MSOE TetraLattice^{TM}

"Applications and Simplified CAD Representation"

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Abstract

Applications for the MSOE TetraLattice^{TM} used for SFF structures are growing. The simplicity of the tetragonal lattice morphology makes this structure useful for composite and metal castings, gradient morphologies, conformal cooling & heating systems, transfer and filtering systems, rapid build style, and stress-reducing build style. As more applications emerge, the need for a simplified 3D CAD representation, with easily adjustable void-solid ratios, has become increasingly important. The volume was determined as a function of node spacing and node size in a simplified, minimal surface representation. Reducing the points required to define the lattice surface is key in reducing processing time and improving synthesis quality. The simplified CAD representation and mathematical model, used to determine percent volume, will be presented followed by TetraLattice applications.

Introduction

Unlike traditional material removal fabrication techniques, such as CNC machining, Solid Freeform Fabrication (SFF) is an additive process, starting with a void, and adding material one layer at a time to create a 3D object. CAD data is used to define the surface and interior of the solid 3D object making complex part geometries, as well as complex internal structures, possible and commonplace. No expensive patterns, molds, or parting-lines are required when SFF is used directly, making it a widely accepted, accelerated prototyping tool.

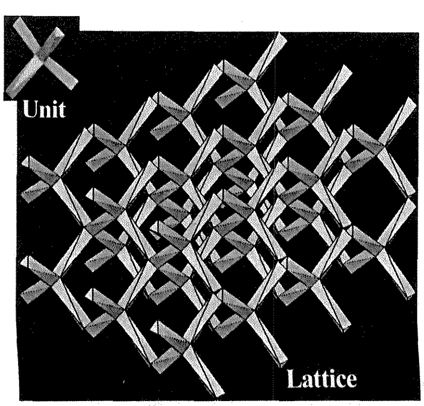


Figure 1. TetraLattice structure with repeating unit shown

With the additive nature of SFF it is possible to create structures never thought possible, such as complex, three dimensional lattice structures. TetraLattice, the topic of this paper, (TL, figure 1) is one of these structures made possible by SFF. TL is modeled after the molecular bond geometry found in diamond -*tetrahedron*- offering a simple repeating lattice unit, that lends itself to SFF. Arrays of TL units form complex structures as well as complex channels within objects, resulting in many useful characteristics. Based on this TL geometry -interconnected in all directions- several applications have emerged and more will likely follow. Initially, TL was used for metal and composite castings. Now, TL applications in the areas of gradient materials, conformal heating and cooling, transfer systems, and stress reduction are being discovered.

As new applications emerged, the CAD design of the early TL was limiting, for some uses, due to the large file size and square geometry. To reduce file sizes as well as improve the TL design, a completely new CAD representation was developed. The new CAD representation is a solid with a minimal number of triangles defining its surface. The volume ratio of TL solid to void space was determined using a tetrahedron breakdown, making this new representation better suited for gradient materials. The volume ratio is determined as a function of several variables related to node spacing and node size.

This paper will discuss three main topics, starting with a detailed description of the new, simplified TL CAD representation. Next, the volume ratio of TL versus Negative TL is represented mathematically and verified. Finally, six TL applications, some benefiting from the simplified CAD representation and volume model, will be discussed.

Simplified TL CAD Representation

Various methods can be employed to define a lattice structure for SFF. Masks could be used, changing masks for each layer until the lattice repeats. Another method would be to form a computer generated hatch within the parameter of the desired part. The hatch would change depending on the current layer. Yet another method is to form a solid three-dimensional model of the desired lattice structure which is boolean intersected with the desired CAD part resulting in lattice structure of the desired form. This portion of the paper will briefly describe several solid CAD representations and a new, simplistic CAD representation.

TL is based on the repeating crystallographic structure of diamond. The repeating or base unit of TL is made up of four "legs", which are joined end to end at specific angles to form an intersection, or "node". Therefore, the shape of the end face of the four legs will dictate the geometry of the intersection. The geometry of the leg also impacts computer file size, as different geometries require different number of facets to create a surface file (.stl format). The TL base unit could be defined utilizing at least four geometric structures: square, hexagonal, rectangular, or triangular as shown in Figure 2.

