IMPLEMENTATION OF A FUNCTIONALLY GRADIENT MATERIAL MODELING AND DESIGN SYSTEM

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Abstract

New advancements in Solid Freeform Fabrication (SFF) processes promise the capability to produce Functionally Gradient Material (FGM) parts, in which the material compositions vary spatially. To realize this potential there is a need for CAD methods and design software to model, design, represent and exchange material information and instructions to the manufacturing process. However, currently available commercial CAD systems are limited to representing and storing only geometric information, which is not adequate for material design purposes. This work presents an extension of a theoretical approach based on Volumetric Multi-Texturing (VMT) and hypertexturing schemes to make the material design process intuitive and user controllable. Inverse distance weighted interpolation is used in conjunction with procedural material functions to accomplish axial or linear material gradient directions from surface to surface across a solid. This offers the capability of specifying fixed material composition values to the faces in the solid and blending them across the interior of the solid. The extension of the proposed approach to the modeling of discrete material domains is also discussed. These material regions can be combined using special sets of operators depending on the form of the material functions. Finally, a design environment has been developed, which allows users to systematically apply material information to geometry and captures design intent.

1 Introduction

Solid Freeform Fabrication encompasses emerging rapid prototyping technologies used in making components with complex geometries without human intervention or part-specific tooling. SFF processes involve the use of a computer-generated model to provide the geometric information required in the fabrication process. In general SFF is based upon layered manufacturing techniques, in which the parts are produced layer by layer in an additive fashion. SFF thus has access to the interiors of solids and provides the most suitable means of building parts with highly accurate local material composition control, which cannot be achieved by conventional fabrication processes. These capabilities enable the use of the technology in the manufacture of parts composed of functionally gradient materials. Functionally gradient material components have smoothly varying material composition. They possess large variations in the volume fractions of their constituents and variations in microstructure because of which they exhibit some useful properties such as reduced thermal and residual stresses at joints, improved thermal and mechanical properties and high specific strength and oxidation properties [4].

The current state of solid and geometric modeling is at an advanced stage of development however existing commercially available CAD modelers do not support material design information. Thus there is a need for new modeling systems that can design and represent heterogeneous material parts. The problem addressed in this work is the implementation of a

design environment for multi-material part modeling and representation. The methodologies used are largely based on implicit procedural and Volumetric Multi-Texturing (VMT) schemes [9]. Issues such as combining and blending the effects due to multiple material constituents at a point, extensibility of smoothly varying material gradients to modeling of discrete material interfaces and achieving multiple directionality of material variation within a part have been addressed. Appropriate data structures for geometry and material attributes have been constructed for material data storage and retrieval and user interface design issues have been discussed, with emphasis on user interaction and input methods to make the design process more intuitive and predictable for users.

1.1 Previous Work

Several researchers have been working to develop heterogeneous material modeling systems. Kumar and Dutta [7] proposed a set-based approach for spatial discretization of the solid interior by associating material density functions with material subsets called r_m sets. Pegna and Safi [12], Kumar and Wood [6] and König and Fadel [5] have used finite element based methods to model and optimize material density distributions given a particular design objective and constraints. Marsan and Dutta [8] and Jackson and Petrikalakis [3] have applied volumetric mesh schemes to describe material data.

Several mesh free methods have also been applied to FGM design, which do not rely on any form of spatial decomposition of the geometry. Hu and Jiang [2], Siu and Tan [15] and Wahlborg and Ganter [16] are some researchers who have adopted heterogeneous solid modeling techniques to material modeling. Wahlborg and Ganter [16] have introduced a heterogeneous implicit solid modeling scheme that makes use of the Boolean operators union, intersection and difference on heterogeneous objects to model both solid as well as material spaces defined as implicit primitives. Biswas and Shapiro [1] discuss the parameterization of the interior of a shape using functions of normalized distance fields from material features with known material properties. The material functions are continuous and constructed using known design and manufacturing constraints and are interpolated using transfinite interpolation, with inverse distance weights to obtain overall material functions.

Representations in the form of spatial discretizations are inefficient due to large data sizes and errors due to approximations. This adds to the cost of computation and to the difficulty in modifying material models. The schemes implemented in this work are based on a mesh free implicit procedural approach and are therefore devoid of the errors that arise from spatial decomposition.

