

Topology Optimization for Anisotropic Thermomechanical Design in Additive Manufacturing

J. S. Ramsey, D. E. Smith

Department of Mechanical Engineering, Baylor University 76706

Abstract

Topology optimization has emerged as an effective design approach that obtains complex geometries suitable for additive manufacturing. However, additively manufactured structures typically have anisotropic material properties, and residual thermal stresses result from nonisothermal processes. This paper presents a new topology optimization-based approach that incorporates both material anisotropy and weakly coupled thermomechanical loading into the material layout computations. An optimality criterion-based update scheme minimizes compliance or strain energy of a design space over material density and orientation where special attention is given to the optimal material orientation computations. The coupled thermomechanical analysis and material direction optimization reflects the anisotropic Young's modulus and thermal stresses present in large-scale polymer deposition. Resultant structures show how thermal loading influences the optimal topology, and how different penalty values determine convergence of the design.

Introduction

Motivation

Additive manufacturing has transitioned from a method for rapid prototyping to a growing manufacturing technique for end-use parts within the past decade. Although early additive manufacturing systems were confined to prototyping applications due to part weakness and print inaccuracy, new developments in material strength and print reliability are overcoming these flaws. The ability to add design complexity for little extra cost opens enormous design freedom, which is extremely useful for lightweight high-strength applications that often require complex geometries.

The traditional part design methods do not fully utilize the design freedom present in additive manufacturing. New methods are under development to optimize the design of parts while considering the near-arbitrary complexity now permitted. Topology optimization methods can model this level of design freedom and can be developed to reflect the unique factors present in the additive manufacturing process.

The topology optimization algorithm presented here considers the design of a two-dimensional anisotropic structure with continuous spatially varying material properties under weakly coupled steady-state thermomechanical loading. The additive manufacturing process is simulated by the anisotropic material properties and thermal loading. The optimality criterion method has been modified to solve the constrained minimization problem over material density and orientation to minimize either compliance or strain energy. Design sensitivities were determined with the adjoint method for both objective functions and the method was implemented in a custom finite element program in Matlab based on Sigmund's 99-line topology

optimization code [1]. The results can be manufactured with existing technology in proof-of-concept parts.

History of Topology Optimization

Topology optimization encompasses a family of optimization methods designed to find the geometry of a design space with the highest performance. The term was first used by Bendsoe and Kikuchi in 1988 [2] to minimize the structural compliance of a two-dimensional part under linear elastic loading by using the homogenization method. Other methods have been developed since, including the optimality criterion method [1], the level-set method [3], and the method of moving asymptotes [4]. These have all been applied to the structural compliance problem, but they have also been used for a diverse set of problems including thermal performance, frequency of vibration, compliant mechanisms, electromechanical systems, anisotropic materials, and multiphase materials. These cases and more are discussed in Bendsoe and Sigmund's book on the subject [5].

Considering anisotropic materials is important due to its application in additive manufacturing. Bendsoe and Sigmund's original work allowed for this by adding rectangular holes of various dimensions and orientation to the part under consideration. More recent work by Hoglund [6] modified a density-based approach to optimize the compliance of anisotropic materials for use in additive manufacturing. This work was extended to three-dimensional parts by Jiang [7], [8], who sought to model big-area additive manufacturing specifically. Their use of a general Matlab optimizer demonstrated that the method is viable and allows it to be used with a variety of different topology optimization methods. A similar technique was used by Luo and Gea [9] in two-dimensional problems.

The thermal response of the material during the additive manufacturing process also significantly affects part performance, as fused-filament fabrication and selective-laser sintering require high localized heating. Isothermal weakly coupled systems thermomechanical systems have been considered, but design-dependent thermal loading has, to the best of the author's knowledge, not been considered for compliance or strain energy optimization. Deaton [10] considered a constant temperature increase in aircraft fuselages and optimized their performance. Pedersen and Pedersen indicated that optimizing compliance can result in a higher maximum von mises stress than optimizing strain energy. This was further supported by Neiford et al [13], who also considered maximum displacement as an objective function.

The parameters present in additive manufacturing have been partially modeled in topology optimization. Material anisotropy has been considered in mechanical compliance optimization but has not yet been expanded to coupled thermomechanical systems. The weakly coupled thermomechanical systems have been modeled for given temperature fields, but not for design-dependent thermal properties. The purpose of this work is to build on the progress already made by modeling design-dependent thermal loading of anisotropic material.

Relevant Developments in Additive Manufacturing

Topology optimized parts often incorporate complex geometries that can only be produced with additive manufacturing. The recent development of high-strength techniques like

fiber reinforcement and expansion to large scale systems allow topology optimization to be applied to new industries. These innovations, while accurately producing end-use parts, can exacerbate the manufacturing challenges inherent in additive manufacturing.

Polymer composite additive manufacturing systems are widely used due to their versatility, ease of use, and cost effectiveness. In techniques such as fused-filament fabrication or screw extrusion deposition, the polymer is melted and then deposited onto the part. The flow of the molten polymer can align molecular strands in the direction of flow, which increases the stiffness of the deposited polymer in the direction of deposition while weakening the stiffness perpendicular to that. The polymer is usually deposited in beads, and imperfect bonding between these beads further decreases elastic stiffness perpendicular to deposition. This anisotropy can cause premature part failure, but the deposition direction can be altered to improve performance.

Adding short chopped fibers, usually of carbon or glass fiber, to a polymer matrix can increase the stiffness and tensile strength of an additively manufactured part. The elastic modulus can double [14] in the direction of deposition, but the fibers align in a similar manner to the polymer strands above. The increase in performance is thus primarily in the direction of extrusion, enhancing the anisotropy. Altering the direction of deposition can drastically affect the performance of fiber-reinforced composite components.

