

## Integration of Voxel Based and Source Based Representation for Computer Aided Design of Functional Gradient Materials

Fangquan Wang<sup>1</sup>, Frank Liou<sup>1</sup>, and Todd Sparks<sup>1</sup>

<sup>1</sup>Department of Manufacturing Engineering, Missouri University of Science and Technology,  
Rolla, MO, 65401

### Abstract

Functionally Gradient Materials (FGMs) feature smooth transition from one material to another within a single object. FGMs modeling is considered to be one of the new challenges in Computer Aided Design area. To overcome this challenge, this paper presents the integration of a source-based and voxel-based approach to model FGMs. The input of STL format can be meshed and voxelized. The material composition and material varying information in each voxel can be generated from control sources. In addition, surface source is used to define default material information, and several filtering methods, including Gaussian filter, Average filter, are applied to blend FGMs for each voxel.

### Introduction

Modeling of FGMs is considered to be one of the challenges in the Computer Aided Design area. One of the limitations is that current CAD software only concerns the geometry information of the desired model, however it does not consider any material information. Additionally, there is no existing software for the users to manipulate material information inside models.

A significant amount of research has been done on modeling of FGMs objects. A variety of models are applied in the manipulation and representation of FGMs material distribution such as volume model, voxel model, control point model, implicit function model, explicit function model, etc. All of the FGMs models can be generalized into two categories, evaluated models and unevaluated models. Evaluated models are discretized representations of an object where each voxel contains material volume fraction information; however unevaluated models are function-based representations of an object where functions are used to calculate the material volume fraction composition [1].

Evaluated models present FGMs objects through dividing the model into small cells. There are mainly two types of methods used in evaluated models: voxel based model and volume mesh based model. In voxel based model, the spatial material distribution is represented in a uniform 3D voxel grid with each voxel indicating the type of material in this unit grid [2]. Volume mesh based model is similar to voxel model. The only difference is that it uses a collection of polyhedrons instead of spatial grids to represent 3D models [3]. Tetrahedron and hexahedrons are the most commonly used cell elements in volume mesh model.

However, unevaluated models represent material distribution of FGMs through analytical mathematical functions instead of spatial subdivision. As a result, unevaluated models are accurate

and precise regardless of resolution. There are many FGMs modeling methods based on unevaluated models. Analytical functional representation applies explicit functions to manipulate the material composition in each point inside the FGMs model in Cartesian coordinate [4]. Siu and Tan [5] proposed a control source representation method. Material composition is based on the distance between the control feature and the point. Moreover, multiple control feature model are applied. Multiple control features of different types are used as the material variation references to define material compositions. Liu [6] proposed a local composition control method by using Laplace equation based approach. Biswas [7] realized a field distant method using implicit source profiles to control material composition. Complex topologies of FGMs models can be represented by this method. Gupta and Tandon [8] create a convolution material based method. It takes the shape of models into control feature to generate the material composition by associated method.

This paper aims to utilize the integration source-based model and voxel-based model to present a new method to design and represent FGMs with the advantages of the both evaluated and unevaluated models. Multiple control source based method is used to develop unevaluated material distribution and voxel-based method is used to represent FGMs for the data format. Control source based method gives accurate calculation of the FGMs distribution. Voxel based method gives an efficient way to store and represent calculated data. In addition, surface source is used to define default material information, and several filtering methods, including Gaussian filter, average filter, are applied to blend FGMs for each voxel.

### Concept and Design Pattern

The design pattern is depicted in the flowchart as shown in Figure 1. At the offset, an STL model is produced from CAD software. The 3D STL model is then sliced into 2D layers along Cartesian axis with the prescribed resolution. First, at each layer, bounding box is identified which is used to contain the whole positions of voxel. Next, the voxel existence in each layer is calculated by Boolean operations of layer contour. Then, default reference features are then selected in the object. And then, FGMs distribution for the model is generated by applying composite methods. Finally, voxel based representation is shown in the last step.

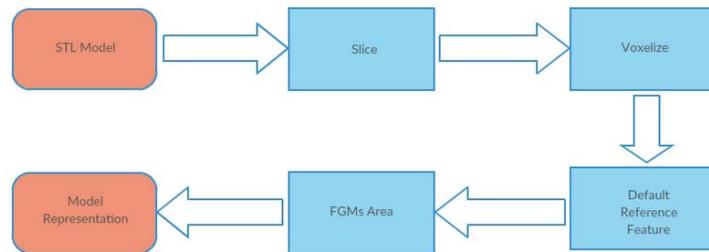


Figure 1. FGMs modeling design pattern

The first task is to model the geometry. Our method is using commercial CAD software to define the physical boundary of the object. The geometric data is exported from CAD software to FGMs modeling system using STL format. Vertex and triangle information are retrieved by analyzing STL file. Using vertex and triangle information, layer contour information is generated by the intersection of layer plane and triangle edges. The bounding box of the layer is identified to store voxels as shown in Figure 2.

