Heterogeneous Solids: Possible Representation Schemes

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Abstract

Solid freeform fabrication processes allow parts to be built with accuracy and mechanical integrity, permitting them to be used in tooling or form and fit applications. There is already a need for multi-color parts for surgical applications, which will eventually lead to multi-material RP machines. Whether for on the spot color deposition or for functionally tailored multiple materials parts, RP machines with such capabilities are becoming available. They will eventually lead to the true promise of Solid Freeform Fabrication: a system that can build a functional mechanism without assembly, and from multiple materials. This paper is aimed at understanding the new challenges raised from representing solids whose material distribution is changing gradually from one material to another (HC), and those made of a collection of discrete materials (HD). Several representation schemes are reviewed and critiqued. Techniques borrowed from medical imaging and geoscience modeling are used to better understand the modeling of heterogeneous and gradient solids, from a geometric standpoint.

1 Problem Statement

Current solid modeling technologies model an object using its boundaries, implicitly making the assumption that the interior of the solid is made of a single, homogeneous material. Recent progress in commercial RP solutions [1] will allow parts to be built with localized mechanical properties. Thus, as more efforts are directed towards material optimization techniques [2], [3] there is a need to shift the focus of a solid representation scheme from the geometry to a more abstract attribute-oriented representation scheme. In such a representation, the geometry of an object as it is conventionally known, would become the residual of all the spaces spanned by the attributes under consideration. Example attributes are: thermal properties, mechanical characteristics, electrical conductivity and density.

2 Rapid Prototyping

The enabling technology behind the creation of heterogeneous objects has to be able to output an artifact representing an object designed with a computer. Rapid Prototyping (RP) seems to be the medium of choice in such applications. RP embodies digital designs into physical mockups in a matter of hours. The techniques behind the RP process rely on the ability to add (and/or in some cases remove), material at any arbitrary location in space. There are two general classes of RP processes (even though combinations do exist): additive-based processes and removal-based processes. The distinction between the processes depends on whether the process adds material to reach the final shape of the artifact, or if it removes material from a bulk to reach that same shape. The following two sections review those two important families of processes. This review is by all means non-exhaustive, and is here only to briefly outline the principles behind them. Further description of RP processes is proposed in [4].

2.1 Additive Processes

Additive processes involve the following or a variation thereof: the object to be manufactured is broken down in layers and the part is built from bottom to top by stacking cross sections of the object. Each layer is manufactured by way of an additive process, one at a time and in a set order, then the object is shifted down to allow the next layer to be added.

Some of the enabling technologies behind additive processes are based on the hardening of a photo-polymer by a laser (SLA) or by ultra-violet floodlights (SGC), or the cutting of an adhesive paper (LOM). Another variation, in powder-based processes, is to sinter a bin of finely grained powder (SLS) [5]. Some processes also rely on a thermoplastic plotting system, akin to an inkjet plotter to deposit the material (FDM, 3Dprinters). Many of those processes require additional support structures to maintain the artifacts' structural integrity throughout the building stage, thus requiring the additional step of part cleanup.

Some RP makers use a color-coded support material whose removal does not risk obliterating a feature from the part. In some other cases, the technological choice of the process prevents from using a secondary material, and the support structures and the part both share the same material, with the exception being that the support structure is attached to the part only temporary by a breakable junction.

2.2 Removal Processes

This class of processes is more on a par with traditional manufacturing technologies. Essentially, it involves milling, grinding or cutting material from a bulk of raw material. Though this technique certainly has more limitations than the additive processes (recessed pockets are one of them), it does provide a valid alternative to obtain mechanical parts with properties and finish that are often unattainable with additive processes.

The bulk material is typically made of a ductile material, which can be cut at a great rate of speed to quicken the process. Example materials are aluminum, steel, Styrofoam and balsa. It is obvious that for manufacturing reasons, the bulk material should be made of a single material, otherwise the rate of cutting will vary from one material to the next, resulting in uneven surfaces, or rips at the interface between the two materials.

