Designing and slicing heterogeneous components for Rapid Prototyping

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Abstract

Many rapid prototyping techniques have the potential for fabricating components whose composition is non-uniform and varies in a desired fashion. A shape and composition modeling technique was developed to enable the representation and design of such heterogeneous components. Techniques for interactively and automatically designing such components are presented. Automatic design is made possible using optimization techniques where the optimal composition distribution is computed based on specified design objective and constraints. Software was also developed to slice 3D heterogeneous solids to generate cross-sectional images as well as composition distribution for each cross-section. Slicing and generation of cross-sectional data are essential to enable rapid prototyping of these components.

1. Introduction

Heterogeneous components are parts whose material composition varies from point to point within the part. Such components are difficult to fabricate using traditional methods. Rapid prototyping systems build parts by adding material layer-by-layer and are in principle capable of varying the composition within manufactured parts. Progress towards composition control of heterogeneous components have been reported recently using SFF processes such as Selective Laser Sintering (SLS) (Jepson et al, 1997), 3D printing (Yoo et al, 1998), Shape Deposition Manufacturing (SDM) (Fessler et al, 1997), Direct Light Fabrication (Lewis and Nemme, 1997), Laser Engineered Net Shaping (LENS) (Griffith et al, 1997) and Electrophotographic Solid Freeform Fabrication (Kumar and Zhang, 1999).

Traditional solid modeling systems do not support the ability to model, analyze or design heterogeneous components. Recently, methods for modeling such components have been explored. Kumar and Dutta (1998) have presented an extension of the traditional solid model representation based on regular sets (r-sets), to define r_m-sets that include material data. They have defined set operations for combining these r_m-sets to define heterogeneous solids. Jackson et al (1998) present an alternate representation where they subdivide a solid model into simpler domains over which composition functions are defined using Bernstein polynomials as basis functions.

In this paper, a representation of composition distribution is presented that is based on a finite element mesh generated over the solid. The composition values at the nodes are interpolated to obtain composition within each element. In section 2, the method for shape and composition representation described. Elements that can be used for both modeling heterogeneous components and analyzing it using the finite element method are described in section. A method for designing these components by optimizing the composition distribution is considered in section 3. A few examples are presented in section 4, where the design of components with varying composition and porosity are considered. Finally, conclusions are presented in section 5.

2. Three-dimensional elements for composition representation

The material composition within a heterogeneous object can vary from point to point in an arbitrary fashion. To represent varying material properties within a solid, we propose a method that relies on a finite element mesh obtained by decomposing the solid into simple elements such as hexahedrons or tetrahedrons. The composition within these elements is obtained by interpolating the values of composition at the nodes of these elements using Lagrange basis functions. Thus by assigning appropriate composition values for the nodes of the finite element mesh we can control the composition within the solid. This method enables arbitrarily complex composition distributions to be specified provided that a sufficiently dense mesh of elements is used. Furthermore, the mesh used to represent the composition distribution can also be used to perform finite element analysis to obtain the mechanical properties of the structure. This approach was introduced in an earlier paper (Kumar and Wood, 1999) where two-dimensional elements were used to illustrate the idea. Here we have described two three-dimensional elements that can be used to model composition distribution in a solid.

Composition distribution is a volumetric property that can be expressed as a scalar or a vector field defined over the volume of the solid. A porous material can be described by defining its density distribution as a scalar field $\phi=\phi(x,y,z)$. If the material is a composite of 'n' component materials, a vector of length n can be used to express the composition at any point. The first n-1 members of the vector can be used to represent the volume fraction of the first n-1 materials that make up the composite and the nth member can represent the void fraction (or density). Additional information may be required for some heterogeneous materials to specify other material properties such as the directions of anisotropy and fiber orientation of composites. All such volumetric properties may be expressed as a vector field $\mathbf{v} = \mathbf{v}(x,y,z)$. Within each element volumetric properties can be expressed as follows by interpolating the values at the nodes of the element.

$$\mathbf{v}(r,s,t) = \sum_{i=1}^{n} \mathbf{v}_i N_i(r,s,t) \tag{2.1}$$

where, $\mathbf{v}_i$ is the volumetric property vector at the node i, N_i are the basis functions used to interpolate the properties, n is the total number of nodes in the element. In this paper, we have used iso-parametric elements (Bathe, 1996) where each element is a perfect cube in the parametric coordinates. The parameters r, s and t vary from -1 to 1 within each element so that each element is a cube of side length 2 in the parametric space. The basis functions mentioned above are also used to define a mapping between the parametric coordinates and the real coordinates x, y and z as defined below.

