2-D Wavelet Analysis of Solid Objects: Applications in Layered Manufacturing

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ABSTRACT

In this paper, we introduce two-dimensional discrete wavelet basis functions and their application in the analysis and modeling of surface topography in layered manufacturing objects. In previous work, a one dimensional wavelet transform technique was developed to generate variable thickness layers. [1] For vertical edge layers Haar wavelet decomposition is used in the slicing direction but is not useful in the slicing plane. For frequency analysis within the slicing plane, biorthogonal wavelets provide the desired analysis ability. When analyzing layered manufacturing with ruled edges a true 2-D transform is appropriate. Two-dimensional wavelet analysis simultaneously controls the layer thickness as well as the density of control points required in the surface definition of each layer edge.

INTRODUCTION

1) What is frequency analysis in layered manufacturing?

It is possible to analyze a 3-D solid object in terms of its spatial frequency content. Using spatial frequency analysis, regions of high complexity appear as high frequencies and flat or gently curved regions appear as low frequencies. Wavelet analysis allows spatially resolved frequency analysis, where the location of the high or low frequencies is identified. By operating on the wavelet representation of an object with custom frequency filters, it is possible to simulate the results of various layered manufacturing processes. [2]

2) Why do we want a frequency analysis technique?

- Simulate the results of layered manufacturing
  - Haar wavelet for vertical edges
  - biorthogonal wavelet for ruled edges
- Control variable layer thickness generation
- Identify appropriate slice direction
- Potential file size reduction

WAVELETS

A wavelet transform (WT) decomposes a function into a summation of scaled and translated basis functions called wavelets. [3] An inverse wavelet transform (IWT) sums the basis functions

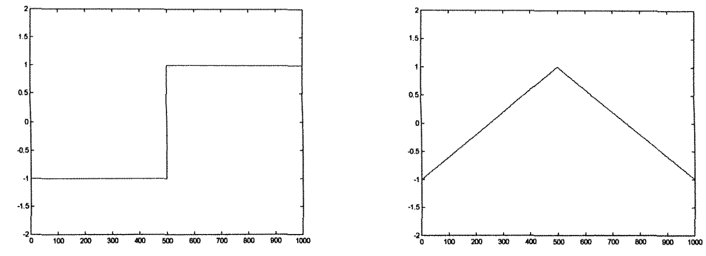


Figure 1. a) Haar wavelet b) biorthogonal wavelet

back together to form the original signal. The wavelets must satisfy certain criteria in order to be valid basis functions. Among these criteria, wavelets must be oscillatory and decay quickly to zero. [4] Two wavelets that have been found useful in layered manufacturing are the Haar wavelet [5] and the biorthogonal wavelet shown in Fig. 1.

Discrete wavelet analysis (DWT, IDWT) assumes that a signal f(x) has been sampled at equally spaced intervals. The sequence length N of the signal being analyzed determines how many wavelet levels can be represented. [6] When $N=2^n$ there are n + 1 wavelet levels. The levels represent different frequencies in the signal. Fig. 2 demonstrates frequency decomposition of a signal into wavelet levels. Level a10 is the lowest frequency level and a1 is the highest frequency level.