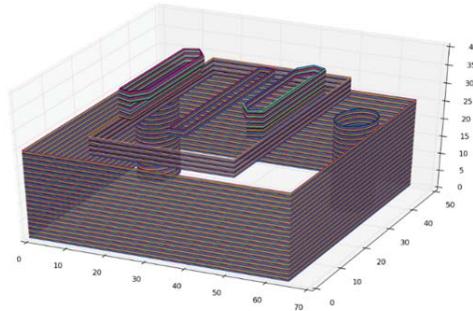


Figure 2. Layer contours of the object

The existence of voxel is determined by the binary operations of object contours. Binary value is assigned to each voxel according to whether it is included in odd times of contour or even times of contour. If the voxel is included in odd times of contour, true value would be assigned and if the voxel is included in even times of contour, false value would be assigned as shown in Figure 3.

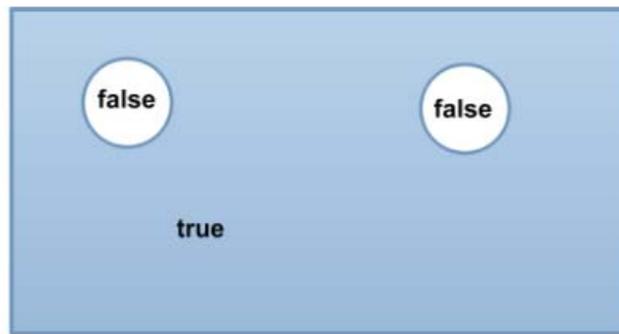


Figure 3. Binary representation of model layer

Models have one of three directions of layer orientations to be sliced which are x, y, z directions in Cartesian coordinate. Voxels in each layer are generated by utilizing the layer contour and Boolean value information. The computation efforts and representation accuracy depend on resolutions of voxels. Since the geometric model is three dimensional and resolution is one dimensional parameter, the computational time complexity could be defined as function of resolution:

$$T = CR^3 \quad (1)$$

Where T is computational time complexity, C is constant and R is resolution. The higher the resolution is, the more computation efforts will be and the more accurate for FGMs representation. Figure 4 shows the relationship of the computation of voxelization time versus resolution in FGMs modeling system.

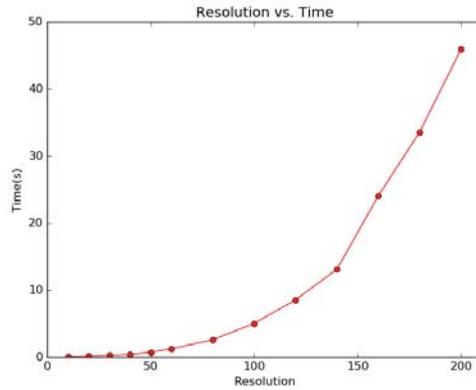


Figure 4. Computation of voxelization time vs. resolution in FGMs modeling system

### Control Source Modeling

#### FGMs Distribution along Axis

Many FGMs models have their gradients along slicing axis, so that each of the single layers would have the same material composition as shown in Figure 5:

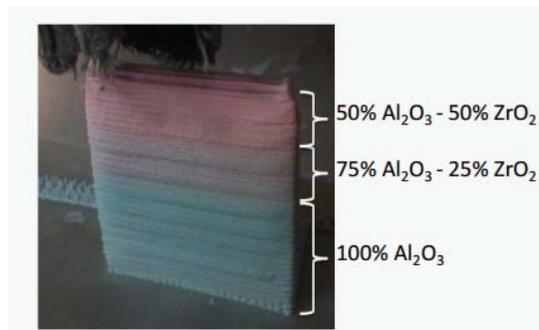


Figure 5. Ceramic gradient materials [9]

Along the slicing axis, the material distribution is defined as linear functions as shown in Figure 6:

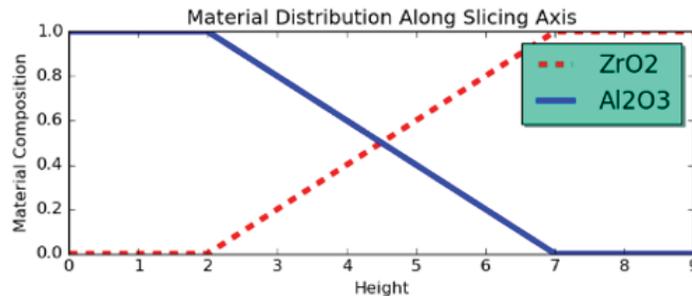


Figure 6.  $ZrO_2$  and  $Al_2O_3$  composition diagram along vertical direction

In our approach, the control sources are a pair of bottom and top surface, front and back surface, left and right surface. Using the linear distribution function in slicing direction, material

composition can be easily assigned which is developed from either pair of the surface. Since FGMs are developed along slicing axis, every voxel in the contour layer of the same height will be assigned the same composition value. Therefore, calculating each composition value for each voxel is easy to be implemented. Examples of models are shown in Figure 7.

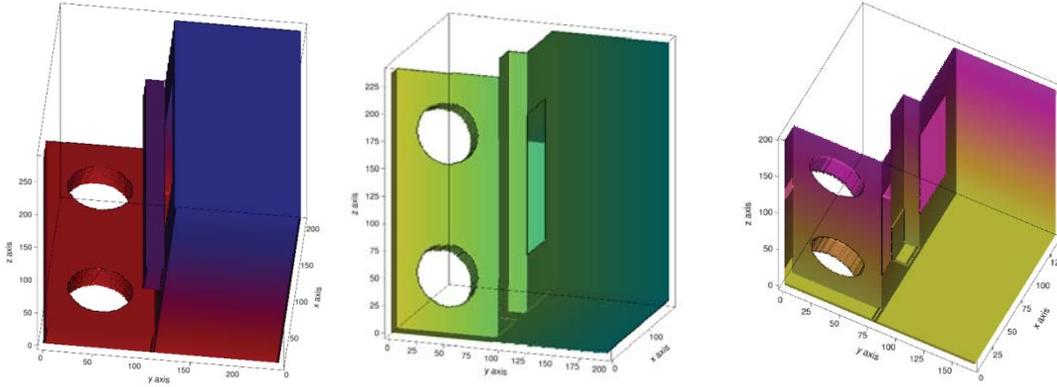


Figure 7. Linear Material Distribution for FGMs along x, y, z axis

### **Multiple Control Sources with Different Types**

Control source (S) is defined to be origin of material distribution. Each control source is assigned with one type of material, and each voxel in the model is influenced by the control source. In each voxel, materials have a set of composition weight values (C) controlled by the distance ( $d$ ) from the voxel point to the material control source. Several types of control sources are defined. For our approach, the three control sources used in the FGM modeling system are planar, point and line.

The material composition weight for each material at one voxel is function of  $\frac{1}{d}$ . There are two regions inside the model for material distribution. One voxel inside the model can fall into only one of the following regions according to the distance with each material control source:

A. Single material region: Only one material describes the region due to the voxel inside the corresponding material control source ( $m$ ). The distance between the material control source and voxel thus is equal to zero. Material composition for each material is  $C_i = 1$  for  $i = m$  and  $C_i = 0$  for  $i \neq m$ .

B. Multiple material region: None of the distance from control source to voxel is equal to zero.

$$C_i = \frac{\frac{1}{d_i}}{\frac{1}{d_1} + \frac{1}{d_2} + \dots + \frac{1}{d_{i-1}} + \frac{1}{d_i} + \frac{1}{d_{i+1}} + \dots + \frac{1}{d_n}} \quad (2)$$

Where  $n$  is the number of material control source. This equation can ensure the summation of control source weight is equal to 100 percent.

As shown in Figure 8 and 9, the material distribution is generated by planar, point and line source. Each control source is with one kind of material indicated by one color. The material composition can be generated by the same type of control sources or different types of control sources based on the distance between the control source and voxel.