$$x(r,s,t) = \sum_{i=1}^{n} x_i N_i(r,s,t), \quad y(r,s,t) = \sum_{i=1}^{n} y_i N_i(r,s,t) \quad \text{and} \quad z(r,s,t) = \sum_{i=1}^{n} z_i N_i(r,s,t) \tag{2.2}$$

In the above equations (x_i, y_i, z_i) are the coordinates of the node i. A variety of elements and their basis functions can be defined that can be used to represent the distribution of volumetric properties within a solid. As the number of nodes per element increases the degree of the basis functions used for the interpolation also increases. Two such elements and their basis functions are defined below.

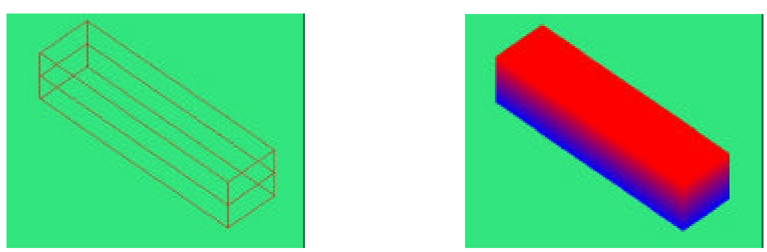
(a) 8-node element (b) 18-node element

Figure 1: 3D hexahedral elements

The basis functions of an 8-node hexahedral element that is commonly used in finite element analysis is defined below. The element is illustrated in figure 1(a) and its eight basis functions can be expressed as:

$$N_i = \frac{1}{8}(1 + r_i r)(1 + s_i s)(1 + t_i t)\,,\ \ i = 1,\ldots 8 \qquad (2.3)$$

where (r_i, s_i, t_i) are the parametric coordinates of the node i. The basis functions and therefore the interpolation within each element are bilinear. Using this type of element, one can define simple piece-wise linear distribution of composition and material properties over a solid. For example, we may want to specify that a solid of material A has a coating of material B such that its composition varies linearly from B to A within a specified depth from the boundary of the solid. This can be achieved by generating a mesh of 8-node hexahedral elements such that the first layer of elements on the boundary is of the appropriate thickness so that a linear variation from pure B to pure A can be represented. Figure 2 illustrates two hexahedral elements with nodal compositions defined such that the composition varies linearly from a red material at the top to a blue material at the bottom.



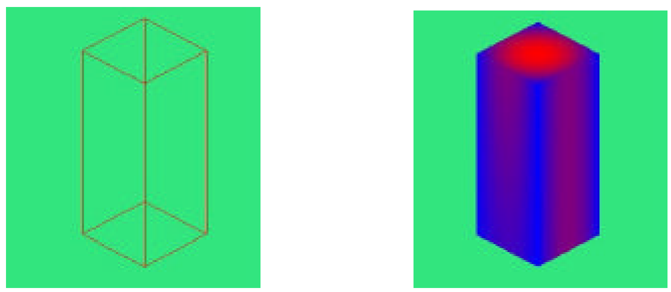
(a) Mesh (b) Composition distribution

Figure 2: Linear variation of composition

More complex distributions require higher order elements. An eighteen-node element can be defined that allows a cylindrical variation of composition. The basis function N_i for the i^{th} node of the 18-node hexahedral element shown in figure 1(b) can be defined as follows:

$$N_i = \frac{(1+r_i r)(1+s_i s)(1+t_i t)(1-r_i^2+r_i r)(1-s_i^2+s_i s)(1-r^2+r_i^2 r^2)(1-s^2+s_i^2 s^2)}{(1+r_i^2)(1+s_i^2)(1+t_i^2)}, i=1,\ldots 18 \quad (2.4)$$

Again (r_i, s_i, t_i) are the parametric coordinates of the node *i*. The composition distribution of a two component non-porous composite can be expressed by defining the nodal values of the volume fraction ξ of the primary material. Figure 3 shows a single 18-node element with nodal values of the volume fraction of the red material ξ defined such that it varies in the radial direction from 100% at the center of the element to zero along the edges where the material is blue. The nodal values are defined such that ξ = 0 for nodes 1-8, ξ = 0.5 at all other nodes except 13 and 18 where ξ = 1.0 (figure 1(b) shows the node numbering used here).