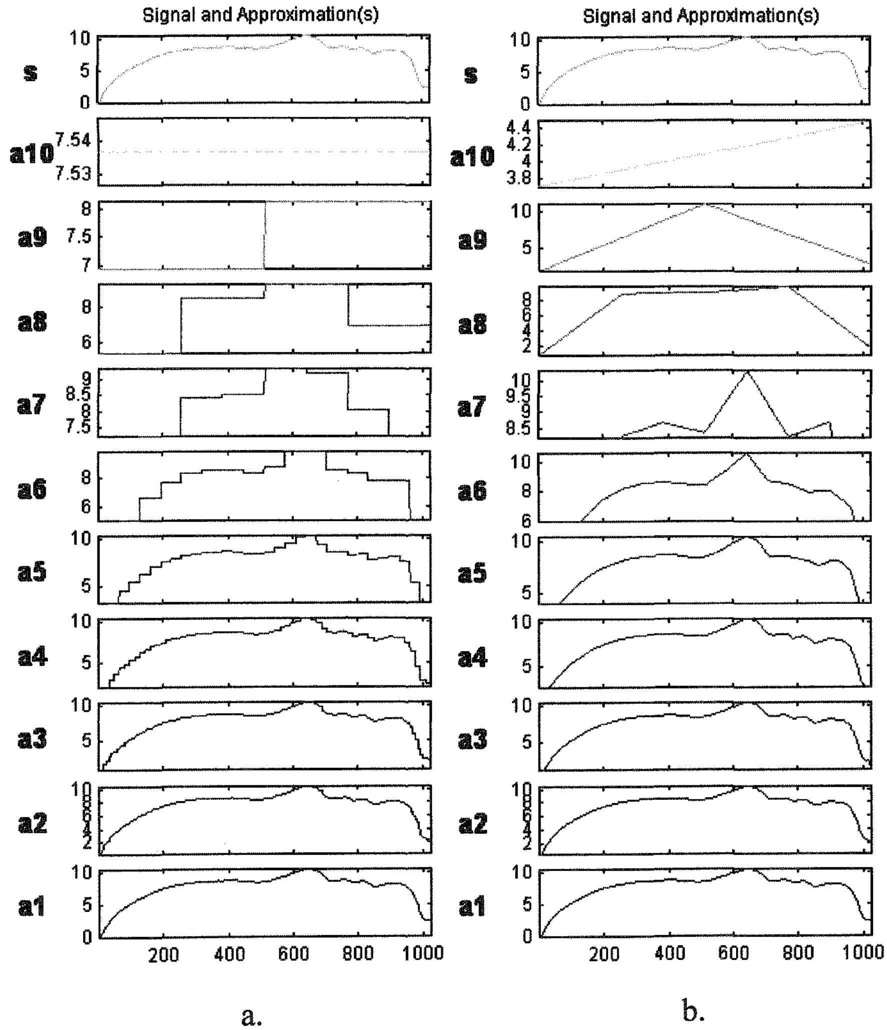


Figure 2. Frequency decomposition of the object. a) Using Haar, b) using biorthogonal wavelets.

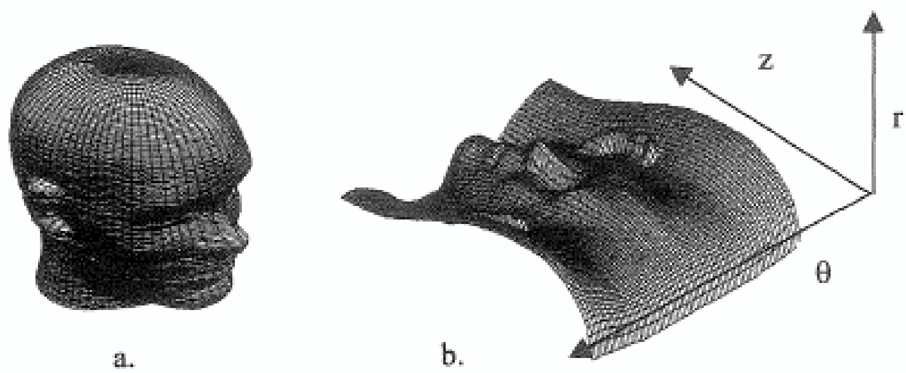


Figure 3. Unwrapping of the object. a) cartesian representation, b) cylindrical presentation.

PROPOSED PROCESS OVERVIEW

The proposed process of analyzing solid objects for frequency content is outlined in Fig. 4. From an STL file, in the cartesian coordinate