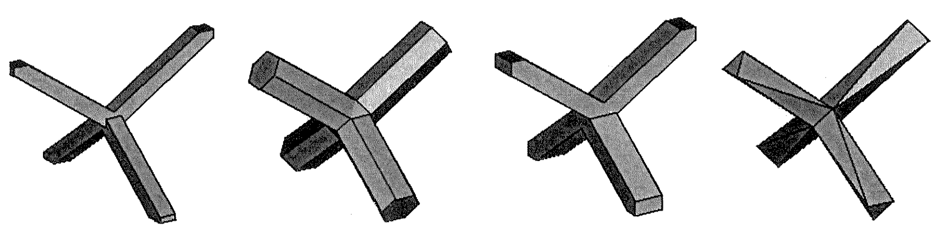


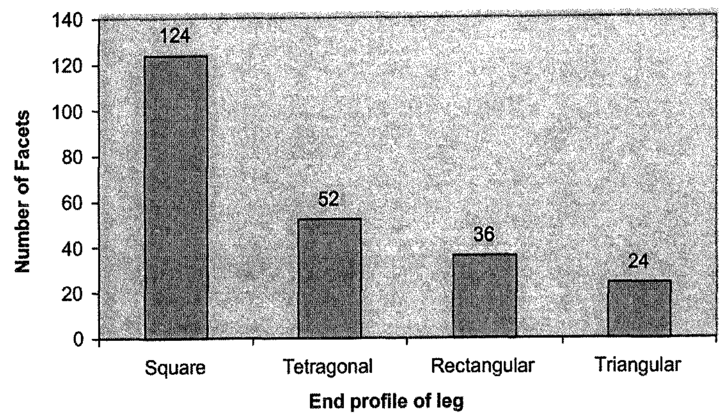
Figure 2. Four possible TetraLattice base unit geometries: square, hexagonal, rectangular, and triangular, respectively

Figure 3 shows the comparison of the number of facets required to define a base unit depending on the base unit geometry. It is shown that using triangular legs to construct the base unit will yield the smallest file sizes. The triangular geometry, shown in figure 2 sues four triangular legs, joining to form a regular tetrahedron node. Legs are twisted 60° to allow fields of units to join with minimal faceting. It should be noted that for certain applications a base unit created using a different geometry may be desired, however this paper will focus on the triangular format.

TL Volume Calculation Methodology

In many of the applications described later, it is beneficial to control the volume ratio of the TL volume to its void space volume. This section will focus on mathematically representing the volume of TL and TL void space. Two variables can be used to describe the volumes, the side lengths of a tetrahedron node, S, and the length of a leg, L. A base unit, created using triangles, is shown in Figure 4.

Figure 3: Number of Facets required to define a base unit with various geometeries



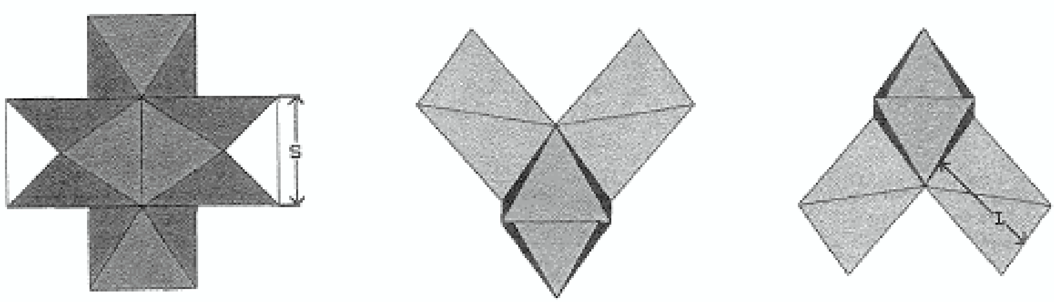


Figure 4: Top, right side, and front of a TetraLattice base or repeating unit. The side length, S, is shown on the top view and the leg length, L, is shown on the right side view.

TL Volume calculation

As stated before, the ends of the four legs join to form a regular tetrahedron node of side length S. The volumes of a regular tetrahedron and the four legs are required to find the volume of a base unit.

The regular tetrahedron belongs to all four legs; hence, one quarter of the tetrahedron volume can be assigned to each of the legs. The same argument can be applied to the other end of each leg. Therefore, ½ of an intersection is assigned to each leg, and two intersections are assigned to a base unit.