(a) Mesh (b) Composition distribution

Figure 3: Cylindrical composition distribution

As mentioned earlier, the finite element mesh used to represent the composition distribution can also be used to perform a finite element analysis to obtain structural and other mechanical properties of the solid. Since the composition varies from point to point within the solid so does the material properties such as elastic constants and density. An appropriate relationship between material properties and the composition must be established in order to compute the material properties at each point. This relation, which we will refer to as the *material property function*, depends on the microstructure and may vary if the component is made using different techniques even if the composition is identical. The true relation between material properties and composition for a composite material can perhaps be established only by experimental studies. Empirical relations may be established by fitting a polynomial over experimentally obtained results. In this paper, we assume the material property functions to be simple polynomials to illustrate how mechanical properties may be computed if the material property function is known.

For a linear elastic non-porous composite material with just two component materials A and B, we assume that the Young's modulus is a linear function of the volume fraction of material A. Therefore, the relation between the Young's modulus of the composite material and the volume fraction of A can be expressed as

$$E = \xi_A E_A + (1-\xi_A) E_B , \quad (2.5)$$

where, ξ_A is the volume fraction of the material A while E_A and E_B are the Young's modulus of the materials A and B respectively.

If the material is porous, it can be thought of as a composite material that is made of material and void. The composition can be expressed using a relative-density field. In this case we assume the material property to be a polynomial function of density. Assuming the material to be linear elastic, we can express the Young's modulus as,

$$E = E_0 \xi^n , \tag{2.6}$$

where, E_0 is the Young's modulus of the fully dense material, n is an integer and ξ is the relative-density or volume fraction of material. This type of material property-density relationship has been used in many previous work on shape and topology optimization where the corresponding material has been referred to as Solid Isotropic Material with Penalization (or SIMP) (Bendsøe 1995, Zhou and Rozvany 1991, Kumar 1993).

During the finite element analysis, we integrate the principle of virtual work over each element to obtain the element stiffness matrix and then assemble these together to obtain the global stiffness matrix. The deformation of the solid due to applied loads is represented by a displacement vector field that is interpolated using the same interpolation functions that is used to represent the composition distribution within each element. This representation yields the following strain-displacement relation for the element.

$$\{\varepsilon^e\} = [\mathbf{B}]\{u_h^e\} , \tag{2.7}$$

where, $\{u_h^e\}$ is the nodal displacement vector corresponding to the finite element. The governing equations can be expressed in general using the following type of weak form of a variational principal.

$$\int_V \{\delta\mathbf{X}\}^T [\mathbf{B}]^T [\mathbf{D}][\mathbf{B}]\{\mathbf{X}\} dV = \int_S \{\delta\mathbf{X}\}^T \{\mathbf{f}\} dS + \int_V \{\delta\mathbf{X}\}^T \{\mathbf{b}\} dV \tag{2.8}$$

The above equation represents the weak form applied to an element. V is the volume of the element, S is the surface area of the element that lies on the boundary of the object, $\{\mathbf{f}\}$ is the traction acting on these boundaries and $\{\mathbf{b}\}$ is the body force acting on the element. The vector $\{\mathbf{X}\}$ contains the nodal variables while the $\{\delta\mathbf{X}\}$ vector contains the corresponding virtual variables. The $[\mathbf{D}]$ matrix is a function of the material properties and therefore is a function of composition. Since composition varies from point to point this matrix is also a function of the spatial coordinates. The left-hand side of equation (2.8) needs to be integrated to determine the stiffness matrix. The integration was performed using Gauss quadrature algorithm (Bathe, 1996).

3. Optimization of composition distribution

The mechanical properties of a heterogeneous component can be determined by finite element analysis as described in the previous section. Alternately, one may want to design a heterogeneous component that has the desired mechanical properties. For example, one may solve for the optimal composition distribution with the objective of maximizing the stiffness of a structural component subject to a limit on its weight or mass. Maximizing the stiffness of a structure is equivalent to minimizing its compliance. The compliance is computed as a product of applied force times the deflection. It is a commonly used objective function in many structural optimization studies and has been used extensively in topology optimization research (Bendsoe and Kikuchi, 1988, Kumar and Gossard, 1996). Below we illustrate the process by designing a simple heterogeneous solid by minimizing its compliance. The solid is represented using the 8-node hexahedral elements described earlier. The material is assumed to be a two-component composite and the volume fraction of the primary component, denoted by ξ, is the design variable that is optimized. The optimization problem may be written as,

$$\text{Minimize } L(\mathbf{u}(\xi)) = \int_{\Omega} \mathbf{f} \cdot \mathbf{u}(\xi) d\Omega + \int_{\Gamma_t} \mathbf{t} \cdot \mathbf{u}(\xi) d\Gamma \tag{3.1}$$

subject to,

$$V(\xi) = \int_{\Omega} \xi d\Omega \le V_0 \, , \tag{3.2}$$

$$\int_{\Omega_0} \{\delta\varepsilon\}^t [\mathbf{D}(\xi)] \{\varepsilon\} d\Omega_0 = L(\delta\mathbf{u}) \tag{3.3}$$