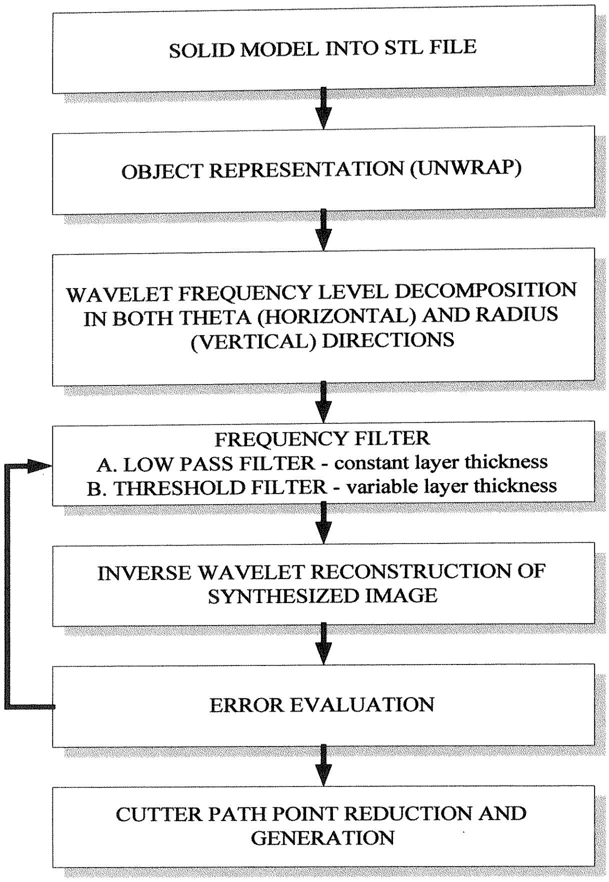


Figure 4. An overview of the 2-D wavelet analysis procedure.

system, 3-D solid objects are sampled at equally spaced intervals in determined sequence lengths N. The object is unwrapped by converting sampled data into cylindrical coordinate representations. Changes in the part radius (r) can therefore be analyzed in two dimensions: vertical (z) and horizontal (θ). Fig. 3 shows how the part is unwrapped. [1] The discrete wavelet transform is extended to two dimensions, which are decomposed and filtered simultaneously. Decisions regarding the type of wavelet basis function and the type of filter are made. After filtering, an inverse discrete wavelet transform is then used to reconstruct the object. From the resulting reconstruction, error is evaluated.

FILTERING

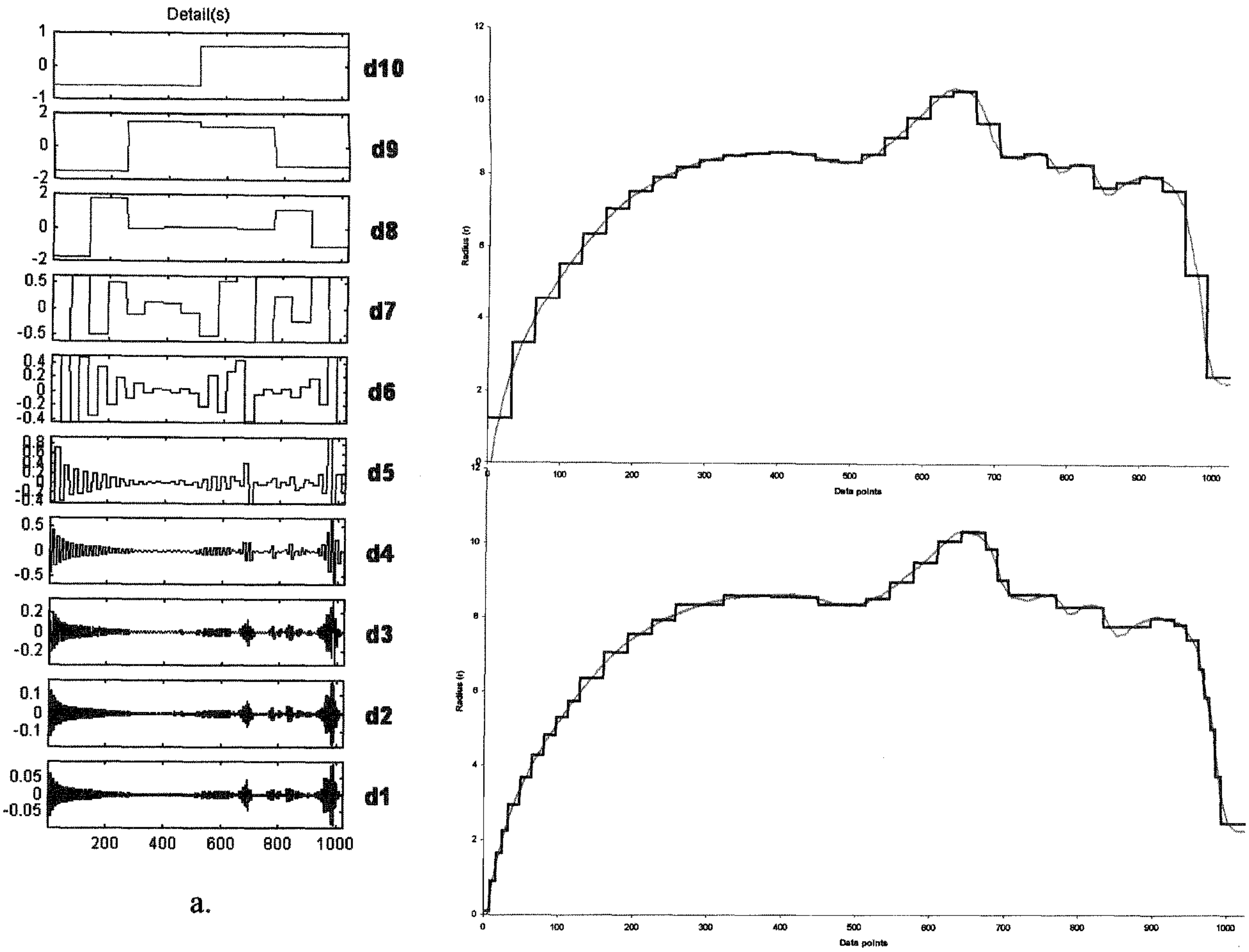
Two types of filters, low pass and threshold, have been examined for application in layered manufacturing.