2 Implicit Procedural Approach

The material modeling scheme used in the implementation of the design interface is largely based on implicit procedural and Volumetric Multi-Texturing techniques that were first introduced by Park et al [9].

2.1 Hypertexture

Hypertexture is a technique in computer graphics that makes use of 3D volume density functions to create effects such as explosions, furry objects and eroded surfaces or objects that have variations in density. The functions that control the density distribution in the fuzzy region are called density modulation functions (DMF) and are chosen to be higher order procedural functions built upon *bias* and *gain* functions as described by Perlin [13]. The *bias* function can be used to control the magnitude of the density value and is defined over a unit interval by a power function as:

$$bias_b(t) = t^{\frac{\ln(b)}{\ln(0.5)}} \tag{1}$$

where $bias_b(0.0) = 0.0, bias_b(0.5) = b$, and $bias_b(1.0) = 1.0$. The *gain* function is used to set the density gradient. It is also defined over a unit interval and is constructed by splining two bias curves:

$$gain_g(t) = \begin{cases} \frac{1}{2} bias_{1-g}(2t), \text{ for t} < 0.5 \\ 1 - \frac{1}{2} bias_{1-g}(2-2t) \text{ otherwise} \end{cases} \tag{2}$$

The function used in this work is simply a product of the *bias* and *gain* functions. It is a monotonically increasing function that lies in the interval [0,1].

$$dmf = bias_b(t) \cdot gain_g(t) \tag{3}$$

2.2 Blending Functions

These are smoothing functions used to approximate Boolean operations on density modulation functions. Ricci [14] introduced smoothing functions to approximate unions and intersections on surfaces describing 3D solids. The functions chosen for intersection, *I* and union, *U* are respectively:

$$\begin{aligned} I_p(f_1,....,f_n) &= \left(f_1^{\,p} + ... + f_n^{\,p}\right)^{1/p} \\ U_p(f_1,...,f_n) &= \left(f_1^{-p} + ... + f_n^{-p}\right)^{-1/p} \end{aligned} \tag{4}$$

where p is a positive number and is the blending factor that controls the degree of smoothness and $(f_1,...,f_n)$ are the n individual functions to be blended. Park [10] has used Ricci's functions with implicit surfaces to construct complex 3D solids for global material spaces. He derives a general form for a union operation from Ricci's intersect function to perform a blending on density modulation functions, which can be generalized for n functions $(f_1,...,f_n)$ as:

$$f_{blend}(\vec{p}_{\text{int}}, a) = \left[1 + (-1)^{n-1} \begin{vmatrix} f_1^a & f_2^a & ... f_{n-1}^a & f_n^a & 1 \\ 1 & f_2^a & ... 1 & f_n^a & 1 \\ . & . & .... & . & . \\ . & . & .... & . & . \\ . & . & .... & . & . \\ 1 & 1 & ... 1 & f_n^a & 1 \\ 1 & 1 & ... 1 & 1 & 1 \end{vmatrix} \right]^{1/a} = \left(1 + (-1)^{n-1} \prod_{i=1}^{n} \left(f_i^a - 1\right) \right)^{1/a} \tag{5}$$

where a is a positive real number [11]. Increasing the value of the blending factor a, has the effect of sharpening the texture of the blend.

2.3 Volumetric Multi-Texturing

Volumetric Multi-Texturing (VMT) is a popular computer graphics technique to represent fuzzy features like clouds, smoke and flames and is extended by Park [11] to the modeling of material clouds. Using the VMT approach any point $\vec{p}_{\text{int}}$ interior to an object in geometric space G^3 can be interrogated to return the material density d_p^i at that point using a set of procedural functions F_m such that for each material i:

$$F_m^i(\vec{p}_{\text{int}}) = d_p^i, \vec{p}_{\text{int}} \in G^3,$$
$$\sum_{i=1}^{n} F_m^i(\vec{p}_{\text{int}}) = \sum_{i=1}^{n} d_p^i = 1 \tag{6}$$

The densities d_p physically represent volume fractions of each material and sum to one at each point for all n materials that compose the part. Therefore only $n-1$ functions, F_m's are needed to fully constrain the model. Each function, F_m is a procedure consisting of several contributing functions, f_{m_i} that are attached to surfaces (faces or group of faces) in the part geometry. These contributing functions are combined together using the blending function described in equation (5) and are functions of the distances of the sample point from the part surfaces and reference geometry such that the closer the sample point is to an entity the greater is the influence of the contributing function associated with that entity in determining the density at that point.

