Level Set Methods for Modeling Laser Melting of Metals

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Abstract

The physical model describing heat transfer and melting taking place during and after the interaction of a laser beam with a semi-infinite metal surface is based on the classical Stefan problem with appropriately chosen boundary conditions to reflect direct selective laser sintering of metals. A level set method for solving this problem is presented in this paper. From the results of these computations, we obtain time evolution of solid-liquid interface and temperature distribution.

INTRODUCTION

Direct selective laser sintering of metals [1] is a process in which a high-energy laser beam directly consolidates a metal powder or powder mixture to full density. Direct selective laser sintering of metals is a complex process exhibiting multiple modes of heat, mass and momentum transfer, and chemical reaction mechanisms. Among them, melting and resolidification processes in direct SLS can have significant effect on the temperature distribution, residual stress, and final microstructure quality of the parts. The inherent complexity of this process imposes serious constraints on the complexity of the models that can be constructed to enable a fundamental understanding of the important physical mechanisms in SLS. This understanding is essential to implement effective process control [2].

There are numerous previous studies for understanding this kind of phase change problem involving moving boundaries. Above all, tracking the motion of a moving front has been of great interest for many researchers. In this paper, a convenient scheme to track moving interfaces using level set theory is extended to the analysis of the Stefan problem. This level set formulation is based on front capturing. In this formulation, the boundary of solid-liquid interface is modeled as the zero set of a smooth function ϕ defined on the entire physical domain. The boundary is then updated by solving a nonlinear equation of the Hamilton-Jacobi type on the whole domain. This level set formulation of the moving interface was introduced by Osher and Sethian [3] and was capable of computing geometric properties of highly complicated boundaries without explicitly tracking the interface [4]. Equation 1 is the level set equation given by Osher and Sethian. For certain forms of the speed function F, one obtains a standard Hamilton-Jacobi equation. Equation 1 describes the time evolution of the level set function ϕ in such a way that the zero level set of this evolving function is always identified with the propagating interface shown in Fig. 1 [5].

$$\phi_t + F|\nabla\phi| = 0 \qquad (1)$$
$$\text{given } \phi(x, t=0)$$

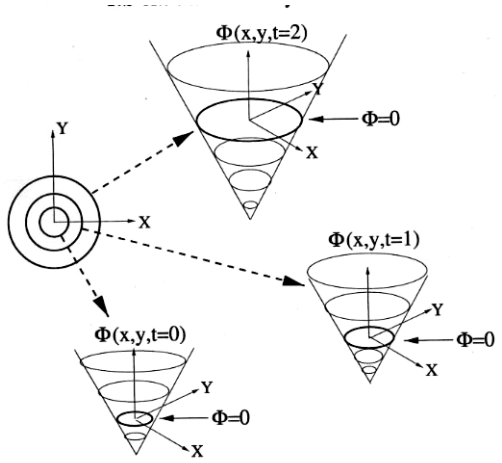

Fig. 1 Transformation of front motion into initial value problem

PHYSICAL MODEL

In this paper, an axisymmertic heat conduction model with phase change in a 5 mm^3 solid is considered. Heat flux from a laser flows in through the top surface during heat up while the other sides are assumed insulated. If heating continues long enough, melting commences and the melt interface moves inward. On the other hand, heat is lost from the top surface only by radiation. The following assumptions are made for developing the model.

- Powder is treated as a solid and no sintering densification occurs during the process.
- Laser beam intensity distribution is uniform across the beam diameter.
- Material properties are independent of temperature in both solid and liquid state.
- No convective heat transfer at top surface (process occurs in a vacuum).
- No melt pool convection, no convective heat transfer at melt interface.
- No evaporative heat loss and no evaporative mass transfer at top surface.
- Top surface is diffuse and gray.

The governing equations are

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t} \tag{2}$$

with boundary conditions on the top surface

$$-k\frac{\partial T}{\partial n}=\alpha_a q''+\varepsilon\sigma(T_\infty^4-T^4) \tag{3}$$

and Stefan condition

$$k_s\frac{\partial T}{\partial n}\bigg|_s-k_l\frac{\partial T}{\partial n}\bigg|_l=\rho V\lambda \tag{4}$$

NUMERICAL SCHEME AND EXPERIMENTAL PARAMETERS

There are a few widely used methods for phase change problems such as enthalpy methods, temperature based equivalent heat capacity methods and front tracking schemes. Each method has some disadvantages as well as some advantages. In this paper, level set method based on the finite elements method was developed. For solving this problem, commercial

software, FEMLAB (by COMSOL) was used. FEMLAB can model virtually any physical phenomena described by partial differential equations (PDEs) including heat transfer, fluid flow, electromagnetics and structural mechanics. FEMLAB also allows users to couple these various transport processes and reactions running simulations on all of them simultaneously [6].

