

Application of Genetic Algorithms in the Design of Multi-Material Structures Manufactured in Rapid Prototyping

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Abstract

Recent developments in the Rapid Prototyping technology establish it as a new manufacturing technique, enabling localized material addition to build a part. Thus, heterogeneous structures, consisting of more than one material can be produced. The aim of this paper is to present an optimization tool to find the best material distribution in a multi-material structure due to given objectives and constraints. The tool is based on genetic algorithms using a discrete material model and FE-analysis to evaluate the objective functions. It can optimize the distribution of different materials in 2D-structures with up to 1500 DOF's at reasonable computational costs. Its performance is shown on a bi-objective optimization of a turbine blade.

1 Introduction

In the past, the intuition and experience of engineers played the key role in designing structures. Recent years have seen the development of numerical tools, which provide conceptual designs for a given design space and specified boundary conditions. The aim of these tools is to support the intuition and the experience of an engineer. In addition, most of these tools are focused on the optimization of traditional structures consisting of one material. Examples of such software tools are the topology optimization method using homogenization introduced by Bendsøe and Kikuchi [1], and the 'Soft-Kill-Option' (SKO) method introduced by Mattheck [2].

The development of new manufacturing technologies and new materials such as Rapid Prototyping [3] or the use of composite structures expands the demands for numerical tools to design structures. These new techniques offer the possibility to manufacture anisotropic and/or multi-material structures. This makes it much more difficult to design parts just by intuition. Numerical optimization algorithms are needed to develop solutions which consider all the different aspects of such a problem. The present project starts at this point, concentrating on the design of multi-material structures manufactured in Rapid Prototyping. This leads to the main research question for this paper:

*How can one optimize the distribution of different materials
in a multi-material structure for given objectives and given constraints?*

The paper presents a methodology based on genetic algorithms to solve this type of optimization problems.

2 Genetic Algorithms

Genetic Algorithms can be described as search algorithms based on the mechanics of natural selection and natural genetics. They belong to a category of stochastic search methods,

with an additional strength that randomized search is conducted in those regions of the design space which offer the most significant potential for gain. The primary monograph on the topic is Holland's "Adaption in Natural and Artificial Systems" (referenced in [4]). The terminology of genetic search and its principal components are discussed in the book from Goldberg [4] and in a paper by Hajela [5].

GA's are not severely limited by discontinuous design spaces like techniques derived from mathematical programming principles. On the other side there is usually a stiff computational requirement associated with the use of this method. Therefore genetic algorithms represent a good solution approach for design problems where standard mathematical programming techniques are inefficient. The main advantages of GA's can be formulated as follows:

- GA's work on function evaluations, not on function derivatives.
- GA's proceed from several points in the design space, this makes it more likely to find global optima.
- GA's work on a coding of the design variables. This allows them to work in design spaces consisting of a mix of continuous, discrete, and integer variables.

2.1 GA's in Automated Design Optimization

Much work has been performed and published as shown below:

The first group of projects discusses general aspects of using genetic search for structural and topology optimization. Chapman and Jakiela [6] apply genetic algorithms to problems of structural topology design. Nair and Keane [7] investigate the combination of approximation concepts with genetic algorithm based structural optimization procedures. A flywheel optimization with a genetic algorithm is presented by the GARAGE [8] group at Michigan State University.

Other projects are more concerned in the application of genetic search on practical problems. For example there are papers investigating multidisciplinary rotor blade design [9], the design of a satellite boom [10] or the optimization of truss structures [11].

Finally, several publications examine the optimization of composite structures with genetic algorithms [12],[13],[14].

All these works establish the use of genetic search in automated design optimization.

2.2 Concept and Additional Functions of GA's

The concept is briefly discussed in this paragraph, a detailed description can be found in the book from Goldberg [4].

After the coding of the design variables an initial population is generated randomly. Then the iteration process starts. The fitness values for each individual in a population are evaluated. The first GA operator which is then applied to the population is reproduction. Individuals are selected for the next generation according to their fitness. The crossover operator mates and crosses the individuals in this newly generated pool of individuals. Mutation as the last operator is the occasional random alteration of the value of a string position. This concludes one iteration of a GA, and a new generation results.

In the following some additional GA features which are used in the project are briefly introduced:

- **Elitist Strategy.** This strategy ensures, that the best individuals stay in the population.
- **Overlapping Populations.** The pool of individuals before reproduction consists of a GA with overlapping populations in the previous population and a specific amount of new individuals. The worst individuals of the entire pool are removed in order to return the population to its

original size. Since only part of the population is generated, this strategy saves computation time.

