An Analysis Technique for Layered Manufacturing Based on Quasi-Wavelet Transforms

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SUMMARY

An analysis technique based on the Wavelet transform (WT) has been recently introduced that allows the spatial frequency content of objects produced by layered manufacturing (LM) techniques to be interpreted in terms of manufacturable features. (Lee and Thomas, 1997) Using Haar's wavelet as a basis function, layers with vertical edges are modeled exactly. Using analysis, a 3D model can be transformed, filtered, and inverse transformed resulting in an image of the part as it would look if constructed from layers of a specific thickness. In order to extend this analysis to construction techniques using higher order edges (ruled surface edges or curved edges), the quasi-wavelet transform (QWT) is introduced. QWT analysis is conceptually the same as WT analysis, except that the basis function can be selected by the user, allowing exact analysis of layered manufacturing techniques using higher order construction algorithms. This work is supported by a grant from the University of Utah Research Foundation.

INTRODUCTION

When a solid model of an object is reproduced physically using a layered manufacturing technique, the result is a double approximation of the electronic model. The first level of approximation occurs when the model is converted to the *.stl format. This triangular tessellation of the part surface clearly results in error between the CAD model and the triangular mesh. The second level of approximation occurs when the part is constructed from layers. Using finite thickness layers, any surface that is not either parallel or perpendicular to the slicing plane has a ridged, stepped appearance. Researchers are attempting to address the stepped edge problem by cutting ruled surface edges on the layers (Lee et al, 1995), but the result is still an approximation to the designed surface.

In order to minimize the error caused by these approximations and maximize the construction speed by increasing the layer thickness, it would be useful to have analysis tools that would assess the manufacturability of a given model using LM. Desirable features of these techniques might include: The ability to compare the geometric complexity of a specific part to a specific devices' ability to create complex geometry. The ability to predict the output of a specific device. The ability to identify the preferred slicing direction for a specific part.

A previous paper addresses these issues and suggests that objects be decomposed in terms of spatial frequency. (Lee, 1997) Fourier analysis was shown to be of little use since it does not identify the location on the part of the high or low frequency information. Wavelet analysis was shown to be quite useful since the frequency information is spatially localized and Haar's wavelet provides an exact match to the stepped edge layers produced by existing LM devices.

This paper provides a brief discussion of Fourier and Wavelet Transform Analysis (FTA and WTA) applied to 3D objects and introduces Quasi-Wavelet Transform Analysis (QWA).

Fourier Analysis

The Fourier transform is currently used in applications such as signal analysis and image processing. It allows frequency analysis and filtering, of an analog signal by decomposing a time domain signal in terms of it's frequency content. A wave form can be filtered and inverse transformed to predict the form of the time domain signal if certain frequencies were missing. Using a 2D FT, images can be analyzed or compressed by deleting the information for frequencies that make little contribution to the image features. (Proakis, 1992)

A 1D FT is often used to examine a time domain signal (amplitude as a function of time). We can examine a slice of an object in terms of spatial frequency by replacing amplitude and time with the two coordinate axes of the slice. Since LM devices in general can create quite complex geometry in the plane of the slice, but are limited by the layer thickness in the slicing direction, one might say these devices are low pass filters in the slicing direction and that they pass all frequencies in the slicing plane. If basis functions can be selected that represent the cutter shape, then Fourier analysis will produce meaningful results for LM.

The limitation of the Fourier transform that lead to the investigation of wavelet transforms is the inability to spatially localize the frequency information. Fourier analysis averages the frequency content over the entire shape under analysis. The transform cannot identify that certain parts of the object are simple and other parts are complex.

Wavelet Analysis

The Wavelet transform decomposes a signal in terms of both frequency and position. While it has similar applications to the FT, it not only identifies the frequency content of a signal, but also keeps track of where in the signal the frequencies occurred. The WT decomposes a signal using basis functions called wavelets. A wavelet is a family of functions derived from a single function, called mother wavelet expressed as:

$$W_{(a,b)}(x)=\frac{1}{\sqrt{|a|}}W\left(\frac{x-b}{a}\right) \tag{1}$$

where 'a' and 'b' adjust the frequency and position of the wavelet. Using the WT, a signal, f(x) can be expressed as: (Newland, 1993, and Tewfik, 1992)