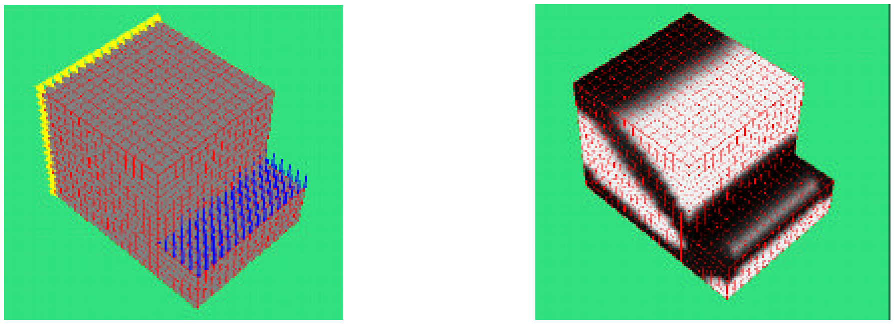
$$0 \le \xi \le 1 \tag{3.4}$$

$L(\mathbf{u})$, the mean compliance, is twice the work done by the applied forces (traction $\mathbf{t}$ and body force $\mathbf{f}$) during the displacement $\mathbf{u}$. Equation (3.2) describes the constraint that the total volume V of the primary material should be less than or equal to V_0. $\{\varepsilon\}$ and $\{\delta\varepsilon\}$ are the strain and virtual strain in the structure caused by the displacement $\mathbf{u}$ and the virtual displacement $\delta\mathbf{u}$ respectively. $[\mathbf{D}(\xi)]$ is the matrix of elasticity constants that relate stresses and strains for a linear elastic material. We assume that these elasticity constants are functions of the composition of the material.

The optimization problem given by the equations (3.1) - (3.4) can be solved using a nonlinear programming algorithm. Such optimization algorithms modify the design variables (composition vector at the nodes) in an iterative fashion searching for the optimal distribution. The objective function value (or compliance) is computed using the finite element method at least once during each iteration. Since each objective function evaluation is expensive, an optimization algorithm that does not require excessive objective function evaluations is desired. In addition, due to the large number of nodes in the mesh used to represent the composition distribution we need algorithms that can handle large number of variables. Our work uses a variation of sequential linear programming, which is described in Kumar (1993).

4. Examples and Results

Two examples of designing heterogeneous parts are presented here. In the first example, the part is assumed to be made of a two-material composite structure whose geometry is fixed but its composition can be varied to maximize its stiffness subject to a constraint on the final percentage volume of the primary material that is used. Figure 4(a) shows the geometry of the part, the applied loads and boundary conditions. The blue arrows represents normal pressure applied to the surface and the structure is supported at the back so that all the nodes on the back face are fixed (denoted using the yellow triangles). The Young's modulus of the primary material (E_p) was assumed to be $2x10^{11}$ units and Poisson's ratio of 0.3. The Young's modulus of the secondary material is assumed to be $0.5E_p$. The material property function was assumed to vary linearly with composition as in equation (2.5). Figure 4(b) shows the optimal composition distribution that was computed using the constraint that the total volume of primary material (black in color) in the final part should less than or equal to 50% the total volume of the structure.



a) Mesh, Load and Boundary conditions b) Optimal composition distribution

Figure 4: Optimal composition distribution for a cantilever

The composition variation within the part can be clearly visualized only by plotting the composition distribution on various section planes that go through the solid. In figure 5, various section planes are plotted superimposed on a semi-transparent image of the part.