- **Scaling the Objective Function.** In order to control the sensitivity of the GA, the objective values are linearly scaled. The aim of a scaling function is to avoid premature convergence in the GA.

2.3 Parameters in Genetic Search

The initiation of genetic search requires specification of some key parameters:

- **Population Size: popsize.** The number of strings processed in each generation must be kept small to minimize the overall computation effort. A population size between 25 and 125 represents a good choice for structural optimization problems [5].
- **Number of Generations: n_{gen} .** Usually a value in the hundreds is needed to make sure that the solution has time to converge.
- **Crossover Probability: p_{cross} .** Values ranging from 0.6 to 0.8 have been used in numerical experiments with very satisfactory results.
- **Mutation Probability: p_{mut} .** Probability values between 0.005 and 0.05 produce in general good results.
- **Overlapping Gap: p_{repl} .** The overlap parameter specifies how many new individuals are created for each generation. A typical value is $p_{repl}=0.5$, meaning that 50 % of the population has to be evaluated new for each generation.

3 Modeling of Materials and Structures

3.1 Discretization of the Design Domain

Any kind of chromosome used as a representation of the design variables in GA's is discrete. Therefore a finite design domain must be selected and discretized into elements.

For a topology optimization the design domain represents the maximal volume in which the structure is to be constructed. For a multi-material optimization, one can either fix the topology and only optimize the material distribution, or both, the topology and material distribution can be unknown.

The representation of the design space in a mathematical way is the space $T = R^3$ [15]. A typical object in the design space is called a class A, while members of this class are noted as r-sets, for example an element of the discretized structure.

3.2 Modeling of Multi-Material Structures

Currently, most solid modeling techniques are capable of capturing only the geometric and topological information of an object. In this project, a modeling method for general heterogeneous objects proposed by Kumar and Dutta [15] is used:

Modeling of Heterogeneous Objects Consisting of a Finite Number of Distinct Materials (HD) [16]. In the previous paragraph, the design space $T = R^3$ was defined in which the geometry of an object can be represented. In order to model multi-material objects, this space is expanded to include a material dimension M. The material dimension is represented with a set of integers I. The product space $T = R^3 \times I$ forms then the new modeling space including both, geometrical topology and material distribution. A new class $A_m = A \times K$ is defined where $K \subset I$ is a finite set of integers. A typical member of this class is then defined as an r_m -set. It is

composed of an r -set describing the geometry and an integer $k \in \mathcal{K}$ defining the material. Each r_m -set, typically an element in this project, is homogeneously filled with one material.

The design variables for the optimization are now defined with the type of material for each element in the discretized design space. The approach provides thus a discrete and finite set of design variables.

3.3 Mapping the Design Variables to GA Chromosomes

The design variables for a GA must be coded, a mapping from the design domain to chromosomes must be defined. Using the HD approach above, the coding results in a binary or integer string which defines the type of material for every element in the design domain. A binary string can be used if there are only two materials to distribute, while an integer string is able to handle a whole set of materials.

Figure 1 shows the mapping for a cantilever structure with three materials mapped into a two dimensional integer string. To consider topology optimization as well as optimal material distribution, the material 1 is defined as void. To prevent numerical singularities, the material void is implemented with very small property values.

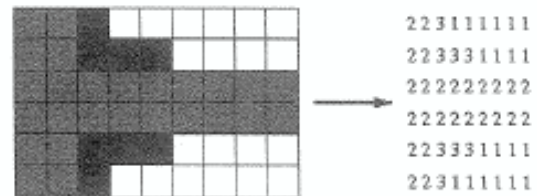


Figure 1: Mapping to GA Chromosome

4 Optimization Procedure

The genetic algorithm library GALib in C++ by Matthew Wall (MIT) provides all the needed functionality and is used in this project.

A GA needs a function which determine the fitness of each individual in a population. For a multi-material optimization this function includes an analysis of the discretized structure. For a typical GA evaluation, the fitness has to be computed for thousands of individuals. Therefore it is imperative that the analysis method has to be fast. This can only be reached if the method for the fitness evaluation has a close interface to the GA.

