Functionally Gradient Material Design and Modeling Using Hypertexture for Solid Freeform Fabrication

Seok-Min Park, Richard H. Crawford, and Joseph J. Beaman

Laboratory for Freeform Fabrication
The University of Texas at Austin

Abstract

SFF technologies have the potential to become manufacturing process that are capable of producing parts that have not been feasible by other techniques. The fabrication of Functionally Gradient Material (FGM) is one of the possible candidate. It is critical to provide three dimensional material gradient data in the solid model to fabricate FGM. The approach is to model spatially varying material density distribution on implicit solid geometry using a hypertexturing scheme and a procedural volumetric modeling. It is essentially an extension of procedural solid texture synthesis, which has been effectively used to model cloud, gas, and flow stream. It will save the amount of information storage especially when the gradient pattern is repeating. Geometric operation becomes convenient since the material data are linked to the geometry only by the reference surfaces.

1 Introduction

SFF technologies have the potential to become manufacturing process that are capable of producing parts that have not been feasible by other techniques. The fabrication of Functionally Gradient Material (FGM) is one of the possible candidates. An FGM is a non-homogeneous solid which exhibits spatially varying volume fractions of the constituents. It has its micro-elements systematically and continuously distributed and controlled so as to provide functionality suited to the intended service environment. Layer based manufacturing becomes a natural choice when material distribution inside a part varies continuously in three dimensions. To accomplish the fabrication of FGMs, SFF processes must be supported by advanced software. The SFF community is currently experiencing a growing need for including additional information, such as material distributions in the solid model.

For fabricating FGMs, it is critical to provide three dimensional material gradient data in the solid model. The material data should be able to exist in discrete, gradient, or mixed form. Predetermined spatial distribution data must easily be represented in the data format. The modeling tool should support procedural modification of the distribution interactively.

2 Approach

2.1 Hypertexturing

I used a hypertexture scheme to model spatially varying material density distributions. Hypertexture is a technique which is intermediate between shape and texture by using space-filling applicative functions to modulate density. The model is essentially an extension of procedural solid texture synthesis that is well known in computer graphics, but evaluated throughout a volumetric region using a Density Modulation Function (DMF) [4].

In hypertexture, a continuous characteristic function which is a mapping from $f : R^3 \rightarrow [0, ..., 1]$ is defined for the solid geometry. All points $\vec{x}$ for which $f(\vec{x})$ is zero are said to be outside the object. All points $\vec{x}$ for which $f(\vec{x})$ is one are said to be strictly inside the object. Finally, all points for which $0 < f(\vec{x}) < 1$ are said to be in the object s fuzzy region. This formulation gives the object surface an appreciable thickness. Finally, the fuzzy shape and the solid become unified once solid texture functions are combined with the function that describes the object s fuzzy region [4].

Procedural texturing or modeling methods can be categorized into two types: explicit and implicit methods. In implicit methods, a texture pattern is defined as a function f of points in the texture space, and the pattern consists of a level set of f, that is, the set of all points at which the function has a particular value. Since implicit models tend to be continuous throughout a region of the modeling space, they are appropriate for continuous density and flow phenomena such as clouds and fog [4]. Thus, the implicit scheme will be chosen to model the material gradient because of their ease of specification and smoothly blending density distributions. The implicit density functions are best defined by summed, weighted, parameterized, primitive implicit surfaces [4].

2.2 Implicit Surfaces

Algebraic implicit formulation was used as a representation scheme for both solid and sliced geometries. Implicit surfaces for solid geometry are two-dimensional, geometric shapes that exist in three dimensional space; they are defined according to a particular mathematical form. Intuitively, an implicit surface consists of those points in three dimensional space that satisfy some particular requirement. The requirement is represented mathematically by a function, generically named f, whose argument is a point $\vec{p}$. By definition, if $f(\vec{p}) = 0$, then $\vec{p}$ is on the surface. The function f does not explicitly describe the surface, but implies its existence [1].

Compared to parametric surfaces, implicit surfaces are receiving increased attention especially with respect to their accurate yet compact depiction of solid objects and their innate blending properties. That is, implicit surface functions naturally describe the interior of an object, whereas a parametric description of an object usually consists of piecewise surface patches and requires additional information for the interior. Geometric queries, such as point classification to determine whether a point is inside, outside, or on the surface, are simpler with implicit surfaces that with parametric surfaces. The ability to enclose volume and to represent blends of volumes endows implicit surfaces with inherent advantages in geometric design and the corresponding fabrication.