The volume of a regular tetrahedron is

$$V_{tetrahedron} = \frac{S^3\sqrt{2}}{12} \tag{1}$$

where S is the side length of the tetrahedron.

The height of the leg, L, is measured perpendicularly between the equilateral end triangles, as seen in Figures 5a and 5b. The leg is a triangular shaped column that twists from one end to the other. While the center points of the end triangles align, the ends are rotated 60° about the center of the leg, as can be seen in the end view in Figure 5c.

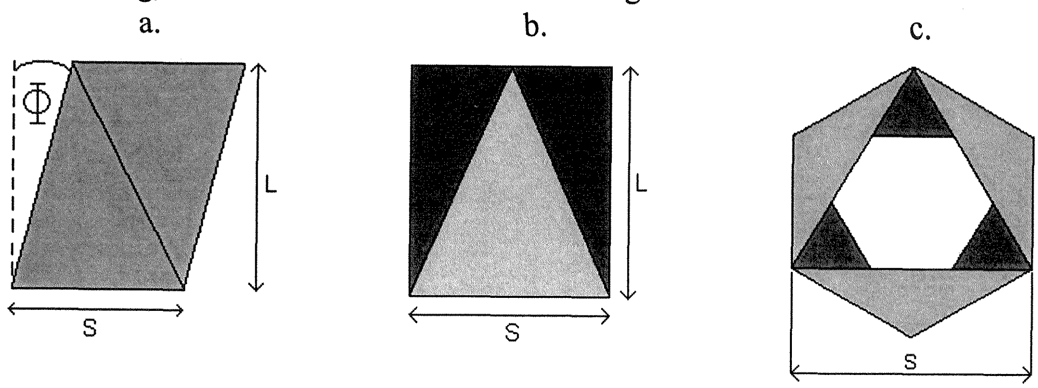


Figure 5: Illustrated is a triangular leg of TetraLattice. Variables, S and L, are used to describe its volume. a. Side view of leg. b. Front view of leg. c. End view of leg.

To find the volume of the TL leg, it was assumed that the leg was a hexagonal tube from which six triangular pyramids have been removed. The volumes of the hexagonal tube and the pyramids were found to be

$$V_{hexagon} = \frac{3S^2L}{2\sqrt{3}} \tag{2}$$

and $V_{pyramid} = \frac{S^2L}{12\sqrt{3}}$ (3)

The leg volume is $V_{hexagon} - 6(V_{pyramid})$ which reduces to

$$V_{leg} = \frac{S^2L}{\sqrt{3}} \tag{4}$$

As explained earlier the TL base unit volume is

$$V_{TetraLattice\_Unit} = 2(V_{tetrahedron}) + 4(V_{leg}) \tag{5}$$

$$V_{TetraLattice\_Unit} = \frac{S^3\sqrt{2}}{6} + \frac{4S^2L}{\sqrt{3}} \tag{6}$$

TL Negative Volume

TL occupies a certain percentage of an object's volume. In order to predict the percent volume a TL field will occupy, the space surrounding each base unit or the void space was mathematically characterized. This will be referred to as Negative TL. The geometry of a negative unit was found by enclosing a space with four base units. The geometry was checked by enclosing one base unit with four negative units. This assured that the spatial relationship between TL and Negative TL is one to one and that the geometry of the volume is correct.

The orientation of the TL legs (the 60° twist) causes the angle between side faces to change as the dimensions of S and L change. If the angle between faces of a tetrahedron is measured from the center point then the angle (α) is a constant of 109.47°. If the legs were not twisted, measuring between the adjacent faces of each leg would also result in this angle. However, as the faces of the tetrahedron are extruded and rotated to form the legs, the angle measured between the side faces changes. The angle Φ (Figure 6) will reflect dimensional changes in both S and L. This relationship is shown in the following equation.

$$\Phi = \tan^{-1}\left(\frac{S}{2L\sqrt{3}}\right) \tag{7}$$

To find Θ, the angle between two adjacent faces, 2Φ is subtracted from α since both of the legs will be causing the reduction in α. This is shown in Figure 6.

$$\Theta = \alpha - 2\Phi = 109.47^0 - 2\tan^{-1}\left(\frac{S}{2L\sqrt{3}}\right) \tag{8}$$