$$f(x)=a_0 u(x)+a_1 W(x)+\begin{bmatrix}a_2 & a_3\end{bmatrix}\begin{bmatrix}W(2x)\\ W(2x-1)\end{bmatrix}+\begin{bmatrix}a_4 & a_5 & a_6 & a_7\end{bmatrix}\begin{bmatrix}W(4x)\\ W(4x-1)\\ W(4x-2)\\ W(4x-3)\end{bmatrix}+\dots$$

$$+\dots+a_{2^j+k}W(2^j x-k)+\dots. \tag{2}$$

and

$$u(x)=\begin{cases}1 & for\ 0\le x<1\\ 0 & otherwise\end{cases}$$

where a_{2^j+k} is a contribution of a wavelet at frequency 2^j with a shift k. At a frequency 2^j, there are 2^j wavelets with independent amplitudes. Fig. 1 shows four levels of wavelet components for a typical transform. The wavelet used here is Haar's D2 wavelet which consists of a step to amplitude a_i followed by a step down to -a_i. Thus the WT to spatially localizes the frequency information.

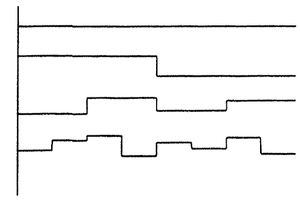

Fig. 1: Wavelet components for a typical transform.

Although wavelets must meet criteria similar to the orthogonality constraints required for basis functions for FT, one wavelet, Haar's D2 wavelet function, consists of steps. Thus, it is perfect for representing the stepped edges of LM. Using Haar's D2 wavelet, the levels of the WT correspond to layer thickness'. The summation of several wavelets of different frequencies results in a stepped surface that can always be reproduced using layers corresponding to the highest of the summed frequencies. Using WT it is possible to transform a 3D object, filter the object, and inverse transform the object producing a prediction of the geometry realized from layers of a selected thickness. (See Fig. 2) (Lee, 1997) Note that the object is first mapped into cylindrical coordinates to get a single valued function.

There are limitations to the wavelet transform: The wavelets double in frequency at each level. Thus, only certain layer thickness' are actually considered in the analysis. If a LM device were capable of varying the layer thickness in an analog fashion, the technique as described would not be able to properly analyze the problem. A second limitation is that there are no known wavelets that match the cutter shapes used by the new higher order construction algorithm LM devices that are currently under development.

QUASI-WAVELET ANALYSIS

The Quasi-Wavelet Transform follows the spirit of the WT, but relaxes the constraints on the basis functions. As a result the QWT can use any function as a basis function. The QWT decomposes a function in terms of quasi-wavelets that are scaled and shifted in a similar fashion to wavelets. The requirement for QWT is that, at the lowest frequency, a single quasi-wavelet is scaled such that the squared error between the quasi-wavelet and the function is minimized. The next level has two quasi-wavelets and each is scaled independently to minimize the squared error between the function and the sum of the two quasi-wavelet levels. At the third level there are four wavelets and each is scaled to minimize the error between the function and the sum of the three levels of wavelets. This process continues until the highest frequency of interest is reached.

A function W(x), made up of a linear combination of general functions g(x), is defined as a quasi-wavelet function, within the interval 0<=x<=1, and is set to zero outside the interval. Thus, an n term quasi-wavelet function can be expressed as:

$$W_n(x)=\sum_{i=1}^{n} g_i(x) \tag{3}$$

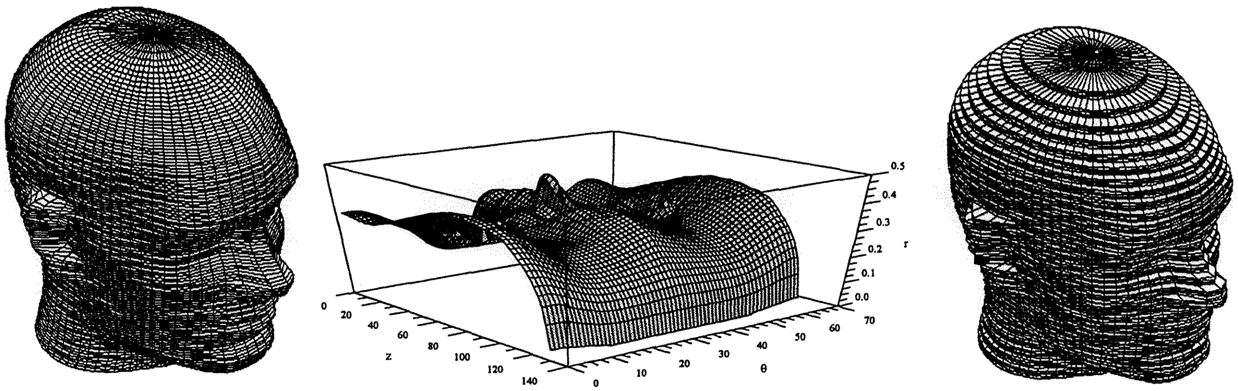

Fig. 2: CAD file, cylindrical coordinate mapping, and filtered inverse transform.

The actual signal, f(x), is:

$$f(x) = \sum_{i=-1}^{m} f_i(x) \tag{4}$$

where each $f_i(x)$ is one level of the transform and is composed of 'r' independent wavelets each with a vector amplitude $\vec{\alpha}_k$:

$$f_i(x) = \sum_{k=1}^{r} \vec{\alpha}_k \cdot \vec{W}_n(rx - k + 1), \qquad r = 2^i \qquad = \sum_{k=1}^{r}\sum_{j=1}^{n} \alpha_{kj} g_j(rx - k + 1) \tag{5}$$

and

$$f_{-1}(x) = \alpha_{-1} u(x)$$

where $\vec{W}_n$ is a quasi-wavelet function expressed as a vector, and $f_{-1}(x)$ is the DC level of f(x).

We define $F_i(x)$ as the sum of levels from level -1 to level i:

$$F_i(x) = \sum_{k=-1}^{i} f_k(x) \tag{6}$$

As a result, $F_i(x)$ is the output of a low pass filter within QWT. The error of $F_i(x)$ is defined as:

$$\varepsilon_i(x) = f(x) - F_i(x) \tag{7}$$

Calculating the QWT involves finding the appropriate coefficient vectors $\vec{\alpha}_k$ in order to minimize this error. Details of this calculation can be found in (Lee, 1997).

Using the quasi-wavelet function $W_n(x)$, the function f(x) should be discretized into $2^{\mu} n$ points, where μ is a positive integer. Here the highest level is μ and there are 2^{μ} layers (or cycles of the basis function) at this level. Therefore, there must be 'n' points for each layer. Since $W_n(x)$ has n coefficients, n points per layer results in a unique set of coefficients without error. Thus, QWT is capable of perfectly reconstructing data points using the highest level.

QWT for Object Analysis

In this analysis the object is first mapped into cylindrical coordinates and unwrapped to produce a single valued function. Complex objects with multiple surfaces must be analyzed one

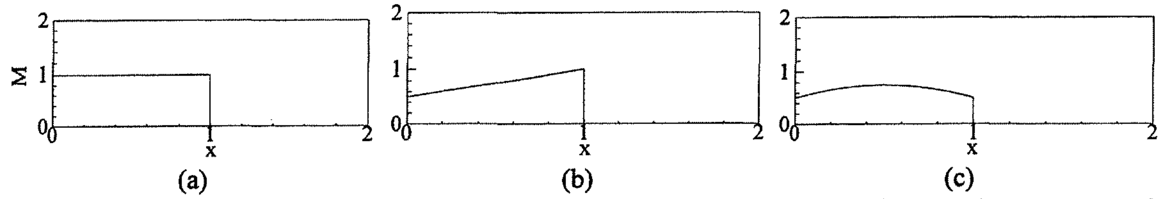

Fig. 3: This shows three quasi-wavelet basis functions. a) Step - This is equivalent to Haar's D2 wavelet. b) Angle - This quasi-wavelet models the ruled cut algorithm for layered manufacturing. c) Curve - This quasi-wavelet models the curved cut algorithm for layered manufacturing.