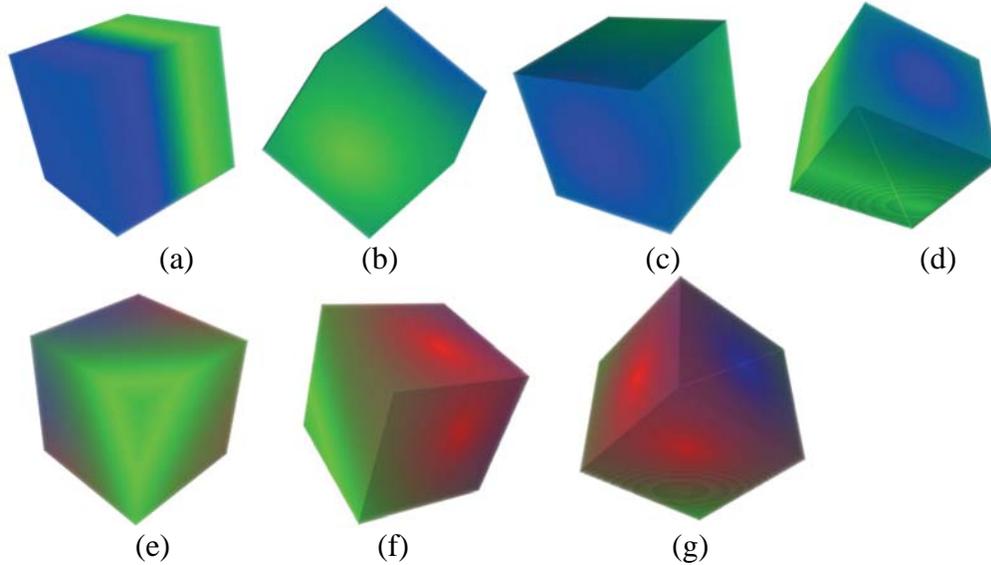


Figure 8. FGMs modeling using different control sources in cubic (a) two planar control sources (b) (c) Two point control sources. (d) Two different types of control sources: one planar and one point control source (e) (f) (g) Three different types of control sources: one planar, one point and one line control source.

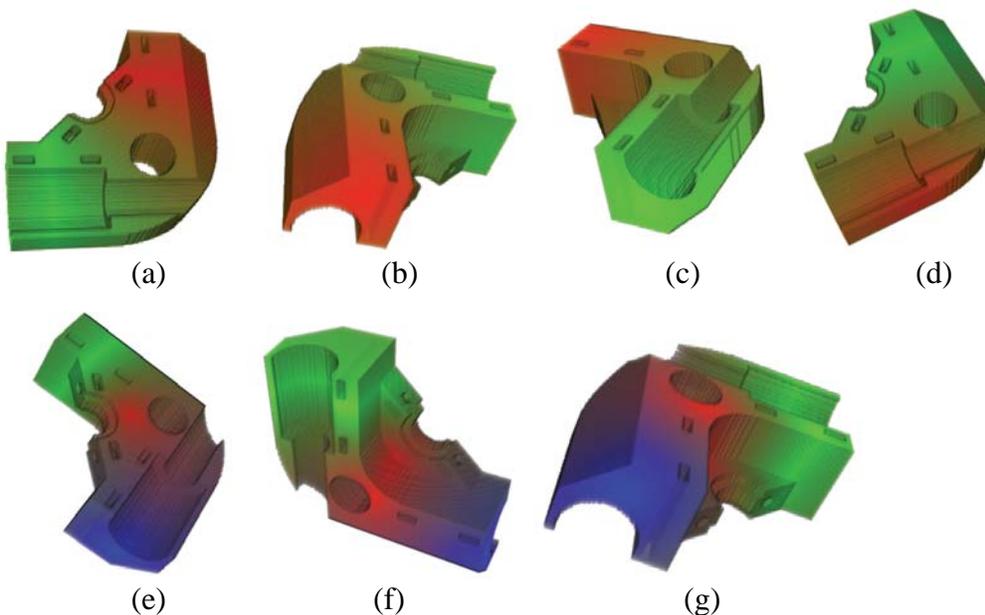


Figure 9. FGMs modeling using different control sources in a part(a) two planar control sources (b) (c) Two point control sources. (d) Two different types of control sources: one planar and one point source. (e) (f) (g) Three different types of control sources: one planar, one point and one line control source.

## LTI Filtering Method

In this section, the process of filtering method used to generate control source FGMs model is discussed. Filtering is the most basic operation in computer vision. In the broadest sense, the term “filtering” is a function of values in the vicinity of a given pixel to determine its final output value [10].