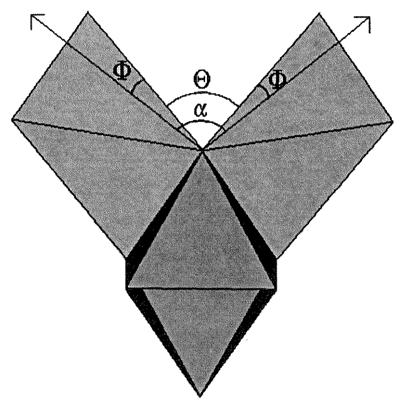


Figure 6: Illustration of TetraLattice base unit showing the relationship between various

Several dimensions of the TL base unit are necessary to calculate dimensions of the negative unit. These dimensions are shown in Figure 5 and the relationships to S and L are found by the Pythagorean theorem.

For simplicity, these new variables will be used in subsequent equations.

$$F = \sqrt{L^2 + \frac{S^2}{12}} \tag{9}$$

$$E = \sqrt{L^2 + \frac{S^2}{3}} \tag{10}$$

Several dimensions must also be obtained from the Negative TL unit. These are shown in Figure 8.

$$T = 2F\sin\left(\frac{\Theta}{2}\right) \tag{11}$$

$$B = \sqrt{F^2 - \frac{T^2}{4}} \tag{12}$$

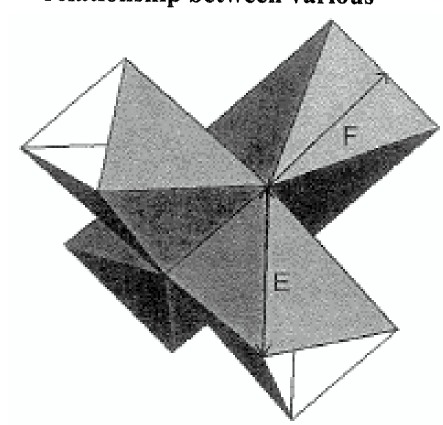


Figure 7: Triangular TetraLattice base unit with E and F shown.

The negative TL unit also has tetrahedrons as the nodes with "legs" connecting them. The legs have a different geometry than that of TL. The side length of the tetrahedron is T, so substituting into Equation (1) we find the volume of the node tetrahedron. Note that, there are two tetrahedron and 4 legs that make up each unit of negative TL.

T can be substituted for S in Equation (4) but the height of the leg is still needed. This height, H, can be calculated as the height of a tetrahedron composed of three isosceles triangles for the sides, with symmetrical sides E and base side, T. Therefore the base is an equilateral triangle of side length T. The height can be found using the Pythagorean Theorem. To do this, the center to corner distance of the equilateral triangle measured along the angular bisector (R) is needed. This is simply,

$$R = \frac{T}{\sqrt{3}} \tag{13}$$

Therefore,

$$H = \sqrt{E^2 - R^2} \tag{14}$$

The negative TL leg volume differs from the TL leg volume by the formation of an additional wedge on each side face of the leg. The volume of the wedge is the volume of a pyramid, which reduces to

$$V_{Wedge} = \frac{BTs}{12} \tag{15}$$

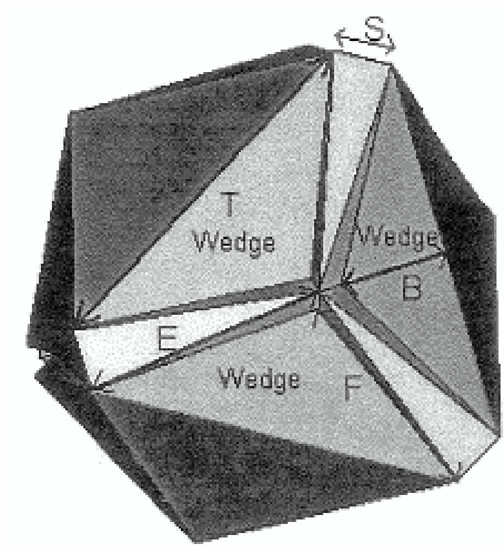


Figure 8: Illustration of negative or void TetraLattice

Substituting into Equation (6) with H for L and T for S, and adding the wedge volume to each face, the leg volume reduces to

$$V_{Neg\_Leg} = \frac{T^2 H}{\sqrt{3}} + \frac{BTS}{2} \tag{16}$$

Finally, the volume of the Negative TL unit is two tetrahedron and 4 legs resulting in the following equation.