surface at a time. The z axis of the cylindrical coordinate system is assumed perpendicular to the slicing plane. When applying QWT, the quasi-wavelet function should be carefully chosen so the basis function can properly represent the LM machine. Quasi-wavelet functions for stepped edges (Stepped Cut Algorithm, SCA), ruled edges (Ruled Cut Algorithm, RCA), and curved edges (Curved Cut Algorithm, CCA), were selected as follows:

$$W_{SCA}(z)=W_1(z)=a_1\,u(z) \tag{8a}$$

$$W_{RCA}(z)=W_2(z)=a_1\,u(z)+a_2\,z \tag{8b}$$

$$W_{CCA}(z)=W_3(z)=a_1\,u(z)+a_2\,z+a_3\,z^2 \tag{8c}$$

where a_i are the coefficients that define the shape of each quasi-wavelet. An example of each is shown in Fig. 3. The QWT coefficients, a_i's, of $W_n(z)$ for SCA, RCA, and CCA for all levels, make up $\{a_i\}$ as follows:

$$QWT_{SCA}:\{a_{-11},a_{01},a_{11},a_{12},\ldots,a_{ij},\ldots,a_{m2^m}\}$$

$$QWT_{RCA}:\{a_{-111},a_{-112},a_{011},a_{012},\ldots,a_{ij1},a_{ij2},\ldots,a_{m2^m1},a_{m2^m2}\}$$

$$QWT_{CCA}:\{a_{-111},a_{-112},a_{-113},a_{011},a_{012},a_{013},\ldots,a_{ij1},a_{ij2},a_{ij3},\ldots,a_{m2^m1},a_{m2^m2},a_{m2^m3}\}$$

where the first subscript identifies the level (frequency), the second subscript identifies the position of the wavelet, and the third subscript identifies the function within the wavelet. Fig. 4 is a pictorial example of the coefficients for QWT_{RCA} at level 1 and 2. In QWT_{SCA}, if there are 2^{m+1} coefficients, then a_{-11} is the average value for all data, and a_{ij} is the coefficient of $u(2^i z-j+1)$. If there are 2^{m+2} coefficients in the QWT_{RCA}, a_{111} is the data average, and a_{-112} is usually zero, where a_{ij1} is the coefficient of $u(2^i z-j+1)$ and a_{ij2} is the coefficient of $(2^i z-j+1)$. In a QWT_{CCA}, if there are $3\times 2^{m+1}$ coefficients, a_{-111} is the data average, and a_{-112} and a_{-113} are usually

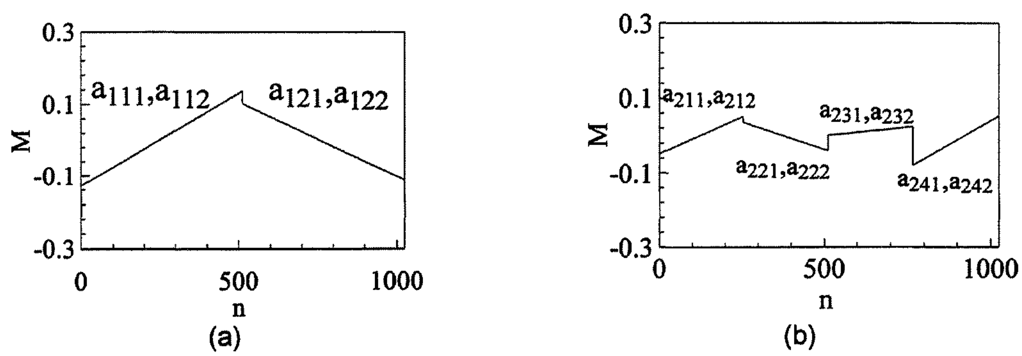

Figure 4: This shows two levels of a transform using the Angle quasi-wavelet. each sloped quasi-wavelet has its own independent coefficients.

zero, where a_{ij1} is the coefficient of $u(2^i z - j + 1)$, a_{ij2} is the coefficient of $(2^i z - j + 1)$, and a_{ij3} is the coefficient of $(2^i z - j + 1)^2$.

APPLICATIONS

Frequency Analysis

In LM, expressing objects in the frequency domain is very meaningful because frequency can be related to the number of layers. The contributions of individual frequencies to the geometry of a part can be found using frequency analysis. At certain cut off frequencies, the loss of geometric information is easily visualized through frequency component charts.