3 Solid Modeling Representation Schemes

Traditional mechanical design encompasses several tasks and disciplines. For instance, designing a car's body involves considerations of structural soundness, ergonomics, aerodynamics, vehicle dynamics, aesthetics and after-life disposability. In addition, modern engineering often professes some form of optimality which insures that a designed product responds ideally to a set of constraints, both from the consumers' and from the manufacturer's perspectives.

It is evident that today's design cycles are complex, demanding and expensive, and that the interplay of different disciplines requires rugged tools to assist the designers and to support collaborative and concurrent work. The widespread use of computers has greatly contributed to reduce the time required to perform the aforementioned activities, leading to a design cycle nearly 100% digital (i.e. performed entirely on computers).

Ideally, the enabling technology behind a digital design cycle is made of tightly integrated software tools revolving around a central application called a CAD (Computer Aided Design) System. Most CAD Systems are mature software products, sometimes resulting from a decade of research or more. Behind all these CAD Systems is the ability to portray a product in a representation allowing the modifications, simulations and variations encountered during the design phase. Since mechanical design has always been concerned with the dimensioning of parts, a natural choice for this representation is based on the geometric features of a product and its dimensions. This is referred to as solid modeling, and the next section will briefly review the representation concepts behind the various modelers found in CAD Systems today.

3.1 Boundary Representation Schemes (B-Rep.)

The B-Rep is a surface boundary representation where solids are defined by a list of their enclosing surface boundaries. This representation originated from the definition of a polyhedra: it is made from polygons, which are made from triangles, which in turn are made from edges and lastly edges are made from vertices. The representation was then further expanded to include arbitrary representations (Beziers patches or NURBS surfaces) for the enclosing surfaces. The surface boundaries are oriented such that given any surface point, the solid interior can be easily determined. For instance, a pyramid with a square base is made of four triangles and a square (Figure 1). This representation is notably good at representing most solids and especially the details embedded in complex shapes, though a known drawback of the B-Rep is the difficulty to verify whether or not a solid is closed or topologically valid.

This difficulty emanates from the task of inferring from the enclosing surfaces (typically surfaces described by two parameters) that a valid, closed 3D entity is defined. A solid is closed when the enclosing boundaries form a closed/fixed volume. A solid is valid if it does not self-intersect. For a more rigorous description of a B-Rep solid topology and its problems, refer to [6].

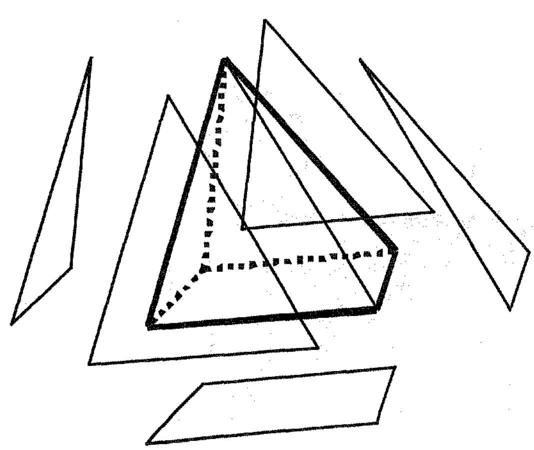


Figure 1: A Boundary Representation for a Pyramid with a Square Base.

3.2 Constructive Solid Geometry (CSG)

The CSG representation [7] defines solids as being made from three-dimensional volumetric primitives (blocks, cylinder, spheres, cones...) associated together using binary operators (intersection, union, subtraction) to form a binary tree (CSG tree) whose end result (the topmost node) is the desired solid. The binary operator requires two arguments to yield a result. A binary tree [8] is defined recursively (up to a sufficient depth) as being constituted by a root node and two binary trees (which in turn are made of two nodes and four binary trees). In the case of a CSG tree, the end leaves are primitive solids and the topmost root node is the desired product, the intermediate nodes being the results of intermediate operations.

Creating complex solids by combining simple primitives is intrinsically appealing when the need to manufacture a product appears, as some operators are homeomorphic to material removal processes: for example, drilling a hole in a block is similar to subtracting a cylinder from that same block.