The union of two algebraic surfaces, unlike the union of two solid models, is usually given by the product of the corresponding algebraic functions. Intuitively, if f is zero, then any multiple of f, including multiplication by another function, is zero. It is called the closure property. Unfortunately, the multiplication $f_1 f_2$ confuses the sense of inside and outside. That is, points that are within both spheres as well as points that are beyond both spheres evaluate positively; only points within one and only one sphere evaluate negatively.

Solid modeling emphasizes operations on volumes rather that on surfaces. Accordingly, it typically defines the union of objects f_1 and f_2 by $min(f_1, f_2)$. Intuitively, if a point is within any sphere it evaluates negatively, regardless of the number of surrounding sphere. Conversely, $max(f_1, f_2)$ evaluates positively if a point is outside any sphere, thus representing the intersection of the volumes.

Analytic expressions approximating union and intersection are given as;

$$\begin{aligned} union_a(f_1, \cdots, f_n) &= (f_1^{-a}+, \cdots, +f_n^{-a})^{\frac{-1}{a}} \\ intersect_a(f_1, \cdots, f_n) &= (f_1^{a}+, \cdots, +f_n^{a})^{\frac{1}{a}} \end{aligned}$$

where $a > 0$, $\lim_{a \to \infty} union_a(f_1, \cdots, f_n) = min$, and $\lim_{a \to \infty} intersect_a(f_1, \cdots, f_n) = max$.

2.3 Density Modulation Functions (DMF)

The geometric gradient information is determined and controlled interactively by a DMF, which is used to modulate an object's density within its material space. It consists of the several procedural functions such as bias, gain, and noise functions which are the base level functions that higher order DMFs are built upon. These functions are used to control some aspect of an object s spatial characteristics. The following briefly introduces these functions.

The bias function, is mainly used to either push up or pull down an object's density around the middle of the fuzzy region. The bias function is typically defined by $t^{\frac{\ln(b)}{\ln(0.5)}}$. The values at three points are fixed such that , $bias(0) = 0$, $bias(0.5) = b$ and $bias(1) = 1$. By decreasing or increasing b, the values in an object's fuzzy region can be biased up or down smoothly [12].

The gain function can be effectively used as an intuitive method to control whether a function spends most of its time near its middle range, or, conversely, near its extremes. As a result, the density distribution can be tweaked to be either flatter or steeper across the fuzzy region. The gain function over the unit interval, for example, can be defined as follows.

$$\begin{aligned} gain(0) &= 0 \\ gain(0.25) &= 0.5 - \frac{g}{2} \\ gain(0.75) &= 0.5 + \frac{g}{2} \\ gain(1) &= 1 \end{aligned}$$

By controlling the value of g, the rate at which the midrange of an object's fuzzy region goes from 0 to 1, can be increased or decreased [12].

In case that the material information is supplied in a discrete data form, for example, directly from FEM analysis, a polynomial interpolation can be exploited to interconnect the data points in a mathematical form for DMF. For graded compositions, analytic functions must be defined, capable of representing smooth variation over the domain of material subspace. There exists a number of effective polynomial bases for interpolation. One of the most popular bases is the *Bernstein-Bézier* basis. The basic formulations are as follows.

$$Tensor : P(x, y, z) = \sum_{i=0}^{m} \sum_{j=0}^{n} \sum_{k=0}^{p} W_{ijk} B_i^m(x) B_j^n(y) B_k^p(z)$$

$$Barycentric : P(x, y, z) = \sum_{i=0}^{m} \sum_{j=0}^{m-i} \sum_{k=0}^{m-i-j} W_{ijk} B_{ijk}^m(x, y, z)$$

$$B_{ijk}^{m}(x,y,z) = \binom{m}{ijk} x^i y^j z^k (1-x-y-z)^{m-i-j-k}$$

The composition at a point can be considered as a blend of the control compositions with their influence determined by the value of their basis functions, analogous to the surface reconstruction with a mesh of control points.

In order to represent objects made of multiple materials using hypertexture, the mathematical space must be modified. The space for the model must now include material subsets apart from the global spatial dimensions that captured the geometry and topology of the object. The material subset also needs to provide its own space for geometry and topology because the hypersurfaces should be in primitive volume and it is convenient to separate homogeneous regions and heterogeneous regions.

3 Results and Discussion

3.1 Discrete Point Data Set

I used a 64 equally spaced discrete point data set in three dimension for demonstration. This data set was scanned into the program to create an implicit polynomial form of continuous DMF by tensor product of trivariate *Bernstein-Bézier* interpolation. It was first mapped onto the pre-determined material subspace domain which contains the graded portion of the secondary material only. If discontinuous or isolated material distribution is needed, more material subspace would be created. One of the surface of the material subspace was mapped onto the surface of the geometry with the prescribed depth. Figure 1 shows an example of material gradient representation on a simple cube and its cross section in z direction.

