Numerical Simulation of Viscous Sintering under Mechanical Loads

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Abstract

This paper describes the use of swelling and creep models in a general purpose finite element program for the simulation of Scherer's constitutive relation for a viscous sintering under uniaxial load. Numerical results of the finite element analysis are presented for validating the models. In the finite element model both the apparent modulus (F) and the apparent Poisson's ratio (N) are used for the stress-strain relation and the stress state considered in the model is multi-dimensional.

Introduction

The growing interest in numerical simulation of sintering processes leads to the recent study of the application of finite element methods to the process [1]. It has been observed that a non-linear general purpose finite element program can be adequately used for modeling the heat conduction in a sintering process. The generality in the finite element approach allows for the study of multi-dimensional problems as well as non-linear material behavior in the process. Based on the assumption that a satisfactory sintering model (e.g. Scherer [2]) is available, the finite element simulations of the sintering process can be expected to include the following areas:

(1) Transient heat conduction problem
(2) Rate dependent elastic stress problem
(3) Rate dependent elastic-plastic stress problem
(4) Coupled thermo-stress problem.

As described in Ref.[1], the study of temperature distributions in laser sintered parts using finite element model has been successful. The sintering models in the study are based on the analyses of Scherer and Mackenzie-Shuttleworth and both the density and conductivity of the particle bed are treated as functions of the void fraction of the bed in the transient finite element heat conduction analyses.

The rate dependent elastic-plastic analysis of a sintering process using finite element simulation has rarely been reported. However, investigation of the hot isostatic pressing (HIP) in powder metallurgy has recently been carried out [3]. The current HIP model assumes visco-plastic representation for the time dependent behavior. It follows classical von Mises yield condition as well as associated flow rule. The yield condition can also be a function of relative density of the powder material. In addition, both the elastic and the thermal constants of the material can be dependent on relative density of the powder material and/or temperatures.

Although finite element coupled thermo-stress analyses have been available for a long time, the author is not aware of any applications of the finite element method to thermo-stress coupling problems in sintering processes. In the conventional coupled thermo-stress analysis, both non-linear material behavior and non-linear boundary conditions can be included in the analysis. Either transient or steady state solution of the problem can be obtainable. With more understanding of the sintering process in terms of thermal and mechanical modeling, it is very possible to perform fully coupled thermo-stress simulation of sintering process sometime in the future.

The attempt in this paper is to carry the finite element modeling one step further into the area of stress analysis of a sintering problem. After reviewing the results of Scherer's [2], it seems to be possible to simulate the rate dependent constitutive equation in the sintering process by swelling and creep models in a general purpose finite element program. In addition, an equation of state approach can be used for the transient solution. It is expected that Scherer's results can be reproduced by the finite element model, and the finite element model can easily be extended to two- or three-dimensional geometries. However, it is not the intent of this paper to discuss the coupling between thermal and mechanical behaviors, as well as inelastic responses such as plasticity, in sintering process.

In the remaining of the paper the sintering model and the constitutive relation described in [2] will be summarized first. Next the swelling and creep models in the finite element analysis will be discussed. Numerical results are included in the subsequent section while an attempt to arrive at some conclusions concludes the paper.

Scherer's Models

In ref.[2] both the free strain rate and the constitutive equations for a porous viscous material are discussed in detail. A summary of the discussions is as follows:

(1) Free Strain Rate

Using Scherer's notations, the appropriate free strain rate can expressed as

$$\dot{\varepsilon}_f = -\frac{K}{\eta}\frac{(3\pi)^{1/3}}{6}\frac{2-3cx}{x^{1/3}(1-cx)^{2/3}} \tag{1}$$

for $\rho/\rho_s < 0.94$, and

$$\dot{\varepsilon}_f = -\left(\frac{K}{\eta}\right)\frac{1}{2}\left(\frac{4\pi}{3}\right)^{1/3}\left(\frac{\rho_s}{\rho}-1\right)^{2/3} \tag{2}$$

for $\rho/\rho_s \geq 0.94$.

Eq.(1) is due to Scherer, using a micro-structural model consisting of cylinders, and Eq.(2) is given by Mackenzie-Shuttleworth.