The thermal behavior of the manufacturing process is also complicated by incorporating reinforcement fibers. The carbon and glass fibers usually have a lower coefficient of thermal expansion than the surrounding polymer matrix and can reduce the thermal stresses in that direction of the fibers. The thermal behavior of the manufacturing process is already anisotropic, as layers are applied and cool sequentially, creating complex stress fields primarily perpendicular to the build plane. These thermal stresses are present once the part has been manufactured and can thus affect its behavior under loading.

The anisotropic material properties and complex thermal loading that arise in additively manufactured parts can significantly complicate the design process. Topology optimization methods are beginning to investigate these aspects and include them in the part design. The material location and orientation of deposition must be considered simultaneously and optimized to best model the anisotropic deposition process, particularly when fiber reinforcement is considered. The thermal cooling process must also be included as a weakly coupled thermomechanical analysis where thermal stresses generate mechanical deformations. The method presented here modifies and extends topology optimization methods for anisotropy and coupled thermal behavior to model both the manufacturing process and the end use of the part.

Methodology

Design Domain Definition

The algorithm presented in this work considers a single part defined in a two-dimensional design space. The design space is discretized into square linear finite elements with identical dimensions, and the constituent equations for linear thermal conduction and linear elasticity are

discretized across the part in the usual manner. Here, the cooling process is modeled by steady state thermal conduction under non-isotropic material properties. It is understood that this does not accurately represent the full complexity of the cooling process, but it is a first step toward incorporating the coupled thermomechanical behavior. The thermal analysis is weakly coupled to the mechanical analysis that represents part use via thermal stresses.

In this model each finite element has two design variables. The density x_i represents the amount of material in element i . It is bounded between $x_{min} = 0.001$ and 1, with $x_i = 1$ representing an element with material and $x_i = 0.001$ representing an empty element. Values of x_i between x_{min} and 1 signify a fractional density which lacks a physical interpretation but is permissible in the algorithm. The lower limit x_{min} is chosen to be small but nonzero to prevent singularities.

The second design variable is the element orientation angle θ_i and it represents the direction of the anisotropic material properties of each element. The material is assumed to be stiffer in the direction of the element orientation and weaker perpendicular to that. It is not bounded, but the element orientation is considered *mod* π so that it can be represented between 0 and π . For fused-filament fabrication, the element orientation corresponds to the direction of extrusion, with an orientation of zero corresponding with an element oriented in the global x -direction. The elemental stiffness matrix K_{Th_i} can be written in terms of the design variables as

$$K_{Th_i} = x_i^{p_{Th}} \int_{\Omega_i} B_{Th}^T R^T(\theta_i) D_{Th} R(\theta_i) B_{Th} d\Omega_i \quad (1)$$

Here, the Th subscript indicates the thermal analysis, D_{Th} is the anisotropic thermal elasticity matrix, and B_{Th} is temperature gradient matrix. The constant p_{Th} is a density penalty parameter used below in the optimality criterion method. The mechanical elemental stiffness matrix is constructed similarly [6].

$$K_{M_i} = x_i^{p_m} \int_{\Omega_i} B_M^T R^T(\theta_i) D_M R(\theta_i) B_M d\Omega_i \quad (2)$$

The M subscript indicates the mechanical analysis and B_M is the displacement gradient matrix. The mechanical penalty constant p_m is a similar constant to the thermal penalty constant p_{Th} , but the two need not be equal. The anisotropic elasticity matrix D_M can be written in terms of the anisotropic Young's Moduli and Poisson's ratios [6].

$$D_M = \begin{bmatrix} \frac{E_x}{1-\nu_{xy}\nu_{yx}} & \frac{\nu_{xy}E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ \frac{\nu_{xy}E_y}{1-\nu_{xy}\nu_{yx}} & \frac{E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \quad (3)$$

The standard rotational tensor $R(\theta_i)$ is only dependent on the element orientation [6].

$$R(\theta_i) = \begin{bmatrix} \cos^2(\theta_i) & \sin^2(\theta_i) & -2 * \sin(\theta_i) * \cos(\theta_i) \\ \sin^2(\theta_i) & \cos^2(\theta_i) & 2 * \sin(\theta_i) * \cos(\theta_i) \\ \sin(\theta_i) * \cos(\theta_i) & -\sin(\theta_i) * \cos(\theta_i) & \cos^2(\theta_i) - \sin^2(\theta_i) \end{bmatrix} \quad (4)$$

The thermal analysis is performed first, which solves the thermal equilibrium to determine the nodal temperatures. These are then used to determine the thermal stresses on each element, which are summed and included in the mechanical analysis. This couples the two models weakly. The thermal load within element i is evaluated with [15].

$$F_{TF_i} = x_{m_i}^p \int_{\Omega_i} B_M^T D_M R^T(\theta_i) \begin{Bmatrix} \alpha_x \\ \alpha_y \\ 0 \end{Bmatrix} R(\theta_i) N^T T_i d\Omega_i \quad (5)$$

The nodal temperatures for element i are denoted T_i , and N is the thermal element shape function. Note that the coefficient of thermal expansion is also anisotropic, with α_x and α_y the values parallel and perpendicular to the element orientation, respectively. The mechanical analysis is then performed to determine the nodal displacements.

The global finite element equations satisfy the linear equilibrium $K_{Th}T = F_{Th}$ and $KU = F$ for the thermal and mechanical systems respectively, with K_{Th} the global thermal stiffness matrix, T the global temperature vector, F_{Th} the global thermal flux vector, K the global mechanical stiffness matrix, U the global displacement vector, and F the global mechanical force vector. The displacement due only to the thermal loading can be written as $KU_{TF} = F_{TF}$.