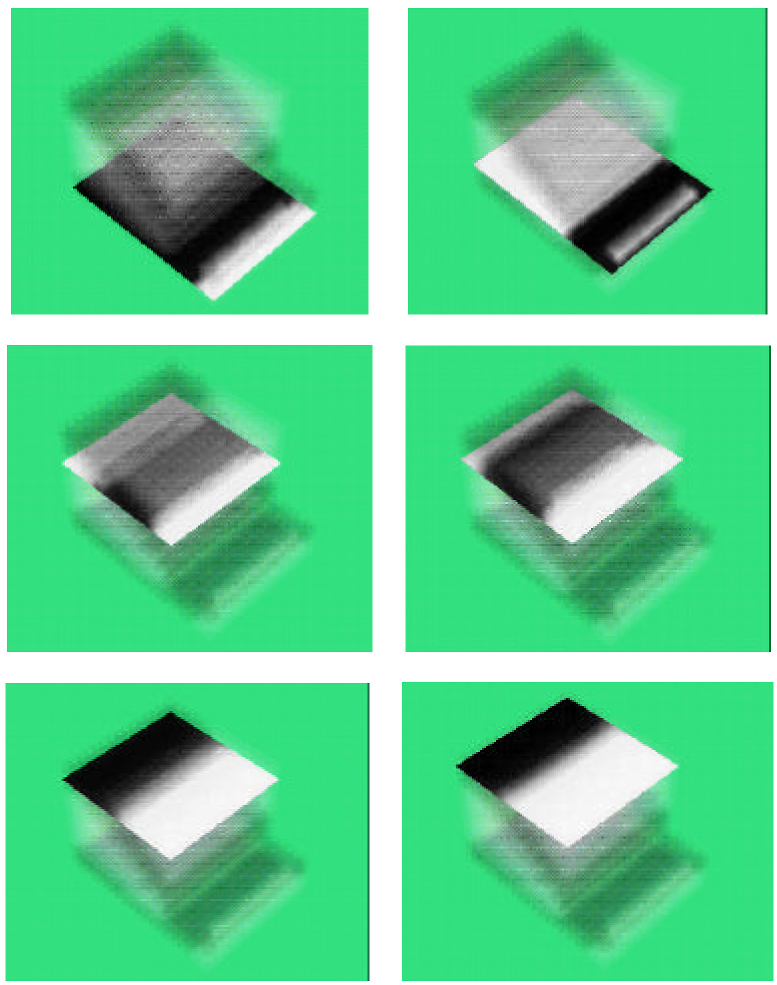
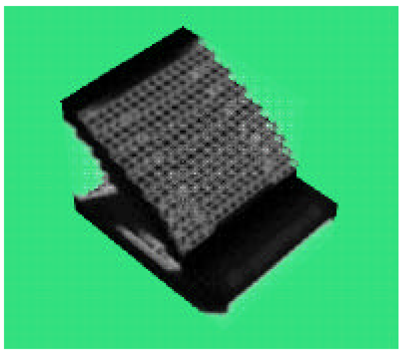
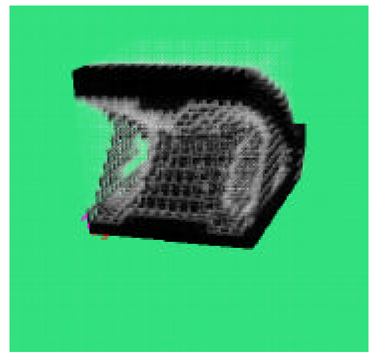


Figure 5: Composition distribution on various section planes

Another example of composition variation was studied where the geometry of the part considered in the previous example is optimized by assuming that the material is porous so that the two materials in the composite are the primary material ($E_p = 2x10^{11}$ units) and void. The volume fraction of the material, ξ, (or relative density) is optimized such that the overall volume is only 50% of the original volume of the part. This approach for geometry optimization has been used in many previous studies on shape and topology optimization. Regions where ξ=1 is fully dense and the regions where ξ=0 has no material. The mesh, the applied loads and boundary conditions shown in figure 4(a) was used in this example also. The material property function was assumed to be quadratic as in equation (2.6) when n = 2.



(a) Viewed from front



(b) Viewed from behind

Figure 6: Optimal geometry

The optimal geometry that was computed is displayed in figure 6, where regions with low value of ξ were made transparent. The transparency was varied such that the material is fully opaque when ξ=1 and fully transparent when ξ=0. The color is also varied such that the material is black in regions where ξ=1 and white where ξ=0 and various shades of gray in between.

5. Conclusion

In this paper, a mathematical representation for shape and composition distribution using a finite element mesh is described. Two hexahedral elements and their interpolation functions are described that can be used to model heterogeneous solids. The variation of the composition within each solid can be controlled interactively by modifying nodal density values or can be computed automatically by optimizing the composition distribution to meet design objectives and constraints.

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7. 8. References

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