(2) Constitutive Equations

For a porous viscous material the following constitutive equations have been proposed by Scherer.

$$\dot{\varepsilon}_r = \dot{\varepsilon}_f + \frac{1}{F}[\sigma_r - N(\sigma_\theta + \sigma_z)] \tag{3a}$$

$$\dot{\varepsilon}_z = \dot{\varepsilon}_f + \frac{1}{F}[\sigma_z - N(\sigma_r + \sigma_\theta)] \tag{3b}$$

$$\dot{\varepsilon}_\theta = \dot{\varepsilon}_f + \frac{1}{F}[\sigma_\theta - N(\sigma_z + \sigma_r)] \tag{3c}$$

Note that Eq.(3c) is not given in [2].
In Eq.(3) σ_r, σ_z, σ_θ are the radial, axial and hoop stresses, respectively. The apparent

modulus (F) and the apparent Poisson's ratio (N) can be approximated by

$$F = \frac{3\eta\rho}{(3\rho_s - 2\rho)} \qquad (4)$$

$$N = \left(\frac{1}{2}\right)\left[\frac{\rho}{(3\rho_s - 2\rho)}\right]^{1/2}$$

In these equations, η is the shear viscosity, $\rho(t)$ is the bulk density, ρ_s is the theoretical density of the pore-free sample. As ρ approaches ρ_s, we have $F \rightarrow 3\eta$ and $N \rightarrow 0.5$, and Eq.(3) reduces to the usual constitutive equations for an incompressible viscous liquid.

Finite Element Models (Swelling and Creep)

In the finite element modeling of stress problems, the total strain vector in the structure is assumed to be a linear combination of various strains

$$\varepsilon = \varepsilon_e + \varepsilon_{sw} + \varepsilon_{cr} \qquad (5)$$

Rewriting Eq.(5) in the rate form we have

$$\dot{\varepsilon} = \dot{\varepsilon}_e + \dot{\varepsilon}_{sw} + \dot{\varepsilon}_{cr} \qquad (6)$$

where $\dot{\varepsilon}$ is the total strain rate, $\dot{\varepsilon}_e$, $\dot{\varepsilon}_{sw}$ and $\dot{\varepsilon}_{cr}$ are total elastic, swelling and creep strain rates, respectively. For a given time step Δt, the assumption of a linear variation of strain in time, yields the following relation of incremental strains

$$\Delta\varepsilon = \dot{\varepsilon}\Delta t = \Delta\varepsilon_e + \Delta\varepsilon_{sw} + \Delta\varepsilon_{cr} \qquad (7)$$

Rewriting Eq.(7), we obtain the expression of incremental elastic strain as

$$\Delta\varepsilon_e = \Delta\varepsilon - \Delta\varepsilon_{sw} - \Delta\varepsilon_{cr} \tag{8}$$

The expression of incremental stress vector is

$$\Delta\sigma = D\Delta\varepsilon_e \tag{9}$$

where D is the constitutive matrix.

Following the principle of virture work, the following expression of incremental equilibrium equation can be obtained.

$$[K]\,\Delta u = \Delta\rho + \int_v B^T D\,(\Delta\varepsilon_{sw} + \Delta\varepsilon_{cr})\,d\nu \tag{10}$$

In Eq.(10), K is the stiffness matrix, and Δu and Δp are incremental displacement and incremental nodal force vectors, respectively. The integral

$$\int_v B^T D\,(\Delta\varepsilon_{sw} + \Delta\varepsilon_{cr})\,d\nu \tag{11}$$

is a pseudo load vector due to the swelling and creep strain increments, in which B is the strain displacement relation and D is the stress-strain relation.

Comparing Eq.(3) with Eq.(6) one observes that the free strain rate $\dot{\varepsilon}_f$ in Eq.(3) can be treated as the swelling strain rate in Eq.(6), while the remaining term in Eq.(3) can be represented as the creep strain rate in Eq.(6). Consequently, it can be seen from Eq.(10) that the incremental swelling and creep strains can be estimated from the strain rate expressions with a given time step. The pseudo force vector can then be evaluated from the volume integral shown in Eq.(11) and added to the load-side of the incremental equilibrium equation (Eq.(10)). Accumulations of the incremental displacements obtained from solution of Eq.(10) represent a deformation history of the structural system. It is noted that the stiffness matrix K in Eq.(10) can be elastic or, inelastic depending on the material behavior of the structure. The finite element swelling and creep models can easily accept material non- linearities providing non-linear material data are available in sintering processes.

Case Study

In order to validate the swelling and creep models in the finite element analysis, for the simulation of a sintering process, two problems were investigated in this study. The first analysis demonstrates that Scherer's results can be reproduced, and the second problem shows an application of the models to a sintering problem of a simple three-dimensional geometry. Finite element results of Analysis one compares well with Ref.[2].

(1) Deformation of a Solid Cylinder

The geometry of a solid cylinder, as discussed in [2], is shown in Fig.1a. In the finite element analysis, both the 4-node two-dimensional axisymmetric element and the 8-node three-dimensional brick element were used. The total creep time was chosen to be 0.8, and a total of 11 time increments were needed for the completion of the creep analysis. The free strain rate equations, Eq.(1) and Eq.(2), were selected for the swelling strain rate in the finite element analysis.