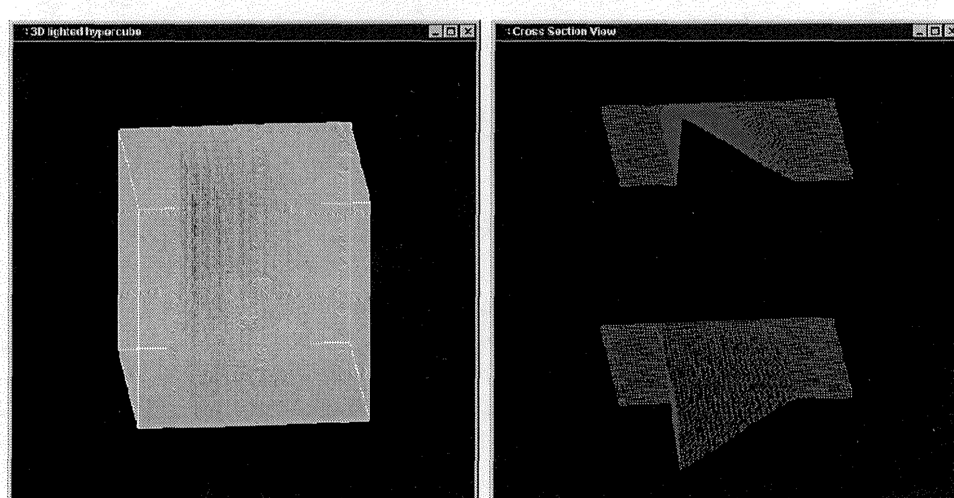

Figure 1: Trivariate *Bernstein-Bézier* interpolation

3.2 Pattern Sketching

When gradient information is based on iterative experiments, designers need to sketch and modify the gradient pattern in the solid model in accordance with the experimental results.

I used the cascaded form of the Bias and Gain functions, which are controlled only by Bias Coefficient, b and Gain Factor, g. The mapping on the material subspace and the solid geometry followed the same procedure as the discrete data set representation. The sketched material gradients with different Bias Coefficient and Gain Factor, are shown in Figure 2