Optimization

The purpose of this algorithm is to maximize the stiffness of the part in the design domain. The design variables are the element densities x_i and the element orientations θ_i . This can be written as a minimization problem, subject to a volume constraint.

$$\text{minimize } f(\bar{x}, \bar{\theta}) \quad (6)$$

$$\text{subject to } \frac{\sum x_i}{N_{el}} \leq V_f$$

This is written for a general optimization function $f(\bar{x}, \bar{\theta})$ because, under thermal and mechanical loading, there are multiple ways to define stiffness and its inverse. The constant N_{el} is the number of elements, and V_f is a constant between 0 and 1 that defines what fraction of the design domain should be filled with material.

Two objective functions are considered here. The first, compliance, is traditionally used as the inverse of stiffness in purely mechanical topology optimization [2]. It can be written in terms of the displacement and force, which are themselves functions of the design variables

$$C(\bar{x}, \bar{\theta}) = U(\bar{x}, \bar{\theta})^T F(\bar{x}, \bar{\theta}) \quad (7)$$

This is regarded as the inverse of stiffness for purely mechanical problems, so minimizing compliance can maximize stiffness. However, in weakly coupled thermomechanical systems the strain energy has been proposed as an alternate objective function that may generate parts with lower maximum stresses [12]. Strain energy can be expressed as

$$S(\bar{\mathbf{x}}, \bar{\boldsymbol{\theta}}) = \frac{1}{2} U(\bar{\mathbf{x}}, \bar{\boldsymbol{\theta}})^T F(\bar{\mathbf{x}}, \bar{\boldsymbol{\theta}}) - U(\bar{\mathbf{x}}, \bar{\boldsymbol{\theta}})^T F_{Th}(\bar{\mathbf{x}}, \bar{\boldsymbol{\theta}}) + \frac{1}{2} U_{Th}^T(\bar{\mathbf{x}}, \bar{\boldsymbol{\theta}}) F_{Th}(\bar{\mathbf{x}}, \bar{\boldsymbol{\theta}}) \quad (8)$$

Recall that simple bounds are applied to the design variables. The densities are constrained to be between $x_{min} = 0.001$ and 1, and the element orientations are bounded between 0 and π , but they are considered modulo π and no must be enforced.

In order for the resultant topology to have physical meaning, it is desired that the element densities approach one of the bounds, that is each element approaches either void ($x_i = x_{min}$) or solid ($x_i = 1$). The penalty parameters p_m and p_{Th} , used in the elemental stiffness matrix and thermal stress formulations, causes elements with fractional densities to be considered artificially weaker, thus encouraging the densities to trend toward the extremes. This penalty method is widely used and is known as SIMP (Solid Isotropic Material with Penalization) when $p_m = p_{Th}$, although other penalty methods can be used. Traditionally both penalty parameters have been set to 3 for two-dimensional problems.

Optimality Criterion Update Method

The problem formulation above could be solved with several different optimization algorithms. This work uses the optimality criterion-based method popularized by Sigmund [1], but other common methods like Dr. Svanberg's Globally Convergent Method of Moving Asymptotes (GCMMA) [4] could easily be substituted. The optimality criterion method was chosen for density optimization because of its per-iteration efficiency and rapid convergence. Under this method, the densities are updated according to [1]:

$$x_i^{new} = \begin{cases} \max(x_{min}, x_i - m), & \text{if } x_i B_i^\eta \leq \max(x_{min}, x_i - m) \\ \min(1, x_i + m), & \text{if } x_i B_i^\eta \geq \min(1, x_i + m) \\ x_i B_i^\eta, & \text{else} \end{cases} \quad (9)$$

The move limit m is set to 0.2 in Sigmund's formulation for compliance optimization of pure mechanical systems, but in this work it is set to 0.05 as the systems are more complex. The cases preserve the move limit and bounds on the densities. The term B_i is defined in terms of the design sensitivities.

$$B_i = \frac{-\frac{df}{dx_i}}{\lambda} \quad (10)$$

The Lagrange multiplier λ is constant for all elements in one iteration and is chosen by a bisection algorithm to preserve the volume fraction constraint. The design sensitivities must be evaluated for the objective function, whether it is compliance or strain energy. A damping constant $\eta = 0.5$ is used to stabilize the convergence, but it is a heuristic parameter and it can be omitted by setting it to 1 the formulation.

The optimality criterion method is a heuristic method designed for updating the densities and is formulated specifically for that problem. The element orientations must them be updated via a different method. Here, each element is considered independently, and the element

orientation is treated as the sole design variable in a sub-optimization routine that uses the Newton-Raphson method to optimize the objective function over that individual element, with the nodal displacements treated as constant. This determines the optimal angle for each element $\theta_{i_{min}}$, which is used to update the element orientations by moving them toward their optimal orientations. Note that, since the element orientations are considered modulo π , no boundaries need be considered, but a move limit L_i is used.

$$\theta_i^{new} = \begin{cases} \theta_i - L_i, & \theta_{i_{min}} < \theta_i - L_i \\ \theta_{i_{min}}, & abs(\theta_{i_{min}} - \theta_i) \leq L_i \\ \theta_i + L_i, & \theta_{i_{min}} > \theta_i + L_i \end{cases} \quad (11)$$

The move limit is defined as a fraction of the distance between each element orientation and the corresponding optimal orientation. Optimizations using a constant move limit have demonstrated convergence issues, but this method allows for smoother convergence.