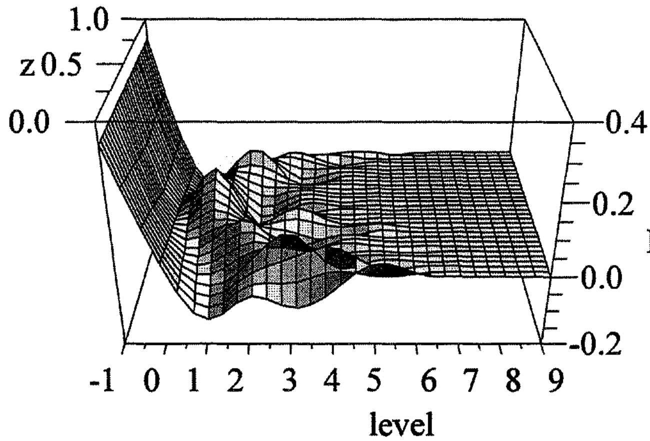

Figure 5: This is the Quasi-Wavelet Transform of a slice of a vase using Angle quasi-wavelets.

Fig. 5 shows the QWT_{RCA}. of a cross section of a vase shown in Fig. 6. The vase has been scaled to unit height with height shown as the z axis. The level axis corresponds to frequency or number of layers with 2^n layers at level 'n'. The magnitude is simply the quasi-wavelet components drawn on the M, z axis in this figure. The figure shows higher frequency components near the bottom and top of the vase. The center of the vase has higher amplitude components at low frequency and little contribution from higher frequencies. This suggests that the base can be constructed from 128 layers corresponding to level 7.

Output Prediction

The output of a SFM device can be predicted by applying a low pass filter to the data from Fig. 5. This is done by setting the magnitudes of all wavelets at levels above the filter cut off to zero. Next, an inverse transform is performed and an image of the object is produced as it would look if constructed using the selected algorithm at the selected layer thickness. Fig. 6 shows examples of output prediction for SCA, RCA, and CCA. As would be expected, the RCA and CCA produce an accurate representation using thicker layers than required for SCA.

Advanced Filtering

Using QWT it is not only possible to filter frequencies, it is possible to create filters that match machine constraints. If an RCA device can cut angled edges on the layers, but the angle is limited, this can be accounted for in the transform. A CCA device might be able to produce curved surfaces as long as the radius of curvature is greater than some minimum.

For Shapemaker II, a LM machine that implements RCA, hardware limitations constrain the maximum angle of cut. The design of Shapemaker II allows for a maximum cut angle of 45 degrees. For some layer thickness', the maximum angle may be reduced from this value. In QWT_{RCA}, cut angles are specified by the coefficient of the z term. By setting a limiting value on the z terms, it is possible to limit the angle of cut. (Lee, 1997)

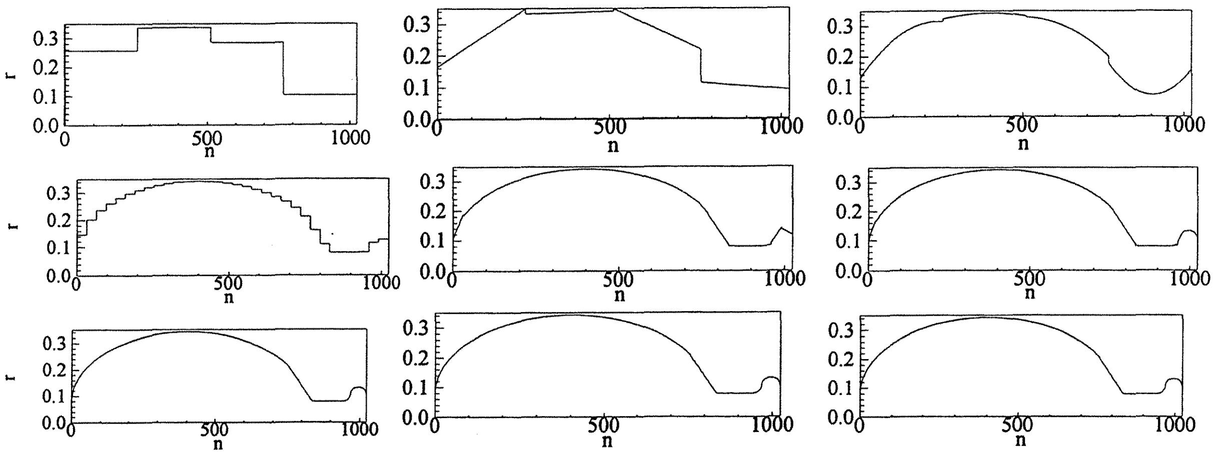

Figure 6: The vase shape has been transformed using Step, Angle, and Curve quasi-wavelets. Then it was filtered at different frequencies and inverse transformed.