$$V_{Negative\_TetraLattice} = \frac{T^3\sqrt{2}}{6} + \frac{4T^2 H}{\sqrt{3}} + 2BTS \tag{17}$$

Final Volume Calculation

Since the sumation of TL volume with negative TL volume results in the total volume occupied by both, the following equation can be written:

$$V_{Total} = V_{TetraLattice\_Unit} + V_{Negative\_TetraLattice} \tag{18}$$

TL Volume Verification

The volume equations for both TL and Inverse TL were validated using a MATLAB program. The program allows the user to input the percent volume of TL desired, as well as the node to node distance (This distance is measured from the centers of the intersection node tetrahedra). Using an iterative solution method, the program determines the dimensions of S and L that are necessary to construct a TL field with the desired volume percentage. These dimensions were used to create a base unit. The base unit was multiplied and arranged to create a field. Several object geometries with known volumes were created. Using Magics RP software, these objects were intersected with the TL field. After the boolean intersection, the remaining part was a TL field having the shape of the object. The volume of this TL object was estimated in software and divided by the volume of the original part. The division resulted in a percent volume of TL for this test. This calculated percent volume was compared to the percent volume input into the MATLAB program.

This test procedure was performed using several object geometries. Five volume percentages were also tested. Results shown in table 1 indicate that the percent difference between the input amount of TL and the actual measured amount varied between 0.0% and 2.1% (Table 1). This was dependent on the orientation of the TL in the object and on the iterative method used. The MATLAB program was written for calculation speed, rather than the highest accuracy. This could be changed depending on the need for accuracy in the application.

Volume Percent desired	Volume Percent achieved	Percent difference
10.00	10.20	2.00
20.00	20.41	2.10
40.00	40.00	0.00
60.00	60.15	0.25
80.00	80.09	0.11

Table 1: Data for Varification of TL equations

TL Applications

Net-shape composite and metal castings were the original applications of TL. Since then, many additional applications have emerged, such as: gradient morphologies, conformal cooling & heating systems, micro lattices, transfer and filtering systems, fast build style, and stress reducing build style. Benefits of TL vary depending on the application. Some applications require a fine, minimal lattice volume with open communication throughout the lattice void space. Others require a TL void where the TL is completely interconnected and allows fluid flow in a controlled yet continuous manner. Although the new simplified TL CAD representation benefits most TL applications the goal of this portion of the paper is to describe where this lattice morphology *—defined using any of the representations—* is useful to the SFF community.

Net-shape Composites and Castings

As mentioned previously, net-shape composite and metal castings were early applications of TL. [1,2] The three-dimensional TL structure is generated inside a hollow shell (the desired pattern geometry) typically using the stereolithographic process. TL legs are thin, providing internal structure for the SLA skin, while adding minimal overall material to the low-density pattern.

Patterns with this unique internal structure currently have two particular applications including rapid composite prototyping and investment casting. Rapid composite prototyping is a process wherein the TL void space is filled with reinforced plastic to form a composite with enhanced mechanical properties. When TL is used for composite casting, TL patterns have three unique characteristics: (1) ease of uncured photopolymer drainage (2) ease of filling with viscous reinforcement plastics, and (3) near-isotropic physical properties after filling.

TL-filled investment casting patterns offer foundries three beneficial characteristics: (1) quick, complete drainage of photopolymer (2) reduced danger of refractory shell cracking and

(3) minimal ash content after burn-out. SLA TL patterns are especially useful when large, thin walls are required. Wall thicknesses of less than 0.040 inches will drain sufficiently for investment casting.

Gradient Materials

Functionally gradient materials[3] (FGMs) have enormous potential in the industrial world today. The ability to fuse two entirely different materials together to form one hybrid material results in the possibility of forming a vast range of new engineering materials. With this technology, engineers can tailor properties within a mechanical component to handle a full spectrum of application requirements, without changing design geometry. FGM's transform from one material (part A) to another (part B), unlike traditionally bonded materials having a distinctive –typically planar– seam.

Gradient TetraLattice[4,5] (GTL) is a three-dimensional FGM morphology, transforming from one material to another, in a controlled manner (this morphology can be likened to M.C. Escher artwork in three dimensions). To create Gradient TL, spacing between standard TL units can remain constant while the branch thickness is increased in a controlled manner, as shown in Figure 9.