$$L_i = 0.1 * |\theta_i - \theta_{i_{min}}| \quad (12)$$

Design Sensitivity Computations

The optimality criterion update method, like most other update methods used for topology optimization, requires the derivatives of the objective function in terms of the design variables for operation. These can be determined using the formulation in Equations 7 and 8 and the adjoint variable method [16] for weakly coupled systems to yield

$$\frac{dC}{dx_i} = -p_m x_i^{p_m-1} U_i^T K_i U_i + 2U_i^T \frac{\partial F_{th}}{\partial x_i} - 2p_{Th} x_i^{p_{Th}-1} \left(\frac{\partial F_{th}}{\partial T} K_t^{-1} \right)_i K_{t_i} T_i \quad (13)$$

The coupled effects of density on temperature and displacement generate a more complex design sensitivity than in the case of pure mechanical loads [1]. The expression above can be computed directly from the displacements and temperatures, which avoids the potential unreliability and computational burden of using finite difference calculations. The strain energy sensitivity formulation is computed similarly.

$$\frac{dS}{dx_i} = p_m x_i^{p_m-1} \left(-\frac{1}{2} U_i^T K_i U_i + U_{th_i}^T K_i U - \frac{1}{2} U_{th_i}^T K_i U_{th_i} \right) \quad (14)$$

The optimality criterion method was designed for compliance minimization under mechanical loads. In that scenario, the design derivatives are all negative, as increasing the density of any element would increase the stiffness of the structure. This is not always the case in thermomechanical loading. It is possible, under high thermal loading that increasing the density of an element can increase the temperature gradient across it and thus decrease the overall stiffness of the structure. The design derivative of compliance can be positive in the cases considered here. However, the optimality criterion method can only operate on negative sensitivities. To circumvent this, the positive sensitivities are mapped to the negative real numbers (including zero).

$$\frac{dC}{dx_i}|_{new} = \begin{cases} \frac{dC}{dx_i} & , \quad \frac{dC}{dx_i} < 0 \\ 0 & , \quad \frac{dC}{dx_i} \geq 0 \end{cases} \quad (15)$$

Several different potential mappings exist. All must map the positive sensitivities to negative values while preserving their ordering relative to each other and the other sensitivities. While this method demonstrates convergence, it is an additional modification to an already heuristic update scheme. The authors intend to investigate other update methods in the future to determine a method that better suits the parameters of the problem, but initial testing suggests that simply setting all positive sensitivities to zero is sufficient in most cases.

Results

The method was applied to optimize the performance of a beam in three-point bending (the MBB beam). A two-dimensional design domain was defined with the dimensions below. Note that the domain takes advantage of the symmetry present in the MBB beam and models only the right half. A symmetry condition is applied to the left edge in both the thermal and mechanical finite element models. The bottom left corner is fixed at zero displacement in the x and y directions, and the bottom edge is set to a temperature increase of 0°C . The domain is discretized into 200×100 square linear finite elements.

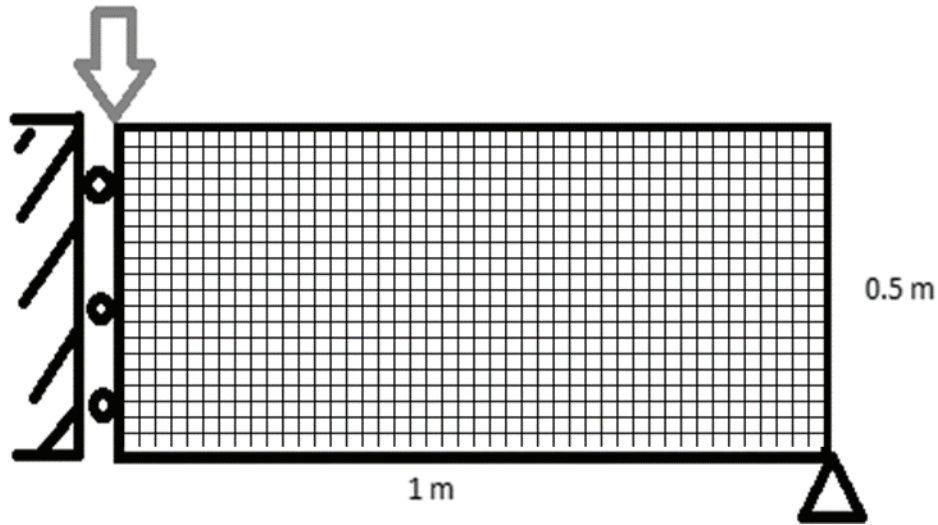


Figure 1: Design Domain

Two separate optimizations were performed, one for compliance and one for strain energy. For both, the penalties were set to $p_m = p_{Th} = 3$ and the material properties of ABS plastic were used. The material was assumed to be ten times stiffer in the direction of deposition than perpendicular to it, i.e. $\frac{E_x}{E_y} = 10$ for each local element. Note that, as elements are oriented in different directions, this does not necessarily correspond to the global coordinates. A force of 500 N and a thermal flux of $1\text{ }\mu\text{W}$ were applied. This generated topologies that were dominated by the thermal loading which is desirable to illustrate the differences between compliance and strain energy as objective functions, as they are equivalent in the absence of thermal loads. Heat

conduction was assumed to be anisotropic according to $\frac{K_x}{K_y} = 2$ and thermal expansion followed $\frac{\alpha_x}{\alpha_y} = 1.2$. A volume fraction of $V_f = 0.5$ was used for both optimizations.

Final Topologies

The results can be presented by showing the densities as opacity, temperature as color, and element orientation as a vector field. The compliance optimization produces the following design after 200 iterations.

