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Thermal Optimization of Injection Molds Produced by Layered Manufacturing Techniques

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Abstract
Producing injection tooling with cooling channels of almost any form seems to be one of the most promising advantages of Layered Manufacturing Techniques (like Selective Laser Sintering). It could be efficiently exploited to achieve higher productivity or better quality parts in injection molding. Unfortunately, at the present time, the lack of data-processing tools to design optimal cooling systems still prevents us from fully benefiting from this new potential.
The first objective of this paper is to present a methodology for the optimal design of cooling systems in three-dimensional injection molds. Our optimization process is based on a finite element model of the mold and on the standard gradient method.
In the second part of this paper, we compare a conventional mold and a mold equipped with a cooling system optimized by means of the proposed methodology. The comparison is carried out thanks to an appropriate protocol. The conclusion is that the optimization of the cooling system doubled the productivity of the mold.

Keywords:
Cooling, Optimization, Layered Manufacturing

1. Introduction

One of the most promising potential of layered manufacturing processes (LMP) is their capacity of producing injection tooling equipped with complex (and therefore well adapted) cooling systems. Tailored cooling systems may help to shorten the mold cycle time. They can also reduce the parts defects, like warpages or residual stresses, which are usually due to highly unfavourable cooling processes [1].

Designing an optimal cooling system is a complicated task. There is no data-processing tool providing a satisfactory response and only a few research works about this subject have been published. In [2], the authors base their analysis on the stationary equation for the average temperature. They evaluate the objective function using boundary elements and they optimize it thanks to a standard gradient algorithm. The model proposed in [1] and [3] is evolutive but 2D. The first transient cycles are simulated by a FE-method until a suitable approximation of the steady cycle is reached. Then, the objective function is evaluated. In [1], the optimization is performed by a genetic algorithm and the Powell's conjugate direction method is applied in [3]. Since they require many evaluations of the objective function, the two methods would be difficult to generalize in 3D.

In this paper, we present an evolutive 3D-model of the mold based on standard assumptions:

- We neglect the part shrinkage as well as the heat diffusion during the cavity filling and the polymer packing stages.
- We assume a simple heat transfer model between the mold and the coolant.

The objective function is evaluated by the FE-method and is optimized by a gradient-like algorithm. The novelty of our approach is to compensate for the numerical complexity of the model, being at the same time three-dimensional and evolutive, by using three mathematical simplifications.

i) The steady cycle is determined immediately without computing the first transient cycles.

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ii) The evaluation of the partial derivatives of the objective function is based on a duality argument and is inexpensive. It does not require any remeshing or any new finite element analysis of the mold.
iii) To avoid meshing problems, parts with thin walls are not considered as volumes but as discontinuity surfaces for the heat fluxes. The jump condition allows for the actual thermal mass of the part.

The optimization algorithm is discussed in section 2. In section 3, we present a comparison between a conventional and an optimized mold for the production of cups. The optimized mold has been designed by mean of our algorithm and built by using Direct Metal Laser Sintering (DMLS). The comparisons will demonstrate a gain in productivity of about 100%.

2. The model and the algorithm

During its first processing cycles, the mold warms up and the produced parts are in principle removed. The mold then reaches a steady cyclic period and its main cooling properties can be deduced from the temperature distribution T_{ej} at ejection. The real-valued function T_{ej} is defined over the mold Ω_m and inside the part Ω_p (see Figure 1). It depends on the cycle time, on the mold material and plastic properties, on the shape of the part, on the location and on the size of cooling channels and on the coolant properties and will be evaluated by the FE-method.

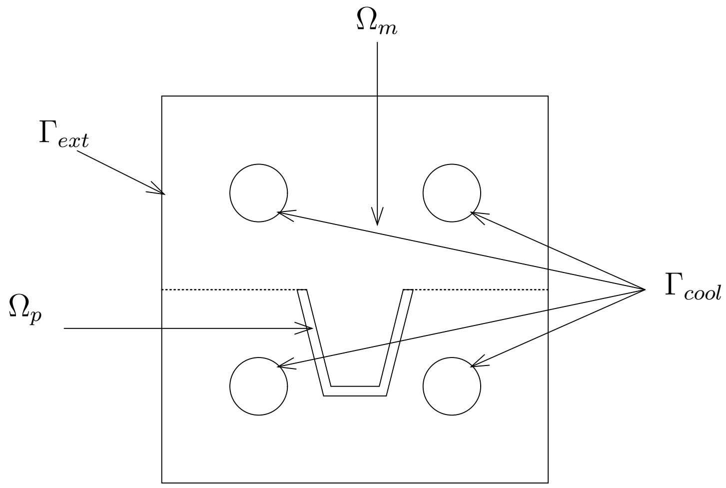

Figure 1. A two-dimensional section of the mold.

The heat exchanges. Following [4], we neglect the natural convection between the mold and the ambient air and we write an adiabatic boundary condition on the exterior surface Γ_{ext} (see Figure 1):

$$-\frac{\partial T}{\partial n} = 0, \quad \text{on } \Gamma_{ext}, \tag{2.1}$$

where n denotes the outward normal vector. According to [3], [1], the heat transfer between the mold and the coolant can be described by mixed boundary conditions along the surface Γ_{cool} of the channels (see Figure 1):

$$-\frac{\partial T}{\partial n} = h_c(T - T_c), \quad \text{on } \Gamma_{cool}. \tag{2.2}$$

In (2.2), h_c is a positive heat transfer coefficient, and T_c is the temperature in the centre of the cooling channels. When the coolant moves sufficiently fast, T_c has a constant value corresponding to the inlet temperature:

$$T_c = T_c^0 \tag{2.3}$$

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The evolution equation. During cooling, the part shrinks inside the cavity and the gap resistance along its interface increases. However, in a first numerical model, this complex process can be neglected ([5] or [1]). We thus assume perfect thermal contact between the mold and the part and the two systems may be considered as an entire computational domain $\Omega = \Omega_m \cup \Omega_p$.

After the cavity filling and the polymer packing stages, the evolution of the temperature distribution in the domain Ω is governed by the boundary conditions (2.1)-(2.2) and by the heat equation:

$$\rho C \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k \frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