Demonstration

The head shown in Fig. 2 was constructed using Shape Maker II. The model, in STL format was sliced in 1.8 mm thick slices. The slices were cut from commercially available 1.22 m by 2.44 m, white polystyrene foam, with an average thickness of approximately 1.8 cm. The size of the 47 layer head is 57 cm by 74 cm by 83 cm.

Using QWT_{RCA}, the shape of the head was predicted for 47 layers with a maximum cutter angle constraint of 45 degrees. Fig. 7(a) shows the inverse QWT_{RCA} of a three dimensional head with 47 layers and a 45 degree constraint on the cut angle. Fig. 7(b) shows a completed head made by SM2. As demonstrated by the predicted head, because of machine constraints, there is obvious error due to the constrained angle near the top of the completed head.

Advantages and Limitations

The QWT uses any linear combination of functions as a basis function. Frequency localization identifies where on an object the frequency components exist. As a result, QWT should be capable of analyzing any LM technique. Using the QWT, the output of LM systems can be predicted and reconstruction errors can be evaluated mathematically. Since the quasi-wavelet transform decomposes a three dimensional surface geometry into frequency components, a frequency analysis can be performed by inspecting the transform coefficients. Optimal construction directions can be found through coordinate rotation and frequency analysis.

Limitations for the application of three dimensional object analysis mainly arise from limitations of the three dimensional surface representation. Currently, the cylindrical representation method is used, which maps the three dimensional surface of an object onto a cylinder. This is done to produce a single valued function for the analysis. However, this method may still produce a multiple valued function for certain types of geometry's.

By nature of the analysis, frequencies analyzed by the QWT are integer powers of two. As a result, building an object from 3 layers or 115 layers is not considered. Most current SFM devices build parts from layers of fixed thickness and, as a result, seldom build parts from layers

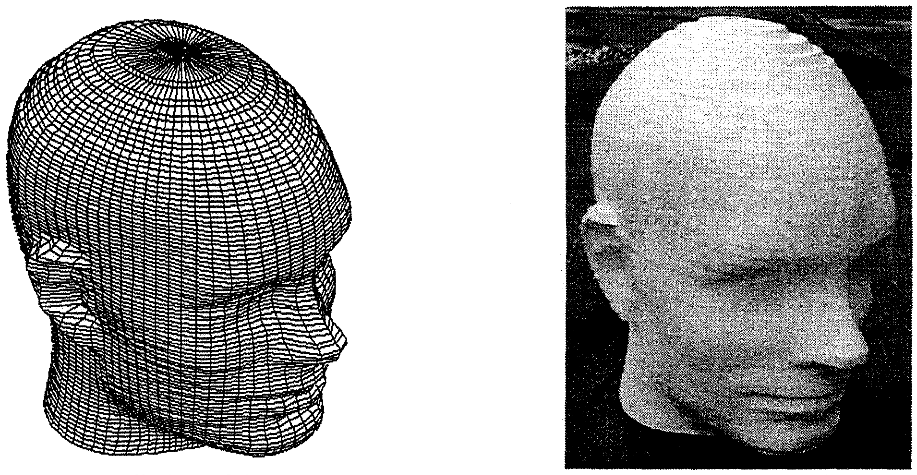

Figure 7: The QWT prediction of the Shapemaker II process is shown compared to a model constructed using Shapemaker II.

that are an integer power of two. This problem can be handled by zero padding. Essentially the model is placed in build volume that is an integer power of two layers thick at the desired layer thickness, and the analysis is done on the entire volume.

CONCLUSIONS

The quasi-wavelet transform is a method for mathematically analyzing the spatial frequency content of three dimensional objects. This technique allows the complexity and manufacturability of a part to be analyzed. Using an inverse transform it is possible to reconstruct the part as it would look if it were manufactured by a specific process. The quasi-wavelet transform is a generalization of the wavelet transform technique that allows any function to be used as a wavelet. This allows any manufacturing process to be modeled by this technique.

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