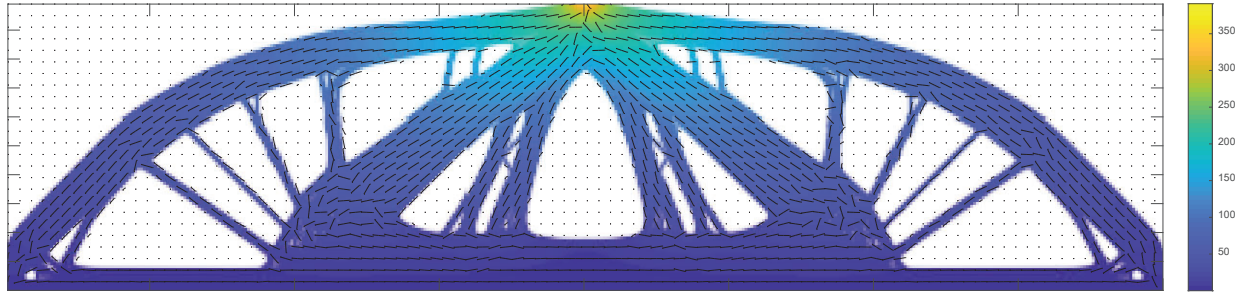


Figure 2: Compliance-Optimized Topology

The final topology has converged to a design composed entirely of densities of x_{min} or 1, showing a fully converged structure, and can thus be feasibly produced. However, if the penalty parameter values are changed this may not always be the case. In the above optimization, both p_m and p_{th} were set to 3, but this need not be the case. A parameter study will evaluate the behavior of the structure under various parameter values.

The penalty for the thermal stresses and mechanical stiffness matrix (Equations 2 and 5) is fixed at 3. The penalty for the thermal elasticity matrix (Equation 1) is set to integers between 1 and 5 in different simulations that are run for 100 iterations. The final topologies from those tests indicate that a higher thermal penalty can will prevent the intermediate densities from remaining.