A first approach to evaluate the fitness of the individuals is to use **Numerical Approximation Concepts**. Keane [7] used this concept to optimize a 10 bar truss structure. It is based on the exact analysis for a limited number of individuals, the fitness for the rest of the population is evaluated using an approximation model. Although this approach can save a lot of computation time, one would have to verify the accuracy of the results carefully, because only a small part of the individuals are evaluated exactly. This method was not pursued any further in this project.

A **Finite Element Analysis** for each individual in the genetic algorithm represents another approach to evaluate the fitness values required by the genetic search. This method is favored for this project, because it provides in general a good accuracy for the solution and adaptations of the method can be found for analyses in a lot of different fields. The computational cost for analyzing each individual in the GA is the main concern. Therefore a big effort was put in minimizing the computational time needed for one single analysis. In order to keep the optimization tool as compact as possible, a FE-code in C++ was developed. With this approach, the optimization procedure can be packed in one single C++ program, the time consuming processes such as storing temporary data on the hard drive are eliminated.

The FE-code was developed to provide analyses for the applications in section 5. Therefore FE-codes for heat conduction and thermal stress analyses had to be implemented. The concepts of the FE-codes can be found in [17],[18],[19], and are not discussed here. The detailed procedure is shown in the flow chart in Figure 2.

The computational performance of this approach was tested for a simple topology optimization of a Cantilever. It represents a small application with 352 DOF's, where not much computation time should be needed to solve it. In a typical optimization for this problem the fitness function has to be evaluated approximately 3000 times. The evaluation on a computational server with 2 CPU's (200 MHz) and 1 GB RAM needs approximately 75s. This is promising for bigger applications.

In an earlier approach Ansys was used for the FE part. But the time consuming processes of storing interface files on the hard disk makes the GA evaluation for the same application about 300 times slower.

The optimization procedure using the FE-Code in C++ is therefore used for all applications.

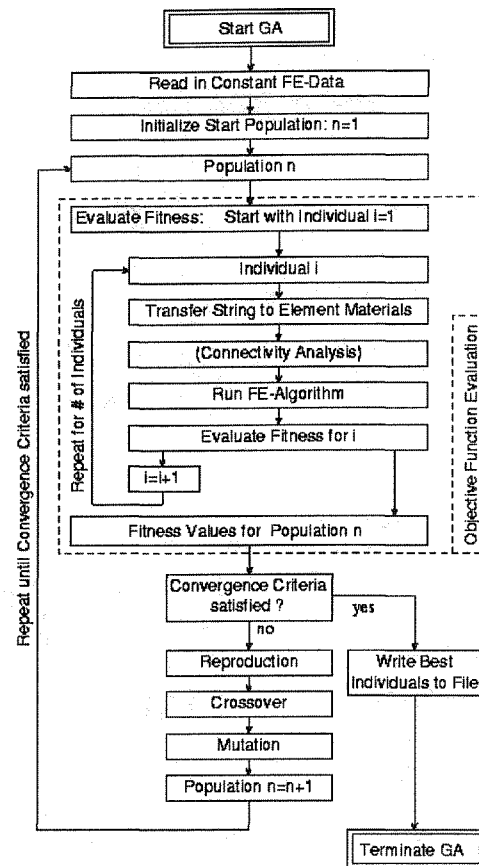


Figure 2: Procedure Using FE-Code in C++

5 Multi-Material Optimization of a Turbine Blade

The application in this section shows the performance of the developed optimization method. The distribution of two materials in a turbine blade shall be optimized.

5.1 Problem Definition

The design domain considered is a 2D cross section of a turbine blade. It is a non-cooled blade and has therefore no internal holes in the structure. This cross section is discretized into 951 linear triangle elements.

Materials. A turbine blade is subject to pressure gradients over the boundary, which introduce high mechanical stresses in the structure. Furthermore, certain regions of the blade in a gas turbine are heated to very high temperatures. This results in the wish to use one material, such as Titanium, to deal with the mechanical stresses in the structure, and a heat withstanding material such as a Ceramic to place at high temperature locations. Therefore, in this example, the materials Titanium Alloy and Silicon Nitride are used.

Boundary Conditions. The temperatures applied on the boundary are based on experimental data described in [20]. Figure 3 shows the resulting temperature distribution for a titanium blade. The purpose of this application is to show the optimization algorithm. Therefore the complex boundary conditions on a real turbine blade were simplified:

- Prescribed temperatures on the boundary replace convection and radiation effects.
- The blade is not cooled, there are no internal holes in the structure.
- Pressure distributions on the boundary are not applied.