Inherently, since the operators applied on the primitives are isomorphic, we are generally guaranteed that the resulting solids are both closed and valid. The cost of having a solid defined as being made from other interrelated solids is associated with the complexity required to manage the CSG tree which may need to be entirely recomputed when updates are made to the model. Also, displaying the solid requires it to be converted into simple display primitives such as triangles, a non-trivial operation.

It should be noted that most modern CSG solid modeling packages do rely at some point on a mixed representation of solids (mixed between B-Rep and CSG), as CSG lacks the capability to perform local operations such as fillet, a typical operation in B-Rep based solid modeling packages. Overall, the CSG representation does an excellent job at representing objects, however, as object complexity grows, the traversal of the binary tree becomes increasingly computationally intensive often resulting in long updates for minor changes.

3.3 Set Theory

This mathematical representation principle lies almost entirely in the following consideration: "S is a solid which can be defined as follows, given a boundary B defined by an implicit equation $f(x,y,z) = 0$, $M(x_M,y_M,z_M)$ is a point such that:

- $f(x_M,y_M,z_M) < 0 \Rightarrow$ the point is outside S
- $f(x_M,y_M,z_M) = 0 \Rightarrow$ the point is on the border of S (=B)
- $f(x_M,y_M,z_M) > 0 \Rightarrow$ the point is inside S

The set of all points inside S and on its border defines the solid. Unfortunately, the strict nature of the membership of a point M in the set (it is either in, on or out of the solid) precludes polyphase objects or gas mixtures to be represented accurately. The need to model these objects is now omnipresent: whether it is for the Finite Element analysis of an engine's explosion cycle or for a feature animation film, there is indeed a need to represent objects completely (which then raises the correct definition of an object).

Used initially for manufacturing, this set-based representation for solids has been outgrown by the demands put on it. An example application for this representation is svLis 3.0, a set theoric based solid modeling kernel [9]. This kernel allows Boolean operations to be performed on solids represented using set theory. Nevertheless, the problem at hand and the advances in modern manufacturing processes are now raising new interest in this representation:

- Ability to have multiple membership for a point (e.g. being part of several materials),
- Allow porous models to be created,
- Allow one (or more) materials to blend gradually
- Allow mechanical/thermal/electrical/… properties to be specified locally

4 Volumetric Representations

4.1 Voxels

Two definitions are commonly found for a voxel, the basic unit of a volumetric representation:

- A voxel, like a pixel, has 0 dimension. Akin to the way that a digital image is represented by an array of pixels, a volumetric dataset is made of voxels laid out on a regular 3D grid. For every voxel in this dataset, a scalar value quantifies the membership of this voxel to a given reference material. Often, such a representation can be termed 'spatial enumeration.'

- A voxel can also be considered to be a cube of small size.

There is no strict definition for a voxel, but we will choose to remain consistent with the definition of a pixel and say that the voxel is a 3D extension of a pixel and thus, has no dimensionality.

4.1.1 Volume Rendering

Thanks to constant increases in the computing power of desktop computers, Volume Rendering, traditionally reserved to medical imaging, is now finding its way in areas such as failure analysis, computational fluid dynamics and meteorology. Though volume rendering [10] merely refers to the act of rendering volumetric data, our primary concern is not with this task, but rather with the underlying representation of the data to be rendered, along with some of the concepts involved in the handling of this data.

Initially, volumetric data was gathered from Magnetic Resonance Imaging (MRI) scanners. These scanners gather data from human organs by measuring the energy received from disrupted atoms as they realign after being subject to an out-of-phase excitation signal. Since tissues of similar composition exhibit similar responses to this out-of-phase, signal consistency is insured in the interpretation of the data. The snapshot of a cross section is developed by using this response as a basis for the computation of the light intensity of the pixels constituting the snapshot. Conceptually, the scanning of the organs is made in all three planes at the same time but a phase shift in the excitation signal (for all three planes) generates a phase shift in the restituting signal, which allows cross section images to be isolated. Without computers, a doctors' attention has to span several cross sections at a time to correctly interpret the 3D dimensional nature of the organs. Stacking the cross sections in 3D space not only helped minimize errors of interpretation but also provided doctors and surgeons with a 3D image that could be panned, zoomed and rotated to better locate individual details.