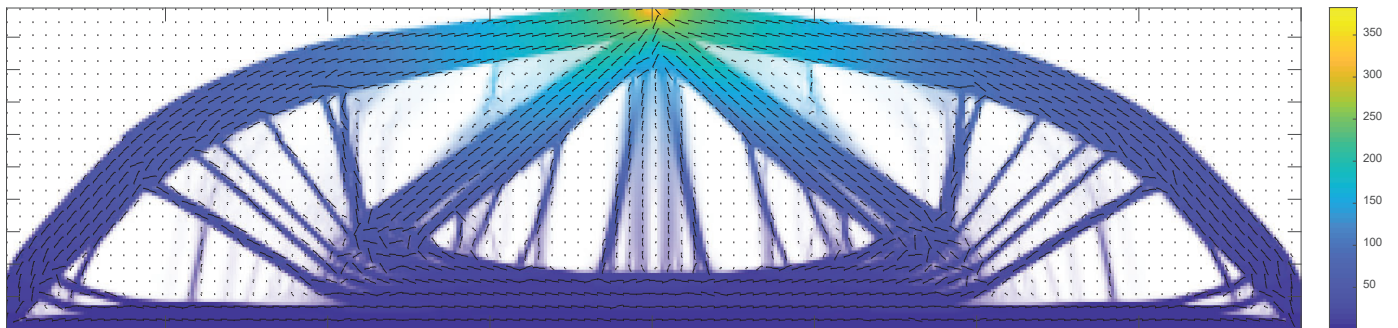


Figure 3: Compliance-Optimized Topology for a Thermal Penalty of 1

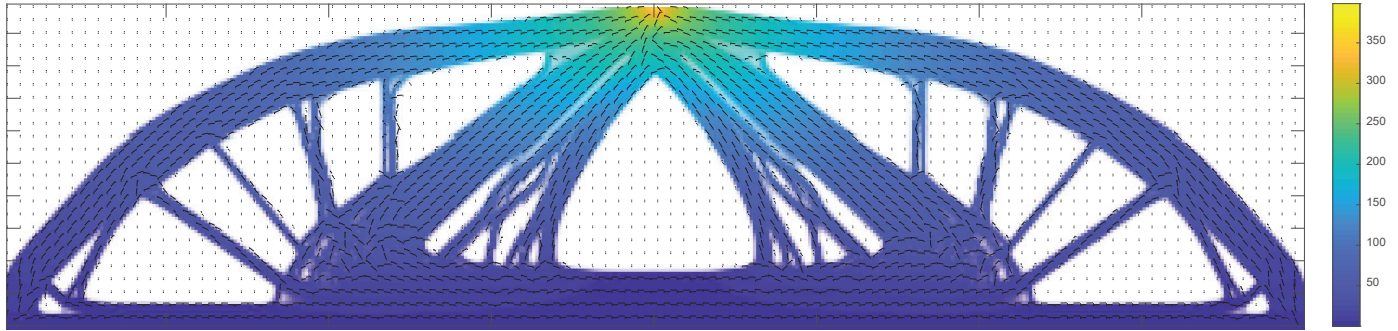


Figure 4: Compliance-Optimized Topology for a Thermal Penalty of 2

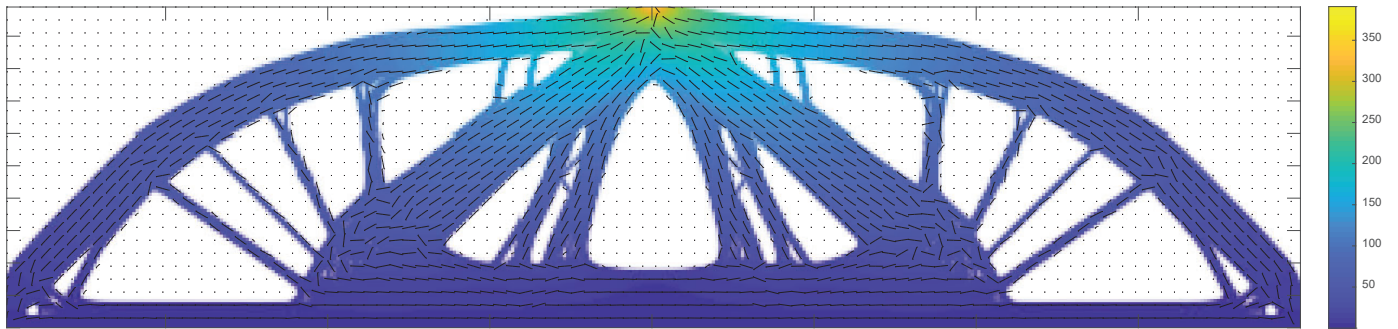


Figure 5: Compliance-Optimized Topology for a Thermal Penalty of 3

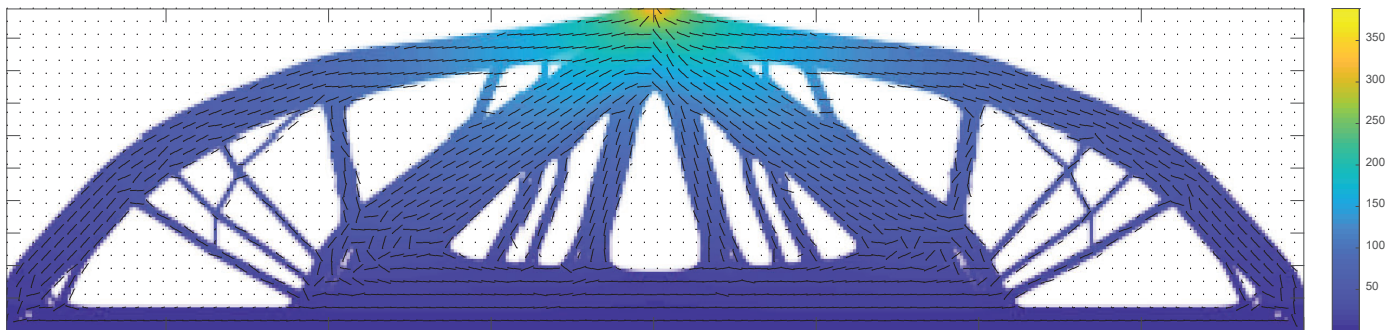


Figure 6: Compliance-Optimized Topology for a Thermal Penalty of 4

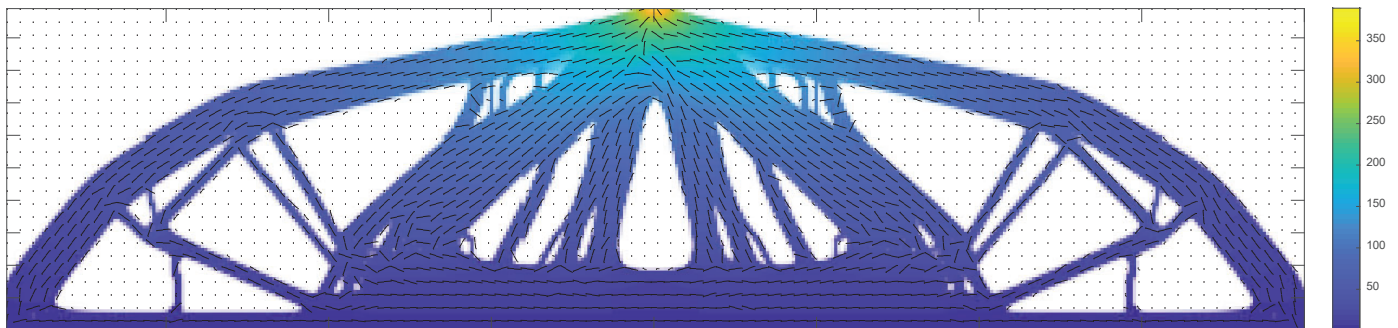


Figure 7: Compliance-Optimized Topology for a Thermal Penalty of 5

Note that the densities demonstrate full convergence in the cases where the thermal penalty is set to at least 3, although there were only large regions of fractional density in the case

where the thermal penalty was set to 1. In Sigmund's 99-line code [1], it was noted that a mechanical penalty of 1 yields poor convergence, and this suggests a similar behavior in the weakly coupled thermomechanical system.

In the strain energy optimization, all parameters were kept the same. No significant regions of fractional densities were encountered, so the mechanical penalty remained at 3 and the thermal penalty was set to 5. This ensures accurate comparison with the compliance optimization in figure 7.

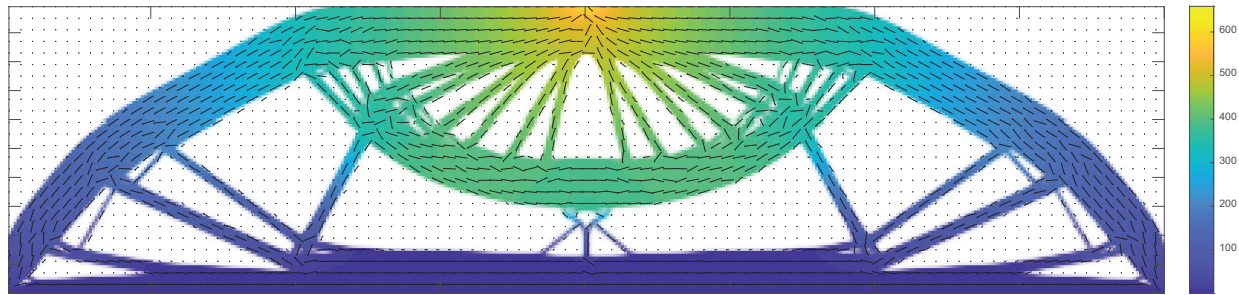


Figure 8: Strain Energy-Optimized Topology

The final topologies of each case are very different, and prior work [17] indicates that the topologies diverge as thermal loads increase. The previous work dealt only with applied temperatures instead of applied heat fluxes, and the convergence issues present in the strain energy optimization did not manifest here.