To deal with the discontinuity of material properties from solid to liquid, we introduced the Heaviside (step) function in our model [7].

$$H = \begin{cases} 0 & if\ \phi \le -\delta \\ \dfrac{\phi+\delta}{2\delta} + \dfrac{\sin(\pi\phi/\delta)}{2\pi} & if\ |\phi| < \delta \\ 1 & if\ \phi \ge \delta \end{cases} \quad (5)$$

Using this function, material properties are defined as following.

$$\rho = \rho_l + (\rho_s - \rho_l)H(\phi) \quad (6)$$

$$C_p = C_{pl} + (C_{ps} - C_{pl})H(\phi) \quad (7)$$

$$k = k_l + (k_s - k_l)H(\phi) \quad (8)$$

Other parameters values used for the computations are shown in table1.

Domain size	Number of Elements	Time step	Laser beam diameter	Laser beam power
5 x5x5 mm^3	3048	1 x 10^{-4} sec	200 μm	1 kW.

Table1: Parameters used for numerical computations

Figure 2 shows mesh structure used for the computations.

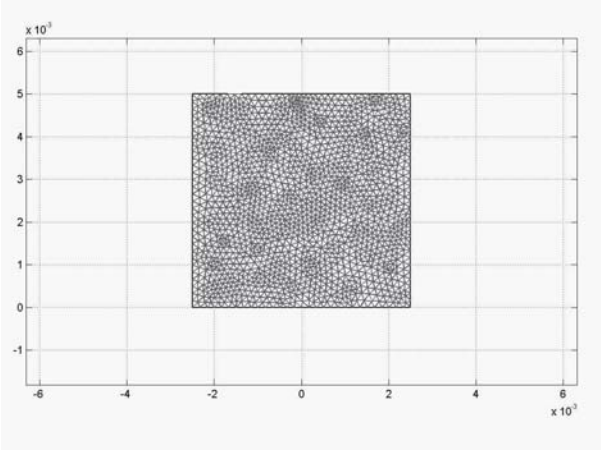

Fig. 2 Mesh structure

RESULTS AND DISCUSSION

Figure 3 and 4 show initial location of solid-liquid interface and temperature distribution. As shown in Fig. 3, zero level set of function which indicates solid-liquid interface is assigned initially as top surface.

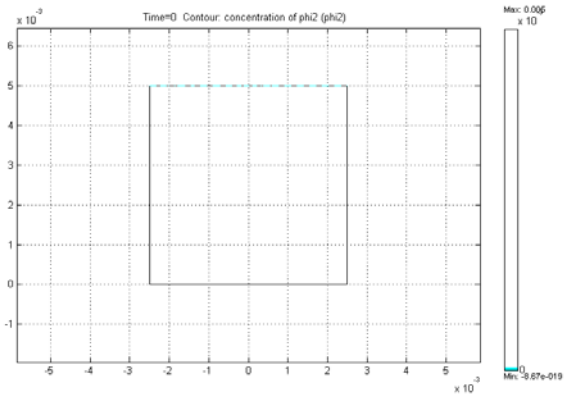

Fig. 3 Location of solid-liquid interface at t = 0 sec

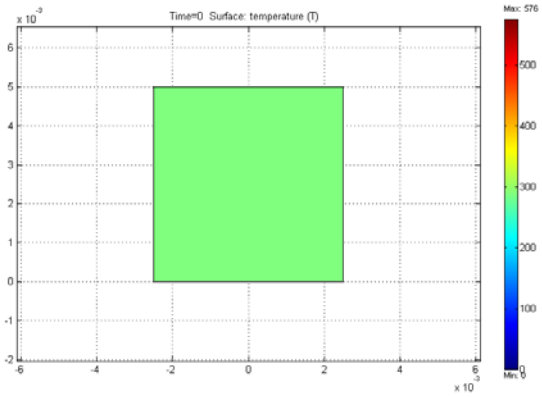

Fig. 4 Temperature distribution at t = 0 sec

As time goes on and heat flux continues to be supplied, the location of solid-liquid interface penetrates deeper and diffuses out axially. Figure 5 and 6 show this phenomenon which can be expected.