The principles of Volume Rendering are similar to those of raytracing: for every pixel on the view plane, a ray of light is cast orthogonal to the view plane and directed toward the objects to be rendered. Each elementary element of volume (termed a voxel) intersecting the ray, is composited (or combined) with other intersecting voxels to provide the final light intensity of the pixel on the view plane. Since this computation is performed repeatedly for every view generated of the volume, a weight can be attributed for voxels of similar absorption to filter out some tissues while emphasizing other. A noteworthy recipient of these techniques is the Visible Human [11], where a man's body was entirely digitized through a similar process, resulting in 1800 cross sections of his body (1000 transverse and coronal MR scans, 1878 transverse scans w/ corresponding photographs).

4.1.2 Voxel Based Representations

Volumetric data sets emanating from human organs are extremely dense, and much of the voxels forming the volume are non-empty. Thus there is little incentive to design space efficient data-structures. The main efforts made to improve the storage of volumetric datasets are aimed at improving the speed of volume rendering algorithms by preprocessing the voxels [12], [13] and skipping empty cells [14]. The use of voxels to represent matter raises an interesting issue when it comes to render the outer surfaces of objects. These surfaces exhibit a shape that is often inappropriately captured by voxels. Nevertheless, these surfaces are rendered as if they where described by a conventional polygonal mesh using a normal estimation algorithm and an interpolation kernel [15]. The improvement over 'naïve' volume rendering is certainly significant, but interpolating the surfaces makes it difficult to use this technique for solid modeling where surfaces are often designed with strict tolerances.

Additionally, since the voxels are a sampling of physical data, the storage requirements for those sets are severe: a model containing a grid made from 512 arrays of 512x512 voxels uses at the very least 128 M-byte of memory (for 1 bit encoding). The data set is usually very large and thus not easily held in main memory without using some form of virtual memory. Another application of a voxel based modeling scheme for solids is also presented in [16], where the aim is to see how a voxel-based modeling fits within an RP enabled manufacturing environment.

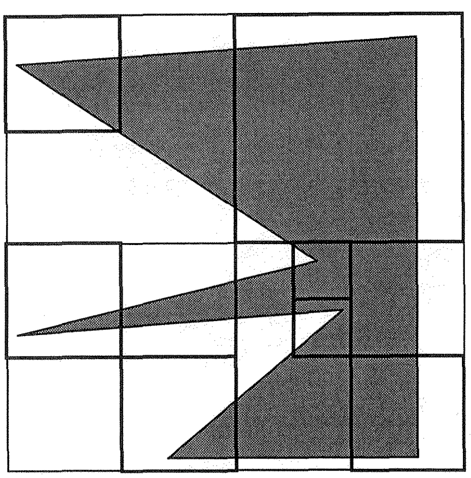
It should be noted that the use of voxels for solid modeling does greatly simplify CSG operations on solids [14]: Boolean operations performed on solids are reduced to plain voxel to voxel operations whose outcomes are extremely simple to compute. For instance, an empty voxel intersecting a full voxel is an empty voxel, an empty voxel 'union' a full voxel is a full voxel.

4.1.3 Octrees

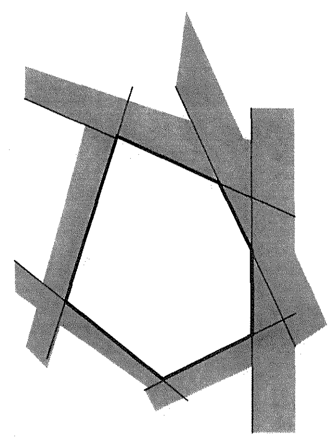
Samet [17] proposed to use a hierarchical space-partitioning scheme in order to store volumetric data, specifically the Octree. The principles of an octree are simple: to recursively subdivide a cube into 8 smaller cells $1/8^{th}$ the size of the original cube, until either the cube is empty or its content is below a set threshold size. Ayala *et al.* [18] then proposed to perform Boolean operations and rigid body motions on octrees. Although the octree encoding has an interesting potential, it is in the termination criteria of the decomposition sequence that the complexity lies. Some example termination criteria, for 3D polyhedra, are:

- A single vertex in the cell,
- A single edge in the cell,
- A single face contained in the cell,
- Several faces sharing the same edge in the cell (which may or may not be in the cell, Fig. 3a).