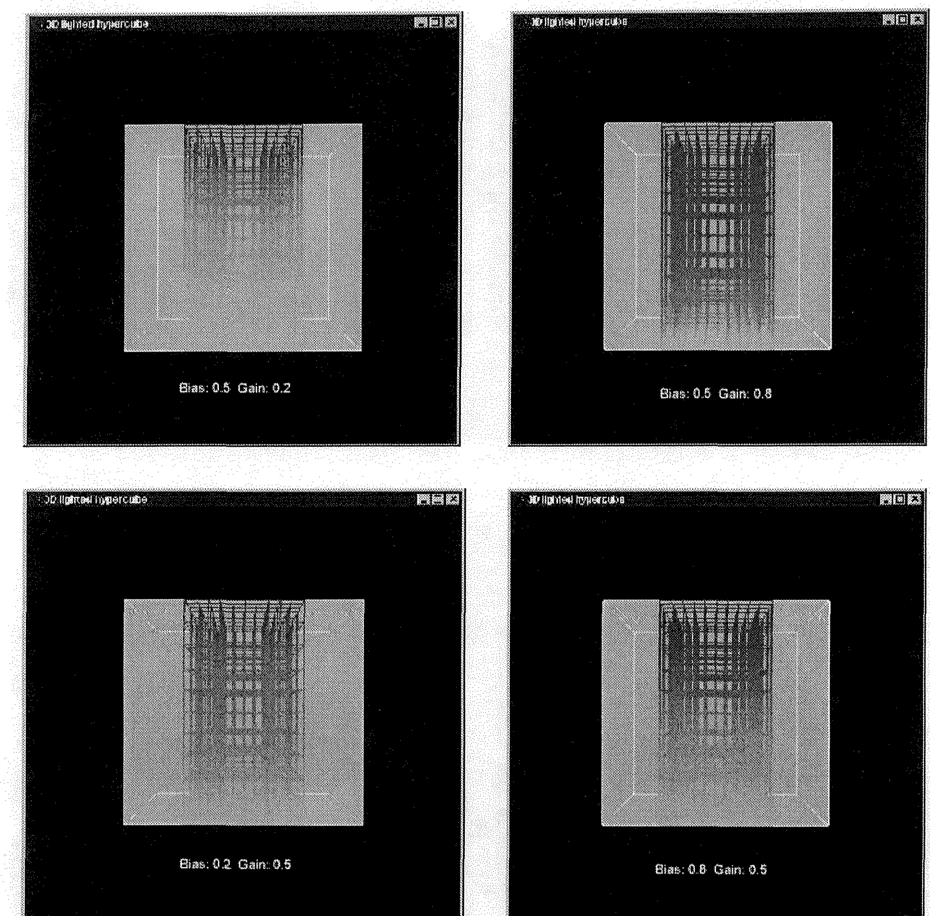

Figure 2: Material Gradient by Bias and Gain Functions

3.3 Applications

The first application chosen for demonstration, is the Wiffle Cube, a rounded cube with a sphere removed. It is defined by $1-(a^2x^2+a^2y^2+a^2z^2)^{-6}-(b^8x^8+b^8y^8+b^8z^8)^6=0$, with $a=0.43$ and $b=0.5$. A sharply edged wedge occurs along each circular opening of the cuboid. As shown in Figure 3, this implicit surface was first polygonized and rendered using the polygnization scheme and the surface normal of each polygon.

The Bias and Gain functions were applied on the material subspace of the spherical coordinates with given range of radius. The highest density at the inner surface of the spherical shell and zero density at the outer surface. Figure 4 shows the material gradient representation and its sliced geometry that were mapped onto the center of the Wiffle Cube. Since both geometry and material data are formulated in implicit algebraic form, they retain the clo-

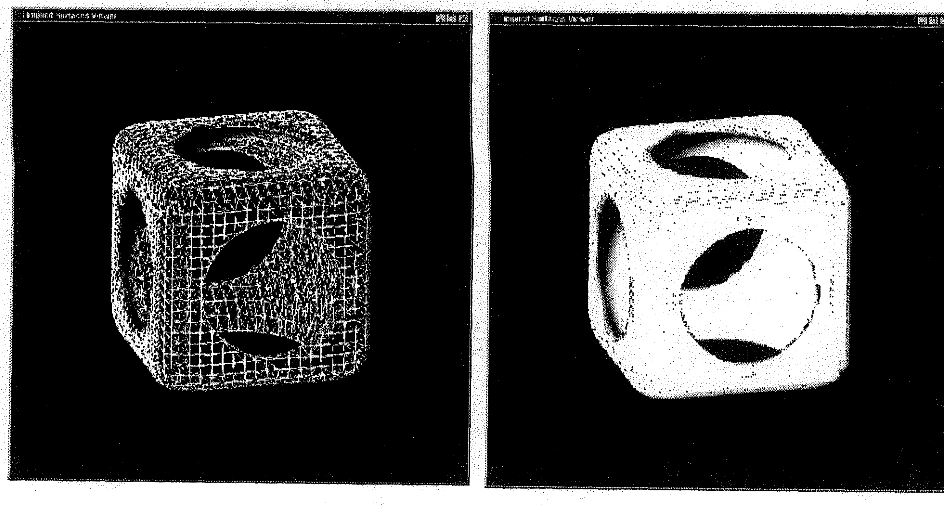

Figure 3: Wiffle Cube

sure property that preserve the implicity of the representation even after geometric operations. Therefore, only by specifying a cutting plane, the slicing of the material and the geometry can be represented in implicit form.

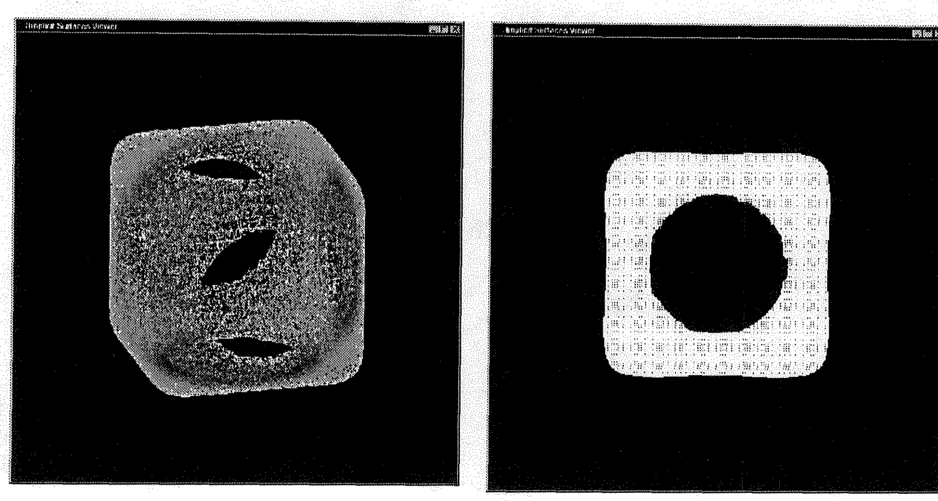

Figure 4: Spherical Material Gradient in Wiffle Cube

FGMs of ceramic and metal can be employed in advanced dental and medical applications, mainly as implants and replacements. As bioceramic materials such as Calcium Phosphate, favorably bioreact with existing bone, they can serve as porous media to support the ingrowth of new bone tissue, which results in permanent bonding with the body. The strength and life of the implants can be even more enhanced by applying biocompatible metal such as titanium for the core of the implant.