Park [11] introduces two classes of gradient functions, namely, global and local (surface) gradients. Global gradients are one-dimensional scalar functions (either density modulation functions or user-defined functions) defined over material spaces called blobs that are mapped onto 3D geometry. The shape of the global material space determines the overall pattern of the material distribution. In this work a B-rep representation of the material space geometry has been added to the existing implicit surface description by Park [10]. If the material function is a function of an independent parameter t, in the case of B-reps, t varies linearly with distance from the boundary surfaces whereas in the case of an implicit surface, t evaluates to the implicit function value and is therefore proportional to the distance from the surface. Material gradient functions similar to those used with global gradients when assigned to the surfaces of the part geometry serve as surface gradients. The global gradient function, when multiplied with the surface gradient, is trimmed to force the net gradient to conform to the boundary surface of the part geometry.

3 Multi-Directional Material Variation

The existing material modeling method focuses on improving surface coatings on components that are exposed to severe thermal and mechanical stress environments. The current approach therefore places importance on material variation from the surfaces of components into the interior and does not support general material distributions in the interior. Further, user-

prescribed boundary material compositions on surface features are not satisfied. Modeling a linear material gradient pattern that starts with one material fraction on one face and blends into another material fraction on another can be achieved by using multiple surface gradient functions for each face, one material function for each material. Consider an object with different material functions assigned to each face in a pair of adjacent faces. A simple blend of the two functions in the overlapping region where each face has an influence on the material properties at points on the adjacent face, would not yield the exact material compositions specified on the faces. For this reason the functions for each material for each face need to be trimmed to the boundary values on the neighboring face using a different approach, as described below.

3.1 Inverse Distance Weighting Function

Consider n faces or surface features and let the material composition of a material k be C_i on the face F_i, such that $0 \le C_i \le 1$. The net material composition of material k at a given point p due to the influence of all n faces is:

$$f_k(p) = f_{blend,k}(p)\left(\sum_{i=1}^{n} C_i W_i(p)\right) \tag{7}$$

where W_i is a weight function controlling the influence of the blended material function, $f_{blend,k}(p)$ with the face F_i. The weight function has the following properties:

- In order for $f_k(p) = C_i$ for points on the face F_i,

$$W_i(p) = \begin{cases} 1, \forall p \in F_i \\ 0, \forall p \in F_j, j = 1,..,n, j \neq i \end{cases} \tag{8}$$

- $W_i(p)$ must form a partition of unity to restrict the sum of the terms on the right hand side of equation 3.12 to one.

$$\sum_{i=1}^{n} W_i(p) = 1, 0 \le W_i(p) \le 1 \tag{9}$$

- The weight functions should be as smooth as the blended function, $f_{blend,k}$.

The weight function is chosen based on distances u_i of points p from the reference surfaces for the material functions (surface gradient functions) such that it is inversely proportional to some power of u. Normalizing the weight function by the sum of all weights yields a function that varies between 0 and 1:

$$W_i(p) = \frac{u_i^{-k}}{\sum_{j=1}^{n} u_j^{-k}} = \frac{\prod_{j=1; j\neq i}^{n} u_j^k}{\sum_{i=1}^{n} \prod_{j=1; j\neq i}^{n} u_j^k} \tag{10}$$

where u_i is the distance of p from the face F_i and the exponent k controls the smoothness of the weight function. These functions are called the inverse distance weighting functions or Shepard's method of data interpolation, as discussed by Biswas and Shapiro [1].

The limitation of this approach is that it does not ensure that the sum of the material densities at a point p for all materials $\sum_{j=1}^{m} f_j(p)$ equals 1, where m is the total number of materials. One way to treat this inadequacy is to consider the remaining quantity $\left(1-\sum_{j=1}^{m} f_j(p)\right)$ to constitute porosity in the part or to constitute the volume fraction of the $(m+1)^{th}$ material. Another way is to normalize the volume fraction due to an individual material k to obtain:

$$f_{k,N}(p) = \frac{f_k(p)}{\sum_{j=1}^{m} f_j(p)} \tag{11}$$

such that

$$\sum_{k=1}^{m} f_{k,N}(p) = 1 \tag{12}$$

As a result of this addition, it is now possible to model multiple material gradient patterns, namely linear as well as concentric or radial material gradations so as to achieve any direction of material variation. Figure 1 shows concentric contours of material gradation between two materials for a cross-section of a bottle mold. It is modeled by assigning a single density function for the primary material to each of the four faces forming the boundary, which blends into the secondary material in the interior. In Figure 2, three faces have been assigned 100% composition of one material each. Using the proposed method, it is now possible to clamp each material composition to be exactly 100% on its respective face and achieve a gradual blend of the three into each other. Here the depth of material influence for each face is set to be greater or equal to the length of a side of the square cross-section.