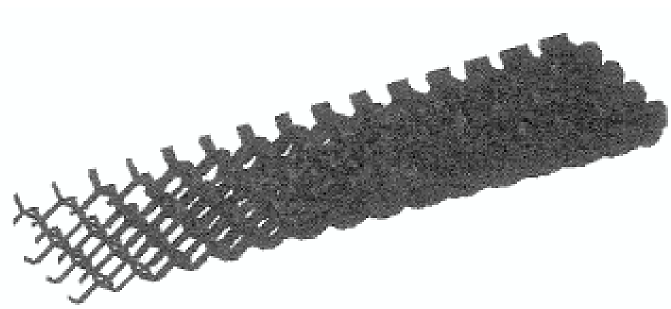


Figure 9: Gradient TetraLattice morphology

This controlled increase in branch thickness results in a proportional change in volume percentage of material combinations. In theory, this volume percentage will transform from 100 percent of material A to 100 % of material B, at any rate, in any direction. With Gradient TL, the transformation is three-dimensional and mechanically inter-linked. GTL becomes two TL with varying branch thickness. Part A is the negative of part B. Another benefit of GTL is the ability to transition from one material to another through a series of layers. Figure 10 illustrates the first Gradient TL sample.

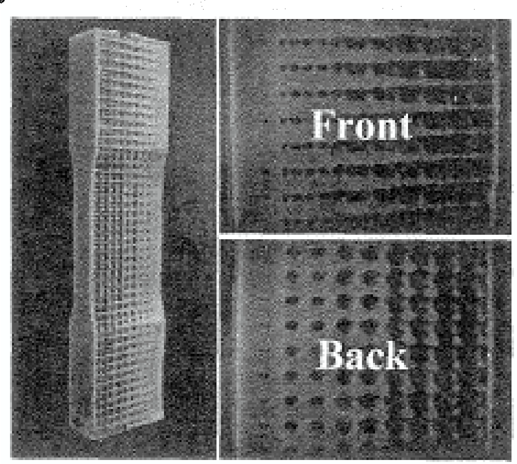


Figure 10: Photopolymer-glass reinforced epoxy gradient TL

Conformal Cooling & Heating

With the layerwise build mode of SFF arises the opportunity to produce cooling and heating systems conformal to complex contoured surfaces, where heat exchange is critical. Applications such as injection molding, where a large percentage of cycle time is spent cooling, benefit significantly from conformal cooling channels.[6,7]

TL can be used for conformal cooling and heating as illustrated in figure 11. In this case a TL void is formed to create the conformal fluid channels. Interconnections between TL nodes make this morphology useful when deep cavities or tall irregular features are present. Leg angles induce beneficial turbulent flow resulting in improved convective heat transfer within the fluid. To ensure even flow over a complex surface or variable heat removal at various locations node size or depth of the TL field can be tuned. Normally the stair-stepping effect of SFF is undesired; here it is beneficial for conformal cooling and may be increased to induce further turbulent flow.

Figure 11: Illustration of Conformal TetraLattice Cooling

Transfer and Filtering Systems

TL can be useful for transfer applications where entities (heat, electrons, ions, molecules, other) need to be moved efficiently from one object to another in a controlled manner. The basic concept here is to intermesh two TLs to form two distinctly separate objects separated by a thin wall. The two objects could be liquid, solid, or gas and the thin wall could be a solid, liquid, gas or vacuum. The simplest approach to describe TL of this form would be the heat exchanger as illustrated in Figure 12. In the case of a heat exchanger two intermeshing TL voids are separated by a thermally conductive wall. The wall transfers heat from one TL void fluid to the other. The TL flow path is non-linear and can be optimized to induce turbulent flow.