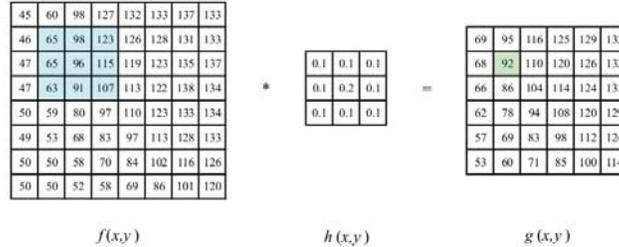


Figure 10. Two dimensional image filtering process [10]

The mathematical general filtering function can be defined as [10]:

$$g = f * h \tag{3}$$

Where  $g$  and  $f$  are new and original pixel value matrix, and  $h$  is filtering kernel. As shown in Figure 10, each pixel value in green multiplies the corresponding value in filter matrix to generate the target pixel value in green in the new pixel value matrix.

Filtering method is first introduced into FGMs modeling in this paper. Applying filtering in control source FGMs modeling would increase the flexibility of FGMs modeling. And the FGMs area can be controlled by the size of filtering kernel and parameters of filters. In the thesis, two linear translation-invariant (LTI) filters, Gaussian Filter and Average Filter are used to generate FGMs area.

### Gaussian Filter

Gaussian equation is three dimensional (3D) since the model is three dimensional. The weight of center voxel and the weights of neighboring voxel can be calculated in the following Gaussian equation:

$$f(x, y, z) = \frac{1}{(2\pi)^{\frac{3}{2}}\sigma^2} e^{-\frac{x^2+y^2+z^2}{3\sigma^2}} \tag{4}$$

Where  $\sigma$  is blur factor and  $e$  is euler number.  $x^2 + y^2 + z^2$  is the squared distance from the center voxel. When the distance increases, the weight for the corresponding voxel will decrease. Given a filter kernel size, the convolution matrix can be generated by the distribution value of the above Gaussian equation. In theory, the Gaussian distribution is non-zero throughout and would require a convolution kernel of infinitely large size, however kernel element values can be set to zero which are more than about three standard deviations from the

mean in practice, so that the kernel can be truncated to the given kernel size. A two dimensional integer-valued convolution kernel that approximates Gaussian function with  $\sigma$  of 1.0 is shown in Figure 11.

	1	4	7	4	1
	4	16	26	16	4
$\frac{1}{273}$	7	26	41	26	7
	4	16	26	16	4
	1	4	7	4	1

Figure 11. Two dimensional Gaussian kernel with approximation of  $\sigma$  of 1.0 [10]

This matrix is applied to the voxel of control source defined default material. Each new voxel value can be calculated by the weighted matrix multiplying neighboring voxel value. Center voxel is the highest weight and the neighbor pixels use weights which are proportional to the distance. In the process of modeling, default materials defined by control source in voxel models is first selected by control source method as shown in Figure 12(a), then the filter parameters is acquired to defined Gaussian Filter. The kernel matrix is applied to each voxel in the object for obtaining the FGMs model by multiplying center and its neighboring voxels with kernel matrix.

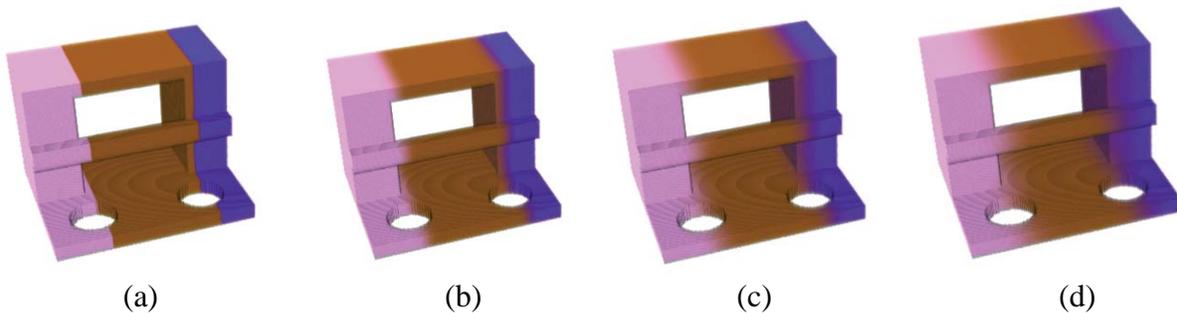


Figure 12. (a) Original default material reference feature defined, voxel resolution at  $100 \times 144 \times 122$  (b) Gaussian filtering with  $\sigma = 5$  and kernel size = 11 (c) Gaussian filtering with  $\sigma = 5$  and kernel size = 21 (d) Gaussian filtering with  $\sigma = 11$  and kernel size = 21

### Average Filter

Second LTI used is Average Filter. Average filtering kernel is much simpler than Gaussian filtering kernel. Each value in the filtering matrix is of equal value as shown in Figure 13. The summation of all the elements in the matrix is one. The neighboring voxels will have more influences on the center voxel, thus will result in a more sparsely distributed FGMs areas.