Low Pass Filter

Here, all wavelet coefficients above a specified frequency level are removed. After inverse transforming, the model contains no frequency components above the cutoff. The resulting model looks as if it were produced by a constant layer thickness LM technique. Fig. 5b. shows these results.

Threshold Filter

Here, all wavelet coefficients below a specified threshold value are removed. The resulting model looks as if it were constructed from variable thickness layers. Fig. 5c. shows this result. Because of the ability to independently vary the filtering of each frequency level, remove wavelet coefficients above a specified frequency level, or perform global filtering of all frequency levels, thousands of possible filter combinations can be constructed.



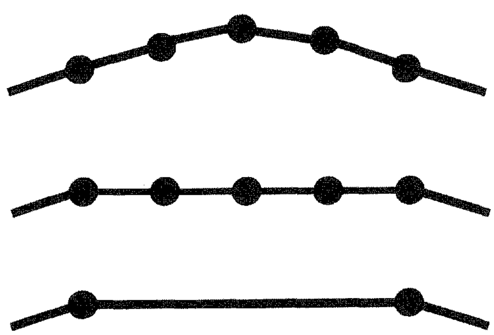
b.

c.

Figure 5. a) This figure shows the Haar wavelet frequency coefficients for a single slice of the head. All ten wavelet levels are shown. Level d1 is the highest frequency. b) Results from low pass filtering, yielding uniform slice thickness. c) Results from threshold filtering, yielding adaptive slice thickness. Filtering was intentionally set high to better illustrate the results.

POINT REDUCTION

When the original model is put into cylindrical format, the file consists of regularly spaced points as shown in Fig. 6a. Threshold filtering forces the points into straight lines, but the total number of points in the file remains constant. Deleting redundant points along each line segment reduces both point density and file size.



a.

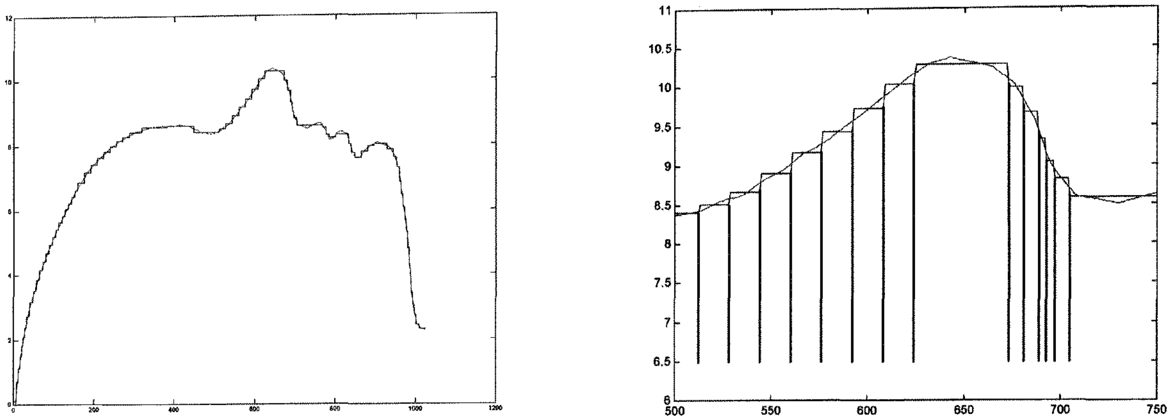
b.

c.