In the case of its 2D counterpart, the quadtree, it is demonstrated that regardless of the termination criteria used, the number of cells is proportional to the perimeter of the object to be decomposed and to the resolution of the decomposition ([17], Chapter 1). Thus, for complex closed hollow 2D polygons, a quadtree decomposition can yield a large number of cells, which restricts its use.



3a. Sample Termination Criteria for QuadTree (single edge sharing the same vertex)



3b. Cell Tree Decomposition of a Convex Polygon

Figure 3a-3b: Sample Termination Criteria for Quadtree & Cell Tree Decomposition

A solid modeling application using an octree based representation scheme is presented in [19]. In this application the termination criteria of the decomposition is to have a single face contained within each cell. Once again, a noteworthy benefit from this volume based decomposition is the simplicity of Boolean operations on solids: though these are slightly more complicated than the operations performed on voxels (cells of different sizes may require additional treatment), their outcome is easily predicted. Equally important, rigid body transformations (rotations, scaling and translations in 3D) are also described without directly applying a transformation matrix to the coordinates of the vertices of the objects and decomposing it (the operation is performed directly on the octree).

4.1.4 The Cell Tree

The Cell Tree [20] is an encoding for general polyhedral point sets of arbitrary dimensions (bounded or unbounded). This encoding represents a polyhedral by the algebraic sum of simpler, convex polyhedra (holes are 'subtracted'). Each convex polyhedral chain is described by the intersection of halfspaces and represented in a vector (Fig. 3b). A halfspace partitions an n-dimensional space in halves, and is a hyperplane of dimension n-1. The representation is assumed to be minimal: a halfspace not intersecting any other halfspace (e.g. empty intersection) is removed from the vector. Also, since the description is minimal, a halfspace is a boundary of the convex polyhedral it is describing. To further normalize this representation, the halfspaces used in the description of a polyhedral are listed in a single location. A convex polyhedral is then represented using a list of 1s, -1s and 0s referring to whether or not this polyhedral is respectively using a given halfspace, its opposite or not using it.

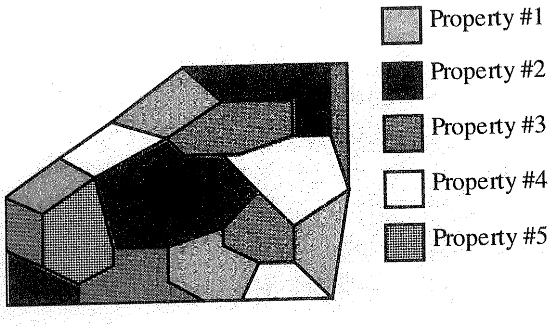
Performing Boolean operations on polyhedra is then a slightly more complicated matter than with the previous representation schemes, though it does not necessitate elaborated algorithms. The union of two polyhedra involves merging the two databases of halfspaces and adding more convex polyhedra to the chain. Subtracting a polyhedral is a similar task, since the description allows unbounded polyhedra, the complementary of the polyhedral to be subtracted is created by negating the list of vectors it uses, and then added to the convex polyhedra chain.