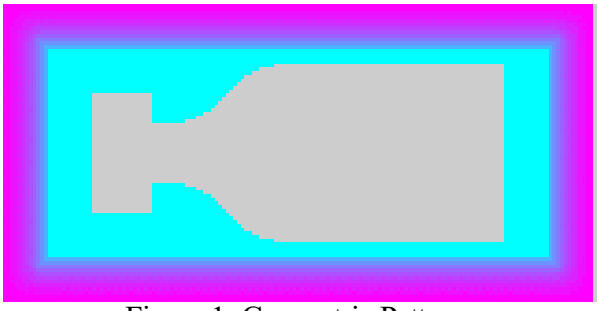


Figure 1: Concentric Pattern

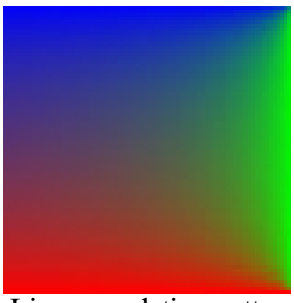


Figure 2: Linear gradation pattern with 3 materials

4 Modeling Discrete Material Domains

The current approach does not account for discrete heterogeneity of materials and hence offers only a restricted case of material modeling. Modeling discrete material regions can be considered as a special case of FGM modeling. The same approach can be applied to discrete material modeling by using constant functions or step functions with the reference geometry. The global material spaces can be used to define discrete material domains confined within the B-rep by assigning constant material fractions to individual blobs. Since assigning a value of 1 to the

bias and gain functions makes the density modulation function in equation 3 a step function at the midpoint of the [0, 1] interval, material compositions could be made to change discretely at these points (see Figure 3). The blobs can then be combined using regularized Boolean operations for B-rep geometry and *min* and *max* functions [16] or equivalent Ricci's functions for implicit solids. Modifications to Boolean operations or equivalent rules must be formulated to combine the corresponding material regions.

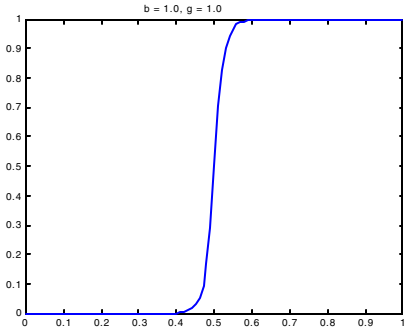


Figure 3: DMF curve for bias = 1.0, gain 1.0

In this work generalized Boolean operations are applied to scalar material functions f and g as in hypertexturing [13]:

$$\begin{aligned} &\text{Intersection: } f(x)\cdot g(x) \\ &\text{Union: } f(x)+g(x)-f(x)\cdot g(x) \\ &\text{Complement: } 1.0-f(x) \\ &\text{Difference: } f(x)-f(x)\cdot g(x) \end{aligned} \tag{13}$$