A simplified human femur implant for hip socket was created using blending and union of implicit spheres and elipsoids. The base shape was formulated in the following form. Detailed features were added by feature based boolean operations. The geometry of the implant is rendered in Figure 5.

```
Implant(x, y, z)
       blend=5.0
       newx=0.5*x*1.414+1-0.5*y
       newy=0.5*x+0.5*(-2+y)*1.414

       tmp=   pow(elipsoid(y,x,z,7.5,0.25,0.25),-blend)
              +pow(elipsoid(newy, newx, z, 0.6, 0.1,0.1), -blend)
              +pow(pow((y-3.4)*4/3,2)+pow((x-0.6)*4/3,2)+pow((z-0.0)*4/3,2), -blend)

       return pow(tmp,-1/blend)-1.0


elipsoid(x, y, z, a, b, c)
       result=x*x/a+y*y/b+z*z/c
       return (result < 0.001?  0.001 :  result)
```

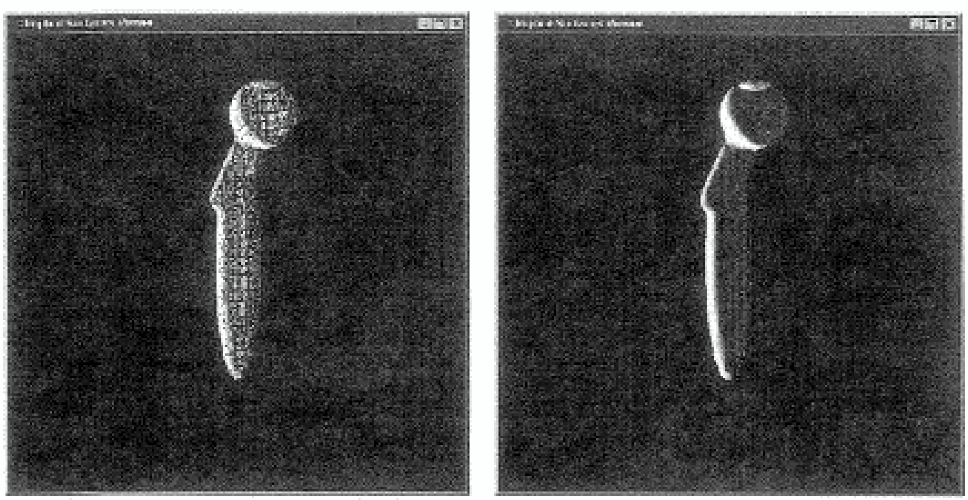

Figure 5: Implicit Model of Femur Implant

The same algebraic surface in implicit form was used for the material subspace. The gradient region in the subspace is specified as a function of radius within the range of the material subspace. Therefore, The material distribution has constant depth of a primary material shell. Localized distribution also can be achieved by modifying the blob model for the material subspace. The material gradient and the cross section along the z axis of the implant are shown in Figure 6.

4 Conclusion and Future Work

I have successfully implemented material gradient varying in three dimension in implicit algebraic surfaces. Material gradient region in the solid can be designed by generating implicit

Figure 6: Material Gradient in Femur Implant

function with the size of either equal to or smaller than the geometry. This method is very effective when the distribution is continuous and smooth. Distribution pattern can be either interpolated from discrete data sets, or sketched using Bias and Gain functions. Implicit scheme was chosen to model both geometry and material gradient. Hence, geometric and material information can be queried at any points without further interpolation.

While it is possible to model a general closed surface as a single implicit surface patches, higher-order algebraic surfaces become difficult to design because there is no readily perceived relation between polynomial coefficients and the shape of a surface. This has prompted the use of piecewise algebraic surfaces, also known as *semi-algebraic sets* or *implicit patches.* Each surface piece is low order and spans a particular cell, usually a tetrahedron, that is defined by a spatial partitioning. The formulation would become more efficient to generate complex shapes by employing implicit algebraic patches.

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