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

Figure 13. Example of average filter with size of 3×3

Average Filter is applied as the same strategies of Gaussian Filter on to the model object. Default materials are first defined by control source method in the model, and later, Average Filter will be applied on every voxel to recalculate the material composition.

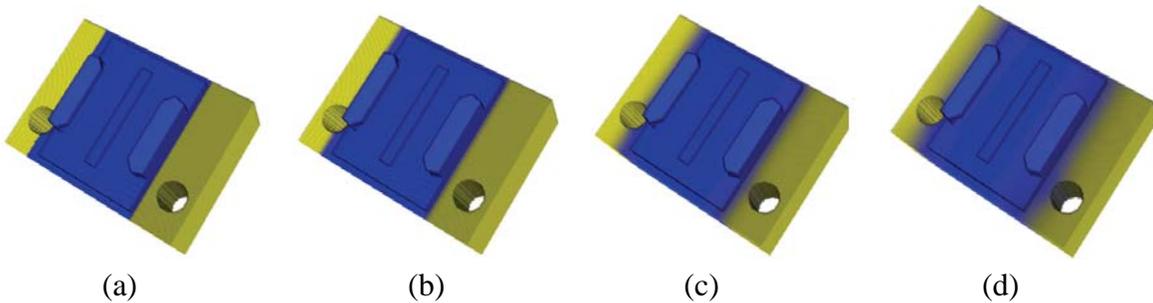


Figure 14. (a) Original default model, voxel resolution at 100×175×135 (b) Average filtering with kernel size = 5 (c) Average filtering with kernel size = 15 (d) Average filtering with kernel size = 25

### Conclusion

A new approach with the composite methods utilizing control source based and voxel based models to manipulate and represent FGMs is presented. Control sources are features introduced to develop FGMs. Different types of control sources can be integrated to develop functionally gradient materials based on distance from control sources. Control sources increase the flexibility of FGMs modeling with the voxel dataset structure. Moreover, although voxels in nature are discrete points in model which will provide inaccurate representation of FGMs, the control source based method to generate voxel information is evaluated, so that FGMs represented in this method can give an accurate representation of FGMs model.

Linear translation-invariant (LTI) filters method is first introduced in FGMs modeling. Applying image filtering method in FGMs modeling in nature improves controlling ability of FGMs areas.

### Reference

- [1] Garland, A., Mocko, G., & Fadel, G., "Challenges in Designing and Manufacturing Fully Optimized Functional Gradient Material Objects." *ASME 2014 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*. American Society of Mechanical Engineers, 2014.

- [2] Chen, M., & Tucker, J. V., "Constructive volume geometry." In *Computer Graphics Forum*, vol. 19, no. 4, pp. 281-293. Blackwell Publishers, 2000.
- [3] Jackson, T. R., "Analysis of functionally graded material object representation methods." PhD diss., Massachusetts Institute of Technology, 2000.
- [4] Zhu, F., "Visualized CAD modeling and layered manufacturing modeling for components made of a multiphase perfect material." PhD diss., University of Hong Kong, 2004.
- [5] Siu, Y.K., & Tan, S.T., "'Source-based' heterogeneous solid modeling." *Computer-Aided Design* 34, no. 1 (2002): 41-55.
- [6] Liu, H., Maekawa, T., Patrikalakis, N.M., Sachs, E.M., & Cho, W., "Methods for feature-based design of heterogeneous solids." *Computer-Aided Design* 36.12 (2004): 1141-1159.
- [7] Biswas, A., Shapiro, V., & Tsukanov, I., "Heterogeneous material modeling with distance fields." *Computer Aided Geometric Design* 21, no. 3 (2004): 215-242.
- [8] Gupta, V., & Tandon, P., "Heterogeneous object modeling with material convolution surfaces." *Computer-Aided Design* 62 (2015): 236-247.
- [9] Leu, M. C., Tang, L., Deuser, B., Landers, R. G., Hilmas, G. E., Zhang, S., & Watts, J., "Freeze-form extrusion fabrication of composite structures." In *Proceedings of the Solid Freeform Fabrication Symposium. Austin, TX*, pp. 111-124. 2011.
- [10] Szeliski, R., *Computer vision: algorithms and applications*. Springer Science & Business Media, 2010.