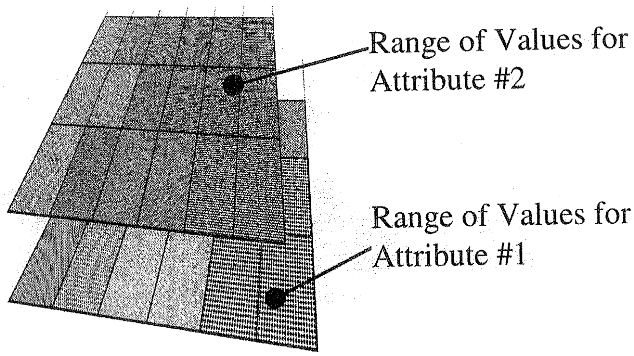
It is interesting to note that this representation offers some features that other representation do not: in the case of 3D polyhedra, there is no need to evaluate the vertices or the edges, strictly speaking, the entire shape is represented using only planes. However, a very stringent requirement in this representation is that every 3D solids must have a convex decomposition.

4.1.5 Geoscience Modeling

Geoscience modeling shares some similitude with mechanical CAD, especially in the area of domain (2D or 3D) representation: 2D geographic domains can be represented by way of polygons, and a 3D hill can be represented with a set of 2D elevation maps. The added dimension of attributes such as soil composition, humidity and erosion tends to make this modeling certainly very relevant for the problem at hand. The critical aspects relevant to us in geoscience modeling [21] are those of uncertainty and fuzziness of the boundaries of the objects described.

According to [22] there are two main geographic data models used: the exact model and the continuous field model (Figure 4a-4b resp.). The exact model is made of adjoining polygons tagged with a number of attributes representing a region with set properties (such as soil composition, pollution…). Conversely, the continuous field model, instead of attaching the attributes to a topology, assumes that each attribute is a continuous and smooth varying function over space. In practice, the function is often discretized on a regularly spaced grid and the overlay of all the grids should provide a meaning similar to the one of the exact object model (with the exception of the boundaries). Note that [23] retained an approach resembling the exact field model in his treatment of multiple material solids.





4a. The Exact Model

4b. The Continuous Model (discretized),

Figure 4a-4b. Geographic Data Models

Strictly speaking, heterogeneous solids do not have any uncertainties associated with their boundary definition. Evidently, the designer of a part knows exactly where a material is and where it isn't. It is interesting to observe how for example a soil boundary is modeled and how the change of a soil (which is often gradual) is treated in terms of fuzziness of the set (the uncertainty, due to measurement errors, is not relevant here). For soils, the "core" region is defined as the region that attains maximal membership in a given attribute, while the boundary is defined as the region attaining between 0 and the maximal membership (the latter interval being open on both ends). The use of fuzzy logic to model the boundary is proposed [24] and a similar approach, pertaining to a gradual change between two materials is certainly an interesting issue. A more elaborate discussion to differentiate a 'Boolean region' from a 'fuzzy region' is presented in [22]. The discussion illustrates why the problem of point membership in a region can not always be a true or false answer in terrain modeling.

5 Heterogeneous Objects Approach at the University of Michigan

Initially, the University of Michigan's efforts were aimed at a theoretical approach to representing heterogeneous solids, [23], [25], [26]. The representation of a solid made from n materials was viewed as an application from the geometrical space to the primary material space [23]. In this context, an n-dimensional vector is associated to every triple (x, y, z) within the solid under consideration. This n-dimensional vector represents the material composition as a combination of the primary materials, such that the sum of all the individual components equals to one. Regularized set operations are then defined on these solids. Furthermore, in another work several blending functions between primary materials are extracted from the literature [28]. Note that no detailed representation structure of the data is proposed to date

It is only recently [27], with the use of a DMD (Direct Material Deposition) apparatus that the attention has shifted to the entire design process: from design to manufacturing while including design homogenization to obtain the ideal microstructure pattern distribution. The material is considered at the microstructure level, which evidently translates into large amounts of data and many variables to consider during the optimization stage. This representation lacks the ability to group together a large number of similar microstructures to reduce the amount of data needed.

6 Conclusion

Several representation schemes have been reviewed. As expected, there is a need to combine the features of traditional mechanical representations with the features of volumetric representations. There is no perfect solution in any of the prevalent representation schemes, and effort must target either the development of novel schemes or the combination of available representations. Furthermore, once the problem of material representation is resolved, these same principles can be applied to other types of attributes such as electrical conductivity or thermal resistance. This will allow heterogeneous solids to be completely and entirely represented.

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