Conclusions

Performance Comparison

It is not surprising that each optimization has minimized their respective objective functions more than the other. The final values can be seen below.

	Final Compliance (kJ)	Final Strain Energy (kJ)	Maximum Von Mises Stress (GPa)
Strain Energy Optimization	32.37	9.826	28.80
Compliance Optimization: $p_{Th} = 1$	14.88	10.14	27.83
Compliance Optimization: $p_{Th} = 2$	15.60	10.37	25.39
Compliance Optimization: $p_{Th} = 3$	15.40	10.52	23.77
Compliance	15.44	10.93	22.04

Optimization: $p_{Th} = 4$			
Compliance Optimization: $p_{Th} = 5$	15.66	11.24	20.69

Table 1: Objective Function Performance of Optimized Parts

It is apparent that the thermal penalty factor affects the shape of the final design and the performance of the part. When the Von Mises stress and other factors are computed, it uses both penalty values and thus the different compliance-optimization tests cannot be compared on the basis of their compliance, strain energy, or Von Mises stress values. However, it is worth noting that when $p_{th} \geq 3$ the design is fully converged, so the penalty parameters do not affect the evaluation of the design. Thus, the last three cases can be compared.

In comparing the performance of compliance and strain energy optimization for weakly coupled isothermal thermomechanical systems, Neiford et al [13] examined the maximum Von Mises stress within the part as a standard. The Von Mises stress is used a failure criterion for some ductile materials, and the stress fields are presented below. The $p_{th} = 5$ compliance optimization has the lowest maximum Von Mises stress value. In the strain energy optimization, a thermal penalty factor of 5 was also used so that valid comparisons can be made between it and the compliance optimization.

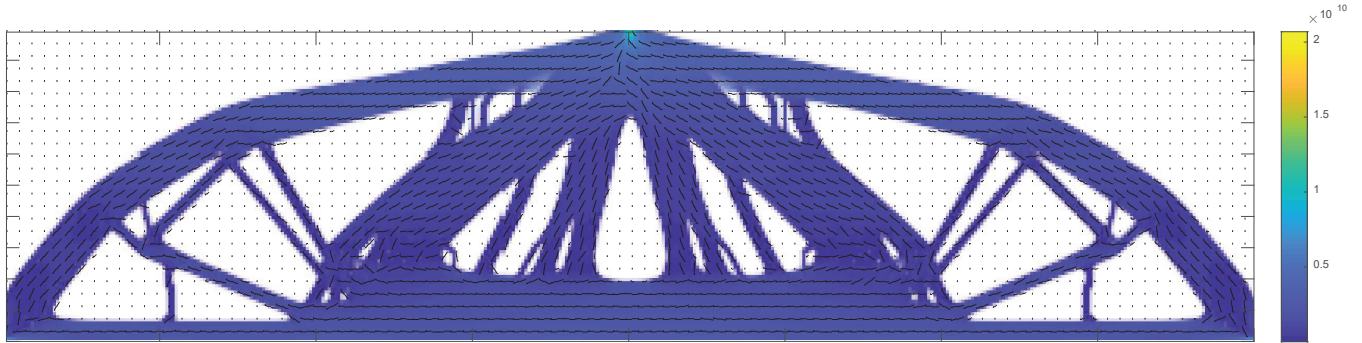


Figure 9: Von Mises Stress Field for Compliance-Optimized Topology, $p_{Th} = 5$

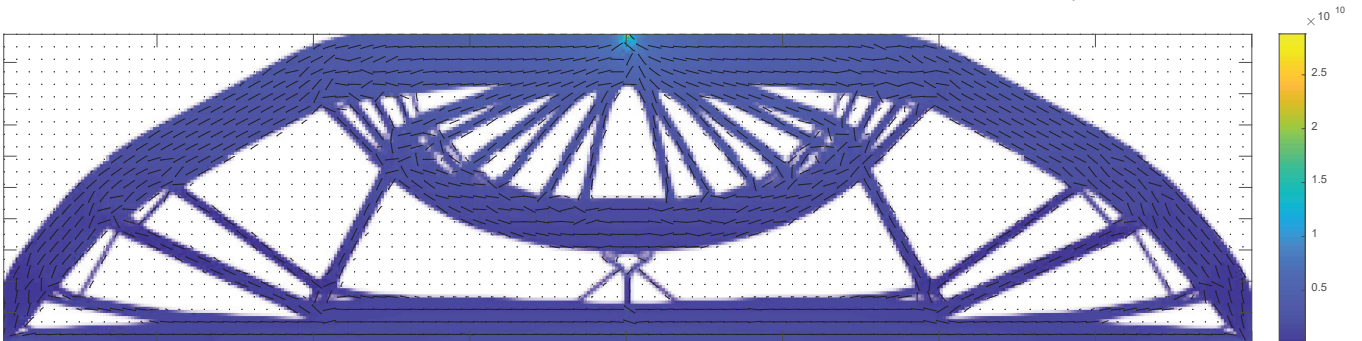


Figure 10: Von Mises Stress Field for Strain Energy-Optimized Topology

The maximum Von Mises stress is 20.69 *GPa* for the compliance case and 28.80 *GPa* for the strain energy case. This is contrary to Neiford's findings that strain energy had a lower

maximum Von Mises stress [13]. It is possible that the relative performance of the methods may be dependent on the loading case in consideration. It is also worth noting that the compliance optimized part has a significantly lower maximum temperature, indicating that it is more effective at distributing the heat flux.