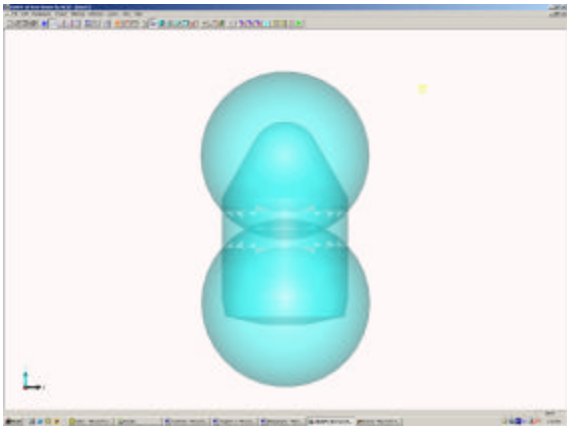


Figure 4: Discrete material regions modeled using global gradient

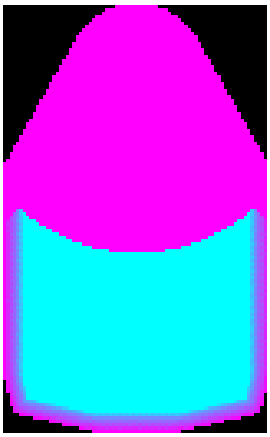


Figure 5: Difference of spheres

Using the above operators on material functions, material regions of arbitrary complexity can be constructed from primitive shapes. Consider the drill insert shown in Figure 4. The two spheres illustrate the use of blobs as global density gradients to define discrete material regions. Figure 5 is obtained after performing a difference of the upper sphere from the bottom one. The net gradient takes the shape expected if a Boolean subtraction were performed on the spheres.

The smooth gradient near the boundary is the result of using smooth functions for the surface gradient.

5 Implementation and Results

The material modeler has been developed and implemented using the 3D ACIS® Modeler and the HOOPS 3D Application Framework. ACIS is a geometric modeling kernel written in C++ that supports modeling of wireframe, surface and solid geometry in a unified data structure. It defines geometric shapes as B-reps and saves model information in the "SAT" format, a format that is slowly becoming a *de facto* standard for the exchange of CAD data. ACIS has been used to build the data structures for the geometry and material information and to perform the required modeling operations. A skeletal application based on HOOPS 3D, a graphics toolkit, has been used to design the user interface for the application.

5.1 Attribute Architecture for Material Information

ACIS supports user-defined attributes that can be attached to geometric and topological entities and can be saved along with them in an SAT file. In this research, user defined attribute classes constructed in ACIS have been used to store the material data associated with the part for use by procedures that generate material distributions. Figure 6 shows the data structure used for modeling the overall density gradient in a part. The blocks in blue represent the user defined attribute classes and the arrows, their associations with each other.

The blob attribute (ATTRIB_BLB) consists of either an ACIS BODY (B-rep) or a user defined implicit function describing an implicit solid primitive since the global density gradient can be modeled in either one of these two ways. This attribute also consists of the *bias* and *gain* parameters for the density modulation function or a user defined material function associated with the blob surface. Finally it specifies the depth of influence of the blob surface over the material composition at an interior point.

The blend attribute (ATTRIB_BND) located one level higher in the hierarchy stores information on how to combine the blob functions using union, intersect or difference operators as described in section 4. It also contains a blending factor, a, which is used to blend the blob functions in the case of a union operation using equation 5. The blend attribute is assigned to the part to be modeled. The blob attributes are stacked together in a doubly linked list such that at all times the blob at the top of the stack is pointed to by the owning entity, which is the blend attribute in this case. The remaining blobs can be accessed using the "next" pointer of the previous one in the list. The blend attribute in turn is owned by the ACIS BODY and can be accessed by it.

The surface gradient data is stored in two separate attribute classes, map and join. The map attribute (ATTRIB_MAP) contains an array of material compositions assigned to a face on the part and arrays of dmf parameters of bias and gain for each material function attached to the face. The owning entity in this case is an ACIS FACE (see Figure 7). There is also a provision for user defined material functions for each material and an array for the depths of influence of the face for the contributing material functions for each material.

