011
0	5515.9	-2.02



Sample size, L	Coefficients			
	$c_3$	$c_4$	$c_5$	$c_6$
12	29.21	7.575E-3	2.09E-11	0.1301
25	6.035	8.623E-3	2.19E-10	0.1139
50	1.543	8.47E-3	8.13E-11	0.112
100	0.3803	8.445E-3	6.62E-11	0.1066

Figure 7 – a) Effect of sample resolution on the average element size for each grayscale value, and b) Effect of grayscale value on the average element size for each sample size.

### Discussion

The resulting relationships shown in Figure 7 are useful for determining the suitable image size and grayscale value. For example, what is the image size required for an average element size of  $1\text{mm}^2$  in high stress (black) regions? This is a minimum element size requirement in the regions of grayscale value of 0 and so using the power law relationship fitted to the plotted results (equation 1) this was calculated to be approximately  $70 \times 70$  pixels.

$$y = 5515.9x^{-2.02} \quad (1)$$

where  $y$  is the average element area ( $\text{mm}^2$ ) and  $x$  is the sample edge length (pixels). So an average minimum element size can be predicted using this relationship, but this modification of the image size also affects the maximum element size. For the  $70 \times 70$  pixel image size, the maximum average element size was calculated using equation 2 to be approximately  $60\text{mm}^2$ .

$$y = 568605x^{-2.149} \quad (2)$$

If this size is too big then the only way to achieve a desired requirement is to modify the grayscale value of this region. This can be achieved using grayscale darkening where the value of the level is changed. The magnitude of the modification can be determined from the fitted relationships between the variables. For this example, the grayscale value required to achieve a maximum element size of  $10\text{mm}^2$  with an image size of  $70 \times 70$  pixels is 225. Therefore, the extent of the modification is -25. This value can be subtracted from the grayscale map to reduce the range of the levels. Because the lowest value of the range is 0, the smallest element size is unchanged. The desired extreme element sizes are therefore defined by modifying the image size for the minimum size and then modifying the grayscale level for the maximum size. Clearly, this could also be achieved in reverse. To validate that this approach translated onto a larger part, the simple test case image was modified based on the image size and darkening requirements. The resulting Delaunay triangular mesh is shown in Figure 8 and the minimum and maximum regions had average element sizes of  $1.08\text{mm}^2$  and  $9.75\text{mm}^2$ , respectively, compared with targets of  $1\text{mm}^2$  and  $10\text{mm}^2$ , respectively. It can be seen from the mesh that there are some heavily distorted elements in the low stress regions. This is due to the meshing constraint of only connecting up the dithered points to the boundary corner points. These distortions do not affect the quality of the FEA as these triangular elements are replaced by 1D beam elements at each edge. However, for buckling or aesthetic reasons, this constraint can be relaxed by adding additional elements to these regions which results in a more uniform mesh.

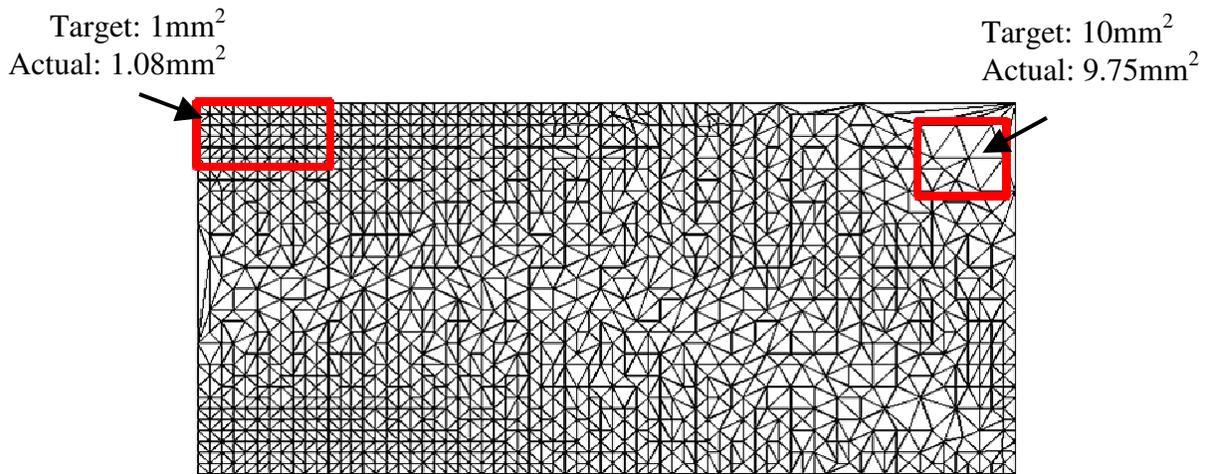


Figure 8 – Good confirmation of resulting minimum and maximum element size targets based on experimental sample results.