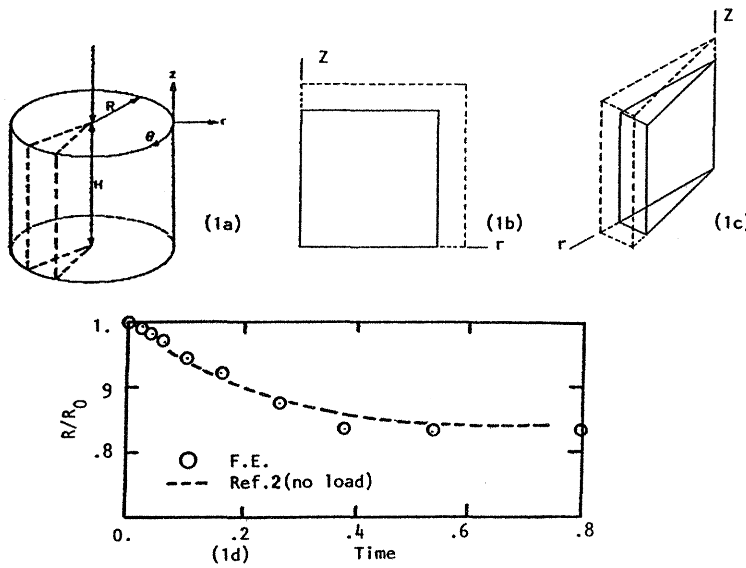

Fig. (1a) Solid Cylinder (Ref.[2]), (1b) Deformed Cylinder (4-node model), (1c) Deformed Cylinder (8-node model), (1d) Radial Reduction vs. Time

Fig.1b shows deformation of the cylinder at the end (t=0.8) of the creep, in the two-dimensional analysis. Final deformation of the three-dimensional analysis is also given in Fig.1c. Favorable comparison of the time variation of radial reduction with Scherer's results (see Fig.1d) is quite evident.

(2) Fixed End Column

Fig.2a shows the geometry of a square column with both ends fixed in space. Due to symmetry, only one eighth of the column was modeled in the finite element analysis. Four 8-node three-dimensional brick elements were used for the finite element model. In this analysis, both the swelling strain rate and the creep time were assumed to be the same as in Analysis one.

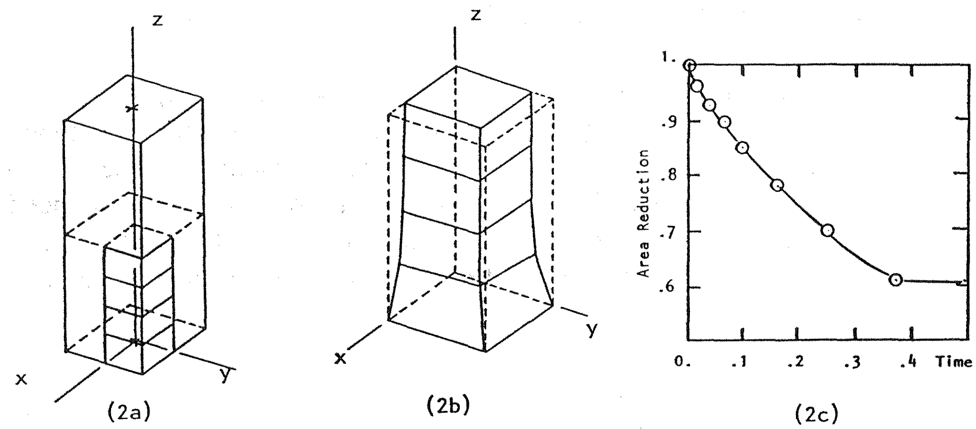

Fig (2a) Square Column, (2b) Deformed Column, (2c) Area Reduction vs. Time

The deformation of the column at end of creep (t=0.8) is shown in Fig.2b. The time variation of the area reduction due to sintering, shown in Fig.2c appears to be satisfactory.

Conclusions

Numerical results presented in the paper clearly demonstrated the validity of using swelling and creep models in the finite element analysis, for the prediction of deformations of sintered structures. In the current study, both the two- and the three-dimensional geometries have been analyzed and the results have been found to be satisfactory. The finite element models have long been recognized to be able to deal with various non-linearities in the problems. The obvious difficulty in a successful simulation of a sintering process remains to be the lack of good understanding of the constitutive relation of the sintered material.

References

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[3] Zhao, K., "Finite Element Analysis in Hot Isostatic Pressing Applications," (to be published), MARC analysis research corp., 1992