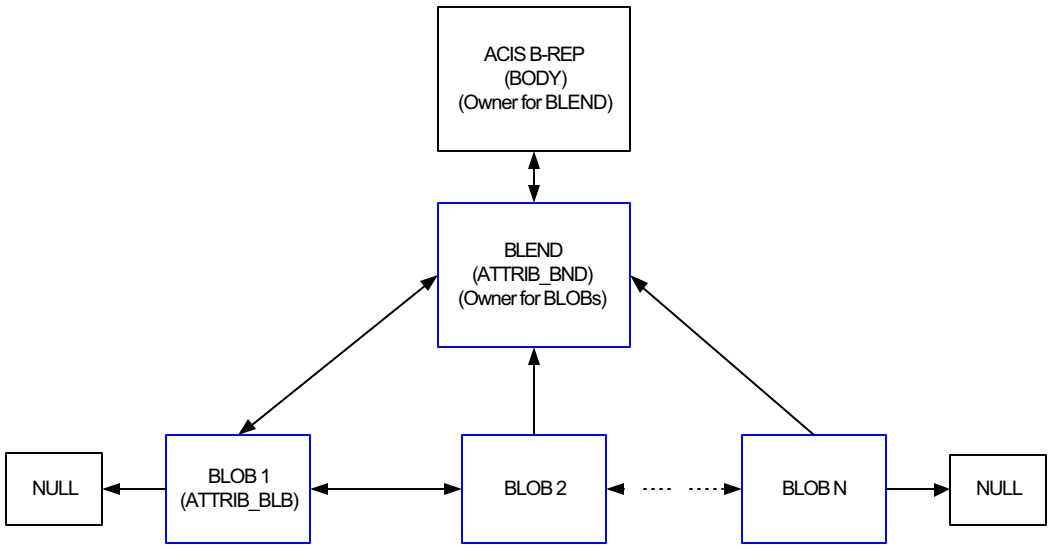


Figure 6: Data Structure for Global Density Gradient

One of the features of the material modeler implementation is the automatic selection of faces to be used to blend associated material functions in the part. The approach used here is that of blending faces that are adjacent to an edge or vertex, using a blending factor unique to that edge or vertex. For this purpose a join attribute class (ATTRIB_JON) containing the value for the blending factor is created and attached to an EDGE or VERTEX owning entity (see Figure 7).

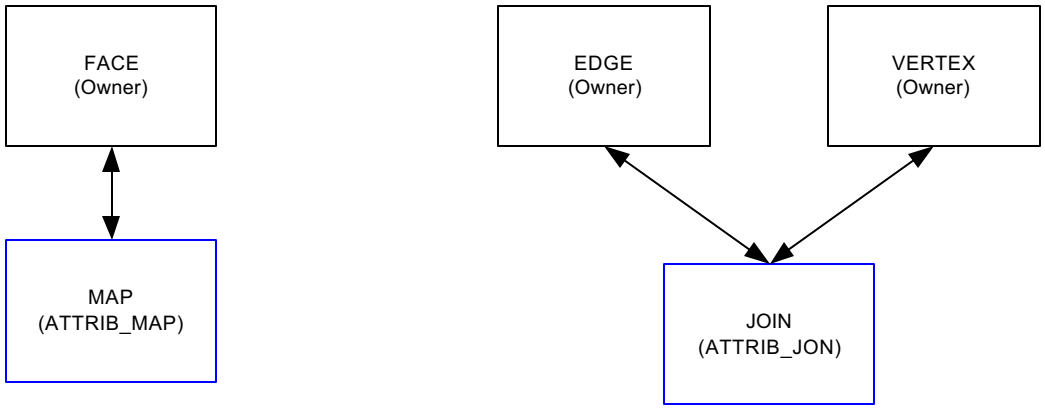


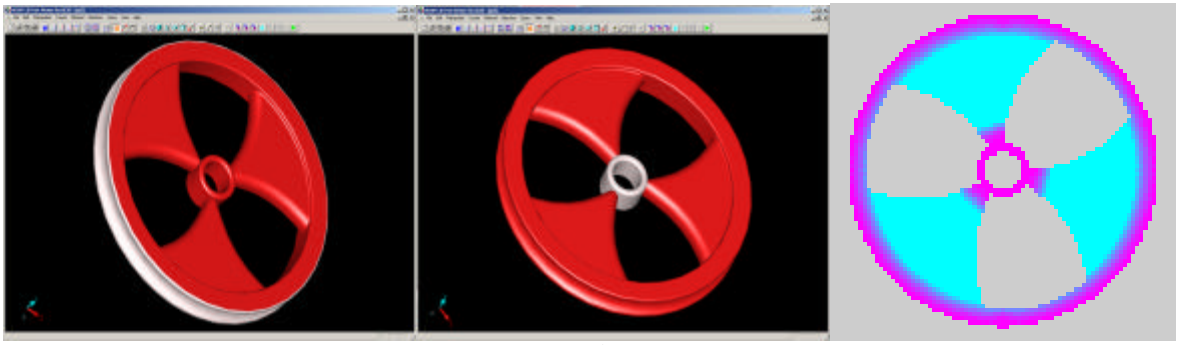
Figure 7: Data Structures for Surface Density Gradients

The user interface (GUI) for the material modeling application has been built using the HOOPS 3D Application Framework (HOOPS/3dAF). It supports the implementation of the material modeling features that have been described above. B-rep models in the ACIS SAT format can be imported in the viewer. The interface has the provision for creating solid primitives and performing Boolean operations on them to serve as complex geometry for the blobs. Implicit functions for the definition of blob geometries and material density functions for both global and surface gradients can then be assigned to the part geometry. Finally geometry and material attributes can be saved and retrieved to and from SAT files. The computed material distribution is displayed as an image.

