performing DEM simulations or experiments to determine what values are appropriate for the packing density to be used in the simulation.

**Table 4: Comparison of packing densities of DEM and Geometric Methods**

| Author | Method Used | Results |
|---|---|---|
| Finney [18] | Experimental | Packing Density 0.636 (with particles of normal distribution) |
| Deng *et. al.* [7] | DEM | Packing Density 0.65 (with particles of singular diameter) |
| Siiria *et. al.* [10] | DEM | Highest Packing Density 0.55 (with particles of singular diameter) |
| Shi *et. al.* [19] | DEM | Stable Packing Density 0.578 (with particles of normal distribution) |
| Jerier *et. al.* [13] | Geometric | Packing Density 0.7-0.4 based on user inputs (with particles of normal distribution) |
| Mueller [14] | Geometric | Packing Density 0.59-0.37 depending on ratio of container height to particle diameter (with particles of singular diameter) |
| Han *et. al.* [16] | Geometric | Packing Density 0.5289 (with particles of normal distribution) |

Table 5 compares the coordination numbers of the various methods. When reviewing a paper such as [20], it becomes obvious that there are multiple different values which have been reported in the literature. This has led to different results based on several parameters, the largest factor is the material used. For this reason, the value of the DEM solutions are usually assumed to be a more accurate estimate of the actual coordination number due to the method of the powder bed generation. As can be seen from the data which is displayed in Table 5, the DEM solutions produce a coordination number which is slightly smaller than that of the geometric models. The difference in these simulations can be again attributed to the methods used to generate the results. In the DEM simulation, the model is a more realistic representation of the actual method of building a powder bed. This results in a bed which has a coordination number more closely aligned with reality. In the geometric method, the coordination number can be varied within the model by changing the initial parameters which determine when the program will end. R.M. German in [20] reports that there can be a wide variance in the reported values based on the material and other variables. Therefore, this parameter should be used as a decisive factor with caution.

**Table 5: Comparison of coordination number of DEM and Geometric Methods**

| Author | Method Used | Results |
|---|---|---|
| Deng *et. al.* [7] | DEM | Coordination Number of 4-6 varying on Particle diameter (10-1000um) |
| Shi *et. al.* [19] | DEM | Stable Packing Coordination Number 5.97 |
| Jerier *et. al.* [13] | Geometric | Average Coordination Number 6 |
| Jerier *et. al.* [17] | Geometric | Coordination Number 4.75-7.5 based on particle sizes |

Due to the uniqueness of each model, various sectors of research rely on each of the models differently. One area of study that relies heavily on DEM simulations is the concrete industry. This area of research is very interested in the stresses that are felt by the particles of the concrete which dictated the use of the DEM simulations to find the state of each particle. On the other hand, for the use in modeling of powder beds for additive manufacturing it is usually only necessary to obtain a proper density and coordination number for the powder bed. Therefore, in most cases, the geometric method would be the best option because it is computationally cheap and has a short time per simulation; which would allow for more time and computation power to be spent on modeling of the thermal processes which are taking place within the system. The main problem with the geometric models is they contain an induced coordination number. In the modeling of the powder bed additive manufacturing systems this is crucial because the coordination number is one of the key factors which dictate the flow of heat through the powder bed. If a powder bed has a higher coordination number then heat will flow faster through the system, since heat travels faster by conduction than convection. This problem can be mitigated by selecting a geometric model which mimics the coordination numbers which are reported in the literature from DEM simulations.

## Other Models

There are a few other models which are not as commonly used as DEM and the geometric method. These methods include, but are not limited to, the ballistic method powder bed generation, and the Monte Carlo simulation of powder beds.

The ballistic method for creating a powder bed can be found in [21], [22] and others. This model is a hybrid between the DEM and the geometric models. In this simulation a particle is selected and given a random radius, and x-y coordinates and placed high above the container. It is then moved down toward the powder bed until contact is made with either another particle or the floor. If contact is made with the floor the particle is considered at its final location. If it hits a particle then it is rotated about the place of contact until another contact is made. After this contact, the particle is rotated about these two points of contact until a third contact is made. This is then considered its final location. If the particle at any time contacts the floor or wall during these rotation it is then considered to be in its final location. This method allows for a much faster simulation than the DEM method and allows for a seemingly more random packing of the particles. Based on the literature presented in [22] and the results presented, this method provides coordination numbers and porosities which are comparable to DEM simulation, geometric simulations, and experimental results.

Another approach to creating a powder bed that has been shown in the literature [23] and [24] is the use of Monte Carlo algorithms. Just as all of the other methods for powder bed creation this method varies depending on the author. In general, this method begins with filling a domain with a specific number of particles. These particles are given a random direction and distance to move. This motion is considered valid if the particle does not come into contact with a wall or another particle. This is considered a Monte Carlo step due to its use of random number generators to find the path and length. This Monte Carlo step is then repeated a given number of times. After the Monte Carlo steps are completed, the minimum distance between spheres is found to be δ. The domain is then scaled down by a factor derived from δ and the simulation is run again. This is method is completed until a specific packing density is created or the packing density between steps does not change more than a given threshold.

## Conclusion

In general, all of these methods have been used to create powder beds which have been validated by experimental results. As was stated previously, the use of each method is dictated by the results which are desired from the simulation. The main factor which will determine the method used is the necessity of the contact forces. If these forces are required, then DEM is the only method which can be used. If these forces are not essential, then computationally it would be more efficient to use the geometric, ballistic, or Monte Carlo simulations. These last simulations would be the best for modeling of additive manufacturing powder bed systems due to their low computational time and accurate model. Overall, Table 2 can be used to help determine which method is the best for any given situation.

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