Figure 6. a) Point density prior to wavelet filtering. b) After wavelet filtering. c) After point reduction.

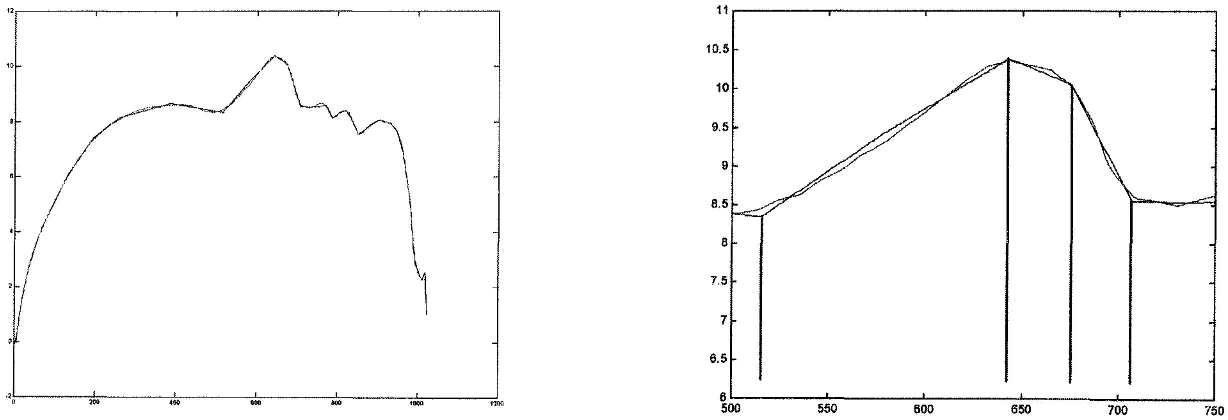
VERTICAL DIRECTION (z) RESULTS

The ability to analyze the object in the vertical direction enables the operator to generate variable thickness layers. In Figures 7 and 8, below, a single contour in the slicing direction was analyzed.



a. b.

Figure 7. a) Vertical slice (side view of front portion of the head) after wavelet filtering. Filtering was intentionally set high to better illustrate the results. b) Close up view of the nose. Notice the variable layer thickness. Gray represents the original contour; black represents the contour after processing. The Haar WT is ideal for vertical edge layered manufacturing. In this example the head was reconstructed from 76 variable thickness layers.

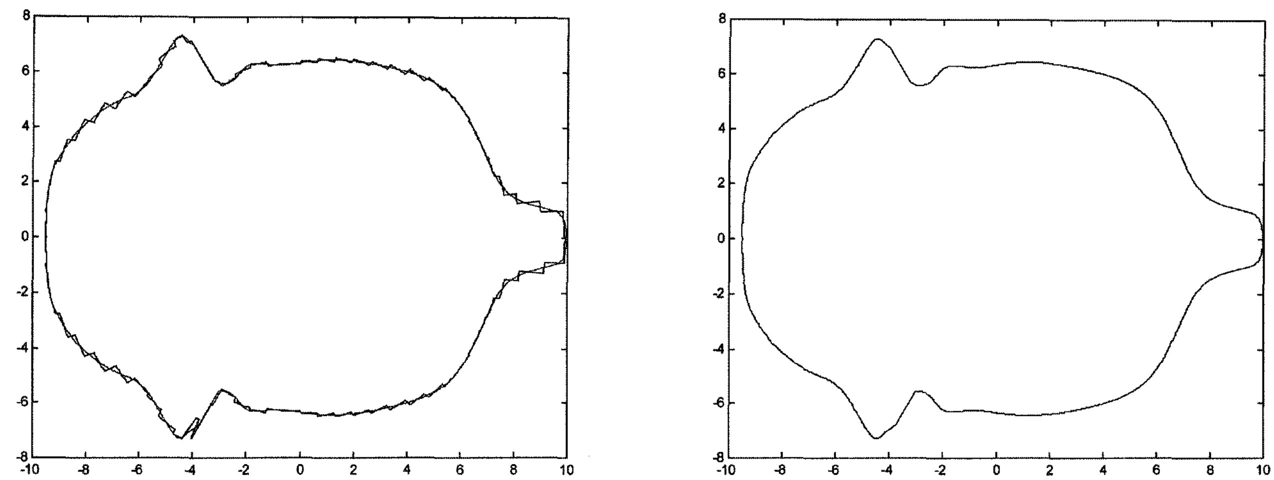


a. b.