5.2 Results

The following example is the FGM design of a pulley shown in Figure 8. Here the design intent is to make the part more wear resistant in areas subject to maximum wear. For example the pulley can be built from stainless steel while having a greater concentration of a wear resistant material like carbide in areas near the hub and the rim [3].

The part is modeled using a surface gradient approach. The material information for only one material (primary material) is entered (carbide in this case) and the secondary material density (stainless steel) is the complement of the first. The faces that compose the surfaces of the rim and hub of the pulley respectively, are selected and assigned a material function (see Figure 8 (a) and 8 (b)). Since these surfaces are composed of the same material (carbide) they are all assigned a 100% material fraction for the same material. Figure 8 (c) displays the resulting material gradient distribution on a cross-section of the pulley passing through its center laterally.



(a) (b) (c)

Figure 8: FGM Design of a pulley

This example illustrates the ease with which the model of the pulley can be designed for FGM variation. The design is shown to be completed in four steps using the material modeling interface. It can also be seen that the material variation conforms to the original design intent of having greater concentration of carbide at the hub and the rim, indicated by the region colored in magenta and a gradually increasing concentration of stainless steel with increasing distance away from the hub and the rim on the spokes.

6 Conclusions and Future Work

New developments in SFF processes promise the capability of fabricating functionally gradient material parts. To realize this potential new CAD methods and design software must be developed to enable designers to model, design, and transmit FGM objects and instructions to the manufacturing process. FGM's are expected to show clear advantages over homogeneous materials in terms of mechanical and thermal properties. The goals of the work presented were to take existing research [9] based on implicit procedural methods and extend it in order to build a tool to design volumetric material information accurately and intuitively. The advantages of the procedural approach are accurate description of three-dimensional material data and compactness in data storage by having to store only the instructions for the procedures and their arguments.

Volume discretization methods are discarded, leading to increased accuracy, concise representation and resolution independence.

Inverse distance weighted functions have been used in conjunction with the procedural material functions to accomplish axial or linear material gradient directions and offers the user the capability of specifying fixed material composition values to the faces in the solid and blending them across the interior of the solid. The definition of global density gradients can be expanded by representing them as both implicit surfaces as well as boundary representations (B-reps). This permits the user to draw or import more complex geometry to define global material spaces. However the use of B-reps to define global gradients significantly increases the amount of data that must be stored compared to a single compact implicit function expression that could define an entire implicit solid. It has been proposed that one-dimensional density functions can be step functions or constant material fractions in order to represent homogeneous material regions. These material regions can be combined using special sets of operators depending on the form of the material functions. Finally, a usable material modeling interface has been designed by which designers can systematically apply material information to B-rep geometry and observe the resulting material distributions in cross-sections of the solid. The user interface captures design intent and is an intuitive tool for designers to use.

6.1 Future Work

Apart from material volume fractions in a component, the macro-structures are equally significant in determining the physical properties of the part. Therefore designing and optimizing the structure variation of materials, e.g. composites, is a valid area of further research. This therefore presents the possibility of designing and optimizing smooth spatial (3D) variations in a structure between two materials by grading reinforcement orientations. Another prospective area for exploration is the formulation of a design methodology for FGM objects that must be placed in the context of an overall design process. Creating such a design methodology would involve investigating how a designer interacts with the model. For FGM modeling, the specification of graded material information is closely related to the geometric design. Ideally, a design system would allow the user to interactively modify the geometry and the composition in any order. Methods for achieving this goal depend on the underlying representation as well as the chosen design methods.

Though geometric data can be exchanged to a limited extent between data formats there is still a need for a new standard data exchange format that is universally supported by most CAD systems to efficiently and accurately translate non-geometric data and attributes from one format to another. In addition to this there would also be the need for new data translators to translate data back and forth between native formats and the new standard format. The next step in multi-material manufacturing is to use the material model and convert it into machine instructions by developing an SFF process-specific process-planning algorithm. The research can only be validated when results in the form of physical products can be produced. Therefore the development of the necessary hardware and the compatible software for tool-path generation is a vital research area that needs to be investigated in the future.

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