5.2 Single Objective Optimization

The objective is to find the optimal material distribution due to given maximal service temperatures for the materials. This is an unconstrained optimization problem. The fitness function for the genetic algorithm is formulated as a maximization problem:

$$\text{Maximize } score = \frac{score_{norm}}{\sum_{i=1}^{Nele} (T_i - T_{service(mat)})^2}$$

The service temperature $T_{service(mat)}$ does not represent a physical property of the material, it is fictive. Since the conductivities for the two materials are different, the temperature distribution in the blade changes for every new chromosome. A heat conduction analysis has to be performed for each individual in the GA. The design variables are represented by the material of each element, they are mapped in a binary string for the GA.

The GA is evaluated for 2000 generations with a population size of 100. This corresponds to 100'000 heat conduction analyses. A typical evaluation takes about 70min. Figure 4 shows the fitness scores for this evaluation, the best individual found is shown in Figure 5.

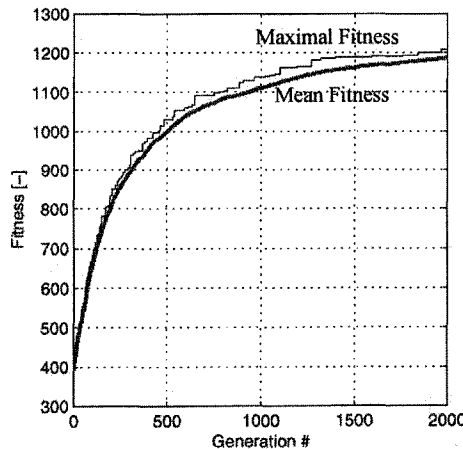


Figure 4: Fitness Scores

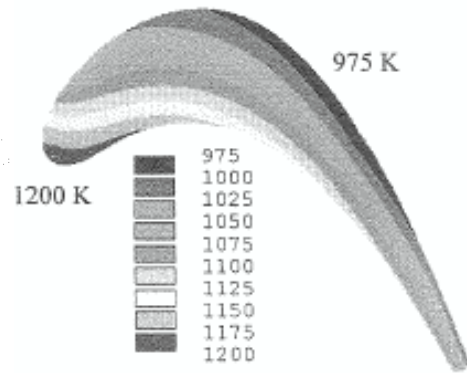


Figure 3: Temperatures in a Ti Blade [K]

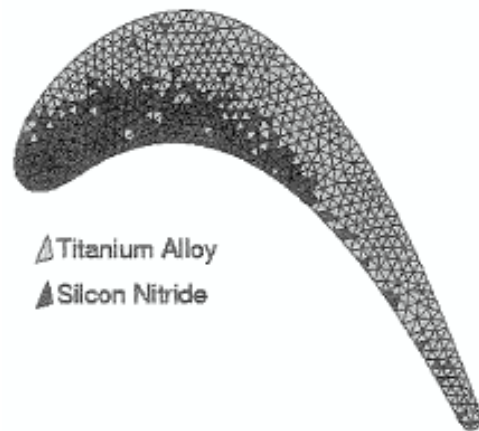


Figure 5: Resulting Material Distribution

5.3 Bi-Objective Optimization

In the solution above, high thermal stresses occur because of the different thermal expansion coefficients of Titanium and Silicon Nitride. Therefore a bi-objective optimization is carried out which combines the temperature objective with a thermal stress objective.

The fitness function is defined as a combination of the two objectives using the weighting method [21]:

$$\text{Maximize } fitness = (w_{temp} \cdot score_{temp} + w_{stress} \cdot score_{stress})^{-1}$$

Where w_{temp} and w_{stress} represent the weighting factors. The objective functions are defined as:

$$score_{temp} = \frac{\sum_{i=1}^{Nele} (T_i - T_{service(mat)})^2}{norm_{temp}}$$

$$score_{stress} = \frac{\sum_{i=1}^{Nele} \sigma_{eq}}{norm_{stress}}$$

The temperature objective is defined similarly to section 5.2, it controls the material distribution according to temperature distribution. The thermal stress objective aims at reducing the thermal stresses in the blade, it is the sum of an equivalent stress σ_{eq} of all elements. To evaluate the fitness of a single individual, a thermal conduction and a thermal stress analysis have to be performed.