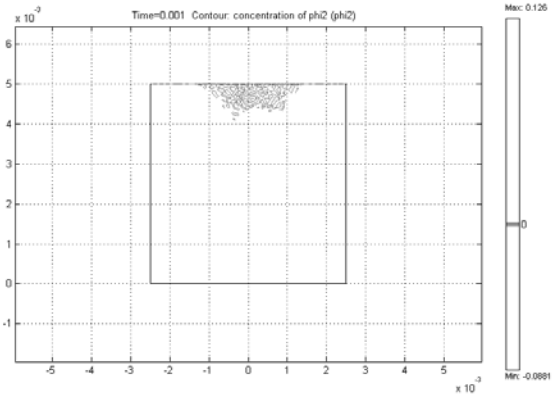

Fig. 5 Location of solid-liquid interface at t = 0.001 sec

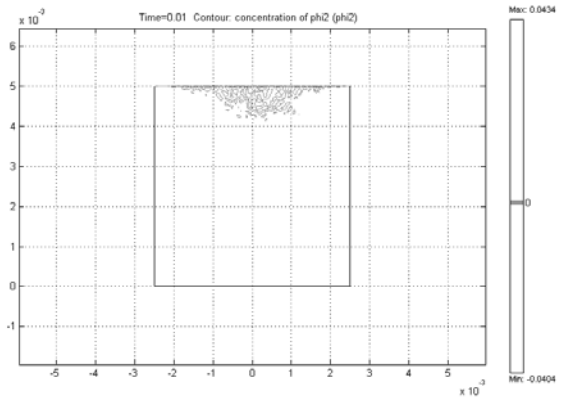

Fig. 6 Location of solid-liquid interface at t = 0.01 sec

Figure 7 and 8 show temperature distribution at corresponding time respectively. Here the peak temperature is far in excess of melting temperature. This is likely a result of not including the effect of latent heat of vaporization.

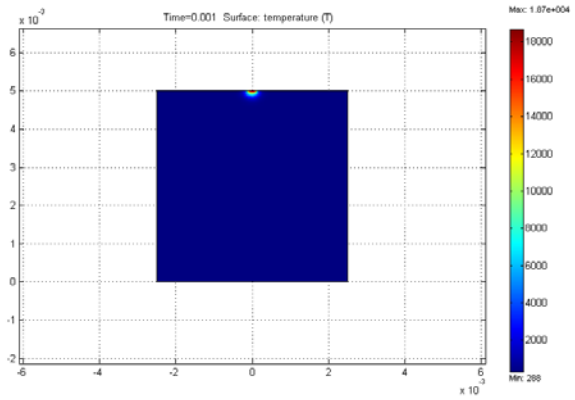

Fig. 7 Temperature distribution at t = 0.001 sec

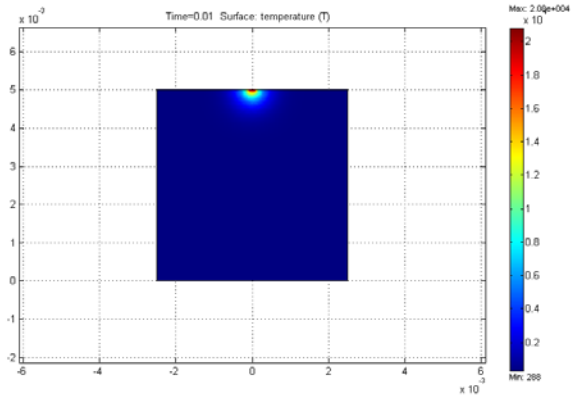

Fig. 8 Temperature distribution at t = 0.01 sec

From this simulation, we proved that the level set method is a useful method for phase change problems. The model presented in this paper will be extended to solidification process for tracking the solid-liquid interface location. Further, we can use an additional level set function to simulate surface deformation due to fluid flow from surface tension gradients simultaneously. Future work will include level set re-initialization process to get more accurate results. Re-initialization is a preferable method to avoid steep or flat gradients developed in the level set function ϕ. The level set function has to keep the exact signed distance from the evolving interfaces. By adding this process, we can acquire more accurate and smoother deformed surface as well as solid-liquid interface.

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NOMENCLATURE

Symbols

c_p = specific heat (J/kgK)
t = time (sec)
k = thermal conductivity (W/mK)
q'' = heat flux (W/m^2)
H = Heaviside function
T = temperature (K)
x = Cartesian coordinate (m)
y = Cartesian coordinate (m)
z = Cartesian coordinate (m)

Greek Letters

α_a = absorptivity of surface
α = thermal diffusivity (m^2/s)
ε = emissivity of surface
δ = small value
λ = latent heat of fusion (kJ/kg)
ρ = density (kg/m^3)
σ = Stefan-Boltzmann constant
ϕ = level set function