Figure 8. a) Vertical slice (side view of front portion of the head) after wavelet filtering. Filtering was intentionally set high to better illustrate the results. b) Close up view of the nose. Notice the variable layer thickness. Gray represents the original contour; black represents the contour after processing. The biorthogonal WT is ideal for ruled edge layered manufacturing. In this example the head was reconstructed from 40 variable thickness layers.

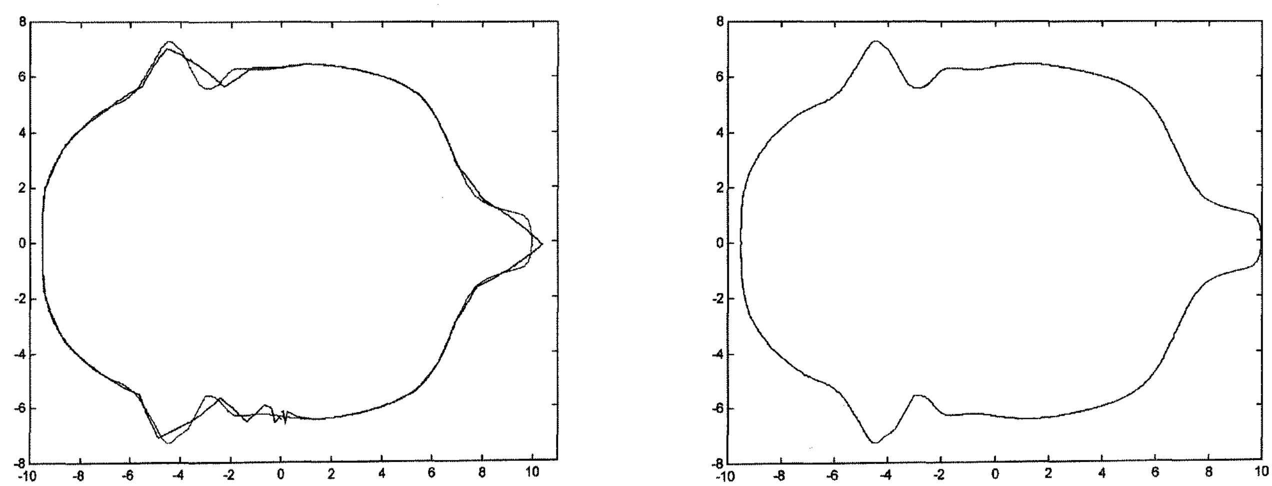
HORIZONTAL (θ) DIRECTION RESULTS

The ability to analyze the object in the θ direction enables the operator to filter unwanted information (high frequency steps) in the cutter path.



a. b.

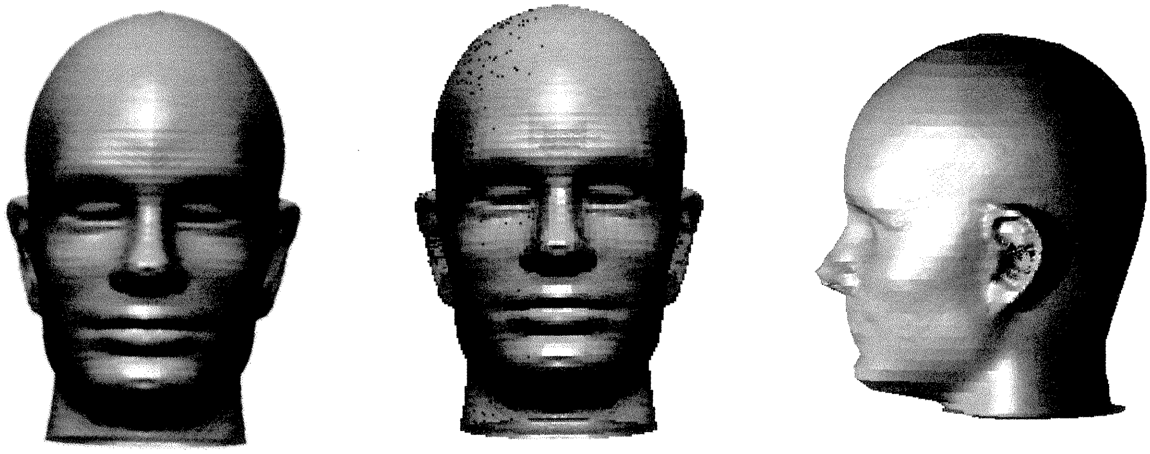
Figure 9. Original slice, gray (1024 points), slice after processing using filtered Haar wavelets and point reduction scheme, black. a) 784 points. b) 1020 points.