In a bi-objective optimization, many optimal solutions can be found. Therefore a Pareto-set [21] of solutions is created in a first stage. The Pareto points result from a systematic variation of the weight factors between 0 and 1. Figure 6 shows the resulting Pareto points for such a variation. Each of these points represents the best population found by the GA in 1500 generations with a population size of 100 and the specified weight factors. The temperature objective is plotted in the x-direction, the stress objective in the y-direction. A typical GA evaluation for this problem takes about 10h, because of the two FE-analyses (thermal conduction & thermal stress) for each fitness evaluation.

A decision making process is necessary to choose a single solution out of this set of optimal Pareto solutions[21]. Figure 7 shows one single solution where the weight factors were chosen as $w_{temp} = 0.3, w_{stress} = 0.7$. In addition the GA for this result is run over 2000 generations with a population size of 150 to get a more converged result.

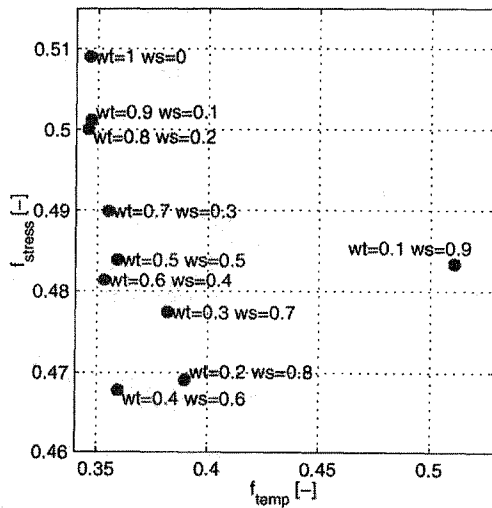


Figure 6: Pareto Points

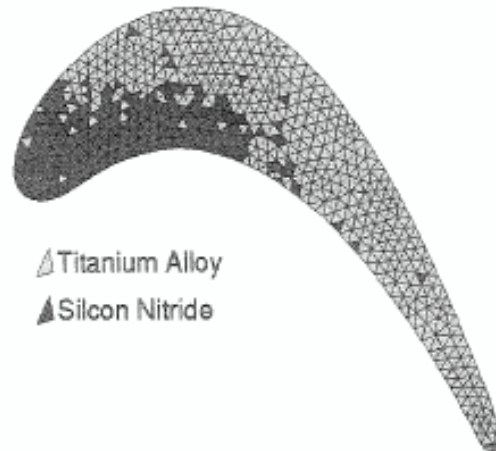


Figure 7: Best Individual for $w_{temp} = 0.3, w_{stress} = 0.3$

6 Discussion and Conclusion

For the single objective optimization in section 5.2, the optimization puts as expected Ceramic at the tip of the blade (see Figure 5), where the highest temperatures occur. The Titanium in the rest of the structure provides the needed strength. Although the GA was evaluated for 2000 generations, it's stochastic concept still influences the result. Several elements have obviously the wrong material (assigned by mutation). Because of the high temperatures and the different thermal expansion coefficients, it is imperative to include thermal stresses in the optimization.

An approach to include thermal stresses is shown in the bi-objective optimization in section 5.3. Figure 6 shows that the weight variation does not result in a convex pareto curve. This indicates that probably the GA evaluations did not fully converge to the optima.

Figure 7 with the best individual for a specific weight combination shows that the Ceramic is smaller. But it is still not clear how the interface between the two materials should be best configured. It was hoped to see a fuzzier mechanical interface between the two materials, but the problem conditions and the way the objectives were formulated prevented this from happening.

An optimization tool for 2D-multi-material structures was developed. It optimizes the distribution of materials in a structure due to given objectives and constraints using genetic algorithms. It can be said, that the developed optimization tool is able to handle structures with as many as 1000 to 1500 DOF's in a reasonable computation time (a few hours). The turbine blade application showed, how essential a fast objective function evaluation for GA's is, a run time of 10h for the bi-objective optimization shows the limits of the tool. Since the solution space for the GA grows exponentially with the number of elements, the number of generations needed to achieve a converged solution increases as well in an exponential way. This project has shown some of the potentials of GA's in structural optimization, as their flexibility, their robustness and ability to find global optima. But it has also demonstrated some of their shortcomings of high computational needs and bad convergence performance towards the end of an optimization. To overcome these shortcomings, future work could be concentrated on introducing parallel genetic algorithms, using gradient based optimizers after the GAs pointed towards possible solutions (memetic algorithms) or using a continuous material model.

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