The end result of this process is a lattice structure that has smaller cells in regions of higher stress where more reinforcement is required and larger cells in regions of lower stress where less reinforcement is required. This is a spatially functionally graded lattice structure that is achieved efficiently as no iterative optimization cycles are required. To ensure that the lattice can withstand the loading requirements, the lattice is next subjected to an optimization of the lattice member dimensions. It is unfeasible to treat each member as a separate design variable, and ideally only a single, or a few variables would be required. The functional grading has been achieved by the dithering method so a single design variable will ensure that the lattice is strong and stiff enough. This will result in an efficient lattice design method.

This whole lattice generation method can be extended to 3D with a few modifications. Firstly, volumetric analysis results are required, which can be obtained from the FEA results by taking slices through the volume of the 3D part at the same resolution as the in plane dimensions. It is important that these resolutions are the same or the lattice will be distorted in a particular dimension. Secondly, as would be expected, the 3D error diffusion filter shown in Figure 2 should be used. Thirdly, a constrained 3D Delaunay mesher is required that can handle non-convex point envelopes and connect interior and boundary dithered points. One such mesher is Tetgen by Hang Si [10]. Fourthly, because the dithered points are distributed through the volume of the component with points not necessarily on the boundary faces, a method is needed to ensure mesh connectivity between lattice and solid non-design regions. This is an important consideration to ensure analyzability of the whole component. As shown in the flowchart in Figure 9, this can be achieved by separating the boundary face and volume points. The boundary points can then be used to generate a surface triangulation mesh to be used as a constraint for the mesher along with the interior points. In parallel, they can be projected onto the solid non-design regions of the component and used as constraints for a solid mesh. In this way, the meshes, when brought back together, can be ensured to be compatible.

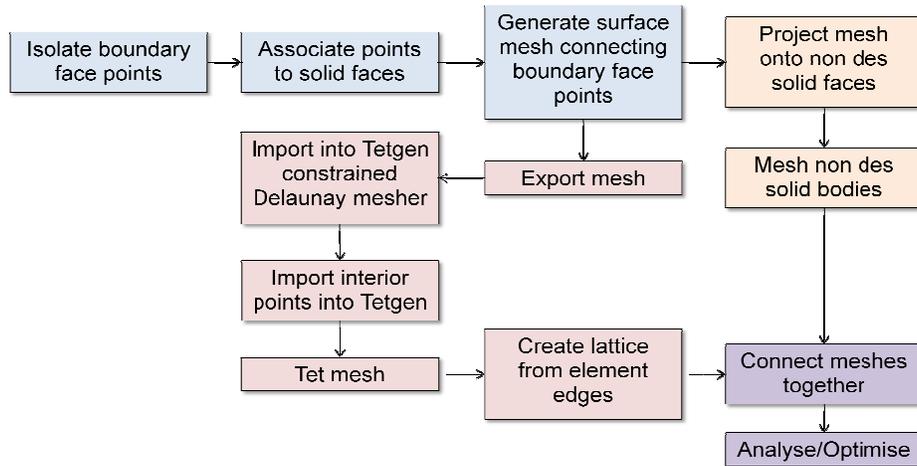


Figure 9 – Flowchart showing stages required to ensure mesh connectivity between the latticed region and the adjoining solid non-design regions.

### Conclusions

This paper has presented a novel method of generating a functionally graded lattice based on the results of finite element analysis. The spatial variation in the area/volume of the lattice cells is achieved using error diffusion to convert a continuous tone image into binary form. Some simple experiments were presented that allow the adjustment of the maximum and minimum lattice cell sizes based on determined parametric relationships. It was found that the image size was related to the lattice cell size by a power relationship,  $A = c_1 L^{c_2}$  where  $A$  is the lattice cell area,  $c_1$  and  $c_2$  are the coefficients and  $L$  is the image edge length. It was also found that the grayscale value was related to the cell size by a double exponential relationship,  $A = c_3 e^{c_4 V} + c_5 e^{c_6 V}$  where  $A$  is the lattice cell area,  $c_3$ ,  $c_4$ ,  $c_5$  and  $c_6$  are the coefficients, and  $V$  is the grayscale value. The use of these relationships was demonstrated with a simple test case example.

This method is efficient as it only requires a single FE analysis to be carried out per load case. It does not require iterative optimization algorithms to vary the cell size. This creates a spatial variation of the lattice which can be used as a basis for subsequent optimization of the lattice member dimensions. In some more complicated cases, it may be required that multiple FE analyses be used to improve reliability. Alternatively, an unpenalized variable density topology optimization method could be used prior to the dithering stage. The current method based on a single FE analysis was demonstrated with a 2D example and the modifications required for extension to 3D explained. Further work will be on determining a parametric relationship between the stress results and greyscale value map to more closely predict lattice element sizes for particular stress levels. This will enable the method to be applied to practical problems.

### **Acknowledgements**

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