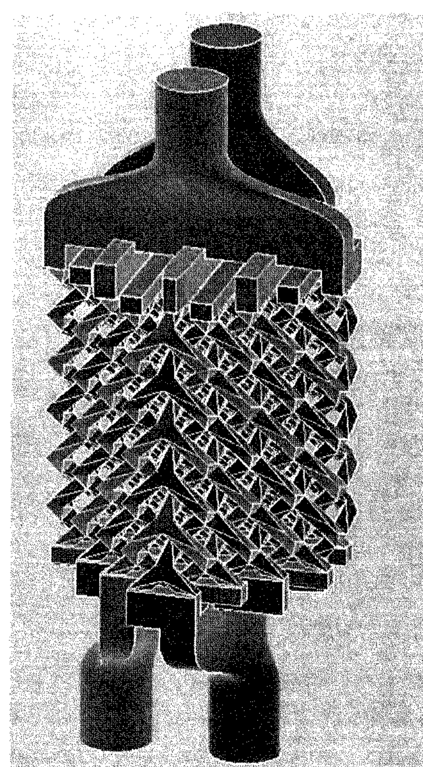


Figure 12: Possible orientation of a liquid to liquid heat exchanger using TetraLattice. The two shaded regions represent the fluid within the heat exchanger (illustration by Adrian Sikorski).

Another application, where the wall separating the two TL channels is a liquid and the TL channels become a solid, would be for a battery cell. One TL solid becomes the anode, the other the cathode and the liquid wall would be an electrolyte.

Listed below are other possible applications of TL based transfer or filtering systems:

- Hemodialysis (A semipermeable membrane separates the two hollow TL regions. In one TC chamber flows the patient's blood, while the other chamber contains a dialysate consisting of water and solutes.)
- Anatomical Micro-circulation modeling (A porous frame work separates the two hollow TL regions.

One TL chamber represents the arterial system in anatomical tissue and the other TL chamber represents the venous system. The separation region or porous region represents the organ bank or tissue specific cells.)

- Filters/separation devices (A porous wall separates the two hollow TLs)

Stress Reduction

In SFF processes where layer-to-layer stresses are locked into objects, resulting in warpage or deflection, TL may be beneficial. This type of warpage is typically due to shrinkage induced stress. When building a solid part, a new layer is added on top of a solid layer and the new layer has a tendency to shrink during solidification and cooling while the previous layer is stable. The result is a planar stress, locked into the new layer. If more layers are added the stress becomes so great that without sufficient support the part will curl or warp. One approach to reduce this stress is to build the part using an array of columns that are independent of one another. The shrinkage is held locally, in the column cross-section, and the stress lines, formed across a large cross-sectional area, are discontinuous. The main problem with building a part consisting of columns is the discontinuity in the X and Y directions (Z being the build direction.) If one TL or two intermeshed TLs are used to fill the desired volume of the object the result will be, in effect, intertwined columns. The cross-section of one layer will be significantly discontinuous. Thereby, breaking the lines of planar stress, but still keeping the part as one entity enclosed by an outer skin.

Build Time Reduction

Quite often rapid prototyping requires a part that will not be used for functional testing nor secondary operations. In many cases the part will be used only as a visual model. In this case a TL internal structure can be used to shorten build time. Stereolithography parts produced with "thick skins" (0.045 inch) often take less than half the time to build as an identical solid model[8]. The option to solidify the part, for improved functionality, via the rapid composite process remains, resulting in a much more efficient and effective usage of RP equipment.

As SFF continues to advance with larger and smaller size capabilities, expanded material capabilities, and multiple material capabilities the aforementioned applications will advance and new applications will likely emerge. SFF enables more than unique objects to be produced —*SFF enables unique objects with complex, functional internal features to be produced*— quite often with negligible added cost.

Conclusions

A new TL CAD representation was developed for faster computing and reduced human intervention. The new representation reduced file sizes and computer processing time significantly when compared to previous CAD representations. A mathematical model of the volume for the new TL CAD was determined. The ability to easily specify volume ratios in gradient TL applications is important and was accomplished and verified. Applications of TL are expanding into unforeseen areas beyond original applications of composite and metal castings to gradient morphologies, conformal cooling & heating systems, transfer and filtering systems, fast build style, and stress reducing build.

Acknowledgments

We gratefully acknowledge the support of the National Science Foundation (REU program – "Undergraduate research in solid freeform materials and technology - #EEC-9619715)

The authors would like to thank Sheku Kamara, Matt Panhans, Robert Crockett, Christopher Kreuser, Doug Cook, Vadim Pikhovich, Darius Daruwala, Jesse Upp, Steven Shaffer, John McCray, and Adrian Sikorski of the Milwaukee School of Engineering for efforts in technical assistance and application development.

Special thanks to the Rapid Prototyping Consortium members for their involvement in this and other applied research.

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