a. b.

Figure 10. Original slice, gray (1024 points), slice after processing using threshold biorthogonal wavelets and point reduction scheme, black. a) 560 points. b) 630 points.

The above figures (Fig. 9,10) show how wavelet filtering provides a reduction in the number of control points defining the layer edge while preserving an accurate edge representation. Figures 9 and 10 demonstrate the advantage of using the biorthogonal wavelet over the Haar in the theta direction. Using Haar, 1020 points were required to reproduce the original slice geometry while only 630 points were required using the biorthogonal wavelet. Based on the results shown, we anticipate this analysis will simplify and smooth the cutter path definition file.



a. b. c.

Figure 11. a) Original 3-D object. b) Predicted head after 2-D WT filtering and IDWT (low resolution in theta direction). c) Predicted head after 2-D WT filtering and IDWT (low resolution in the stacking direction).

Fig. 11 shows the original head and the reconstructed head after unwrapping, biorthogonal wavelet transform decomposition, threshold filtering, inverse transforming, and re-wrapping. Fig. 11 (a) is the original STL file. It is assumed that the apparent layering effect is an artifact produced when the head was scanned. In Fig. 11 (b) the filters were set for fine resolution in the layering direction (z) and low resolution around the circumference direction (θ). Note that there is no apparent change in resolution in the z direction. The nose is broadened and the definition around the ears is poor due to the more aggressive filter in the θ direction. In Fig. 11 (c) the filters were set for low resolution in the layering direction (z) and fine resolution around the circumference direction (θ). Now thicker layers are apparent in the z direction. As expected, the low resolution filtering greatly affected the accuracy of the part causing the lips to be completely removed.

LIMITATIONS

There are two primary limitations for the process outlined in this paper. 1) Currently, this technique works only for single-value functions. Decomposing complex geometry into simple sub-parts prior to processing can eliminate this limitation. 2) The sequence length N is chosen so that f(x) is a desirable representation of the original data. For simple, low-resolution parts, a lower value of N (say, 128) is adequate. For higher resolution parts, a larger N (say, 1024) is recommended. Although a larger value of N gives a better approximation, the value of this integer is limited by the physical constraints of the decomposition software or by the desire to perform computations effectively and efficiently. [5]

CONCLUSIONS

The 2-D wavelet transform is a useful tool for mathematical analysis of the spatial frequency content of three-dimensional objects. This analysis enables the prediction of appropriate layer thickness as well as reducing the cutter path point load for layered manufacturing processes. In order to perform wavelet analysis, 3-D objects are decomposed (unwrapped) into a cylindrical coordinate representation. This single valued representation is then wavelet transformed, filtered, and inverse transformed. The choice of filter (low-pass or threshold) is motivated by the application. Uniform layer thickness part decomposition is created using low-pass filtering. Threshold filtering produces adaptively sliced representations.

As expected, there is a direct relationship between filtering and error. Under-sampling and over-filtering were the most common sources of error. The Haar wavelet is useful for analyzing vertical edge layered manufacturing while the biorthogonal wavelet proved useful for ruled edges. In addition, the 2-D wavelet representation of the part is quite compact and may be useful for file compression.

FUTURE WORK

Several areas have been identified for further work. These areas include techniques for error quantification and filter optimization. These techniques will allow user input of maximum error desired in both directions and will adjust the frequency filtering accordingly. Future work includes the development of more sophisticated software tools for analyzing, decomposing, and filtering stereolithography files and the creation of cutter paths. Future software will automate the process outlined in this paper.

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