The objective function that performs best may depend on the application. A low maximum displacement, maximum temperature, maximum Von Mises stress, or some other criterion may be desired, and the approach that best fits that may depend on the loading. More testing of different loading scenarios is needed to more accurately compare methods.

Future Steps

The design-dependent thermal loading used here is a step toward modeling the cooling process present in additive manufacturing, but it does not capture the full complexity. A transient analysis would better represent this system. Researchers at Oak Ridge's big area printer have proposed a one-dimensional model of the cooling of a single polymer bead [19], which could potentially be extended to represent more complex systems.

More testing needs to be done to determine the different behaviors of the objective functions. Simulating a wider array of geometries and loading conditions will give a broader understanding of their relative performance and applying the method to a three-dimensional system will more accurately model real-world part design. At this point, only proof-of-concept scale parts have been produced on standard desktop 3D printers. Printing and testing full-scale parts on Baylor's large-area polymer printer will allow for testing of parts to validate the model and determine if additional effects should be considered.

Bibliography

1. Sigmund, O. "A 99 Line Topology Optimization Code Written in Matlab." *Structural and Multidisciplinary Optimization* (2001): 120-127. DOI: 10.1007/s001580050176
2. Bendsoe, Martin Philip, Kikuchi, Noboru. "Generating Optimal Topologies in Structural Design Using a Homogenization Method." *Computer Methods in Applied Mechanics and Engineering* (1988): 197-224. DOI: 10.1016/0045-7825(88)90086-2
3. Wang, Michael Yu, Wang, Xiaoming, Guo, Dongming. "A Level Set Method for Structural Topology Optimization." *Computer Methods in Applied Mechanics and Engineering* (2003): 227-246. DOI: 10.1016/S0045-7825(02)00559-5
4. Svanberg, Krister. "The Method of Moving Asymptotes – a New Method for Structural Optimization." *International Journal for Numerical Methods in Engineering* (1987): 359-373. doi:10.1002/nme.1620240207
5. Bendsoe, M. P., Sigmund, O. *Topology Optimization Theory, Methods, and Applications*. Germany: Springer-Verlag Berlin Heidelberg 2003
6. Hoglund, Robert. "An Anisotropic Topology Optimization Method For Carbon Fiber-Reinforced Fused Filament Fabrication." Masters Thesis, Baylor University.
7. Jiang, Dale. "Three Dimensional Topology Optimization with Orthotropic Material Orientation Design for Additive Manufacturing Structures." Masters Thesis, Baylor University.

8. Jiang, Delin, Hoglund, Robert, Smith, Douglas E. "Continuous Fiber Angle Topology Optimization for Polymer Composite Deposition Additive Manufacturing Applications." *Fibers* (2019): DOI: 10.3390/fib7020014
9. Luo, J. H., Gea, H. C. "Optimal Orientation of Orthotropic Materials Using an Energy Based Method." *Structural Optimization* (1998): 230-236. DOI: 10.1007/BF01203536
10. Deaton, Joshua. "Design of Thermal Structures Using Topology Optimization." Dissertation for Doctor of Philosophy, Wright State University 2009.
11. Pederson, Pauli, Pederson, Niels. "Strength Optimized Designs of Thermoelastic Structures." *Structural and Multidisciplinary Optimization* (2010): 681-691. DOI: 10.1007/s00158-010-0535-5
12. Zhang, Weihong, Yang, Jungang, Xu, Yingjie, Gao, Tong. "Topology Optimization of Thermoelastic Structures: Mean Compliance Minimization or Elastic Strain Energy Minimization." *Structural and Multidisciplinary Optimization* (2014): 417-429. DOI: 10.1007/s00158-013-0991-9
13. Neiford, David John, Grandhi, Ramana V., Deaton, Joshua D., Beran, Philip S. "Level-Set Topology Optimization of Thermoelastic Structures – a Comparison of Compliance, Strain Energy, and Stress Objectives." 2018 Multidisciplinary Analysis and Optimization Conference. Atlanta, Georgia, June 25-29 2018. AIAA Aviation Forum. DOI: 10.2514/6.2018-3577
14. H. L. Tekinalp, V. Kunc, G. M. Velez-Garcia, C. E. Duty, L. J. Love, A. K. Naskar, C. A. Blue, and S. Ozcan, "Highly oriented carbon fiber–polymer composites via additive manufacturing," *Compos. Sci. Technol.*, vol. 105, pp. 144–150, Dec. 2014.
15. Dechaumphai, Pramote, Thornton, Earl. "Enhanced Thermal-Structural Analysis by Integrated Finite Elements," *Flight Dynamics Laboratory*. 1984.
16. D. A. Tortorelli & P. Michaleris. "Design sensitivity analysis: Overview and review, Inverse Problems in Engineering," 1:1, 71-105, (1994) DOI: 10.1080/174159794088027573
17. Ramsey, Jackson S., Smith, Douglas E. "Topology Optimization of Coupled Thermomechanical Analysis for Additive Manufacturing." *SAMPE Conference* (2019).
18. Ma, Keith. "Evenly Spaced Streamlines," *Mathworks File Exchange*, 06-Oct-2016. Accessed on 06-Jun-2019 from <https://www.mathworks.com/matlabcentral/fileexchange/59476-evenly-spaced-streamlines>
19. Compton, Brett G., Post, Brian K., Duty, Chad E., Love, Lonnie, Kunc, Vlastimil. "Thermal Analysis of Additive Manufacturing of Large-Scale Thermoplastic Polymer Composites." *Additive Manufacturing* (2017): 77-86. DOI: 10.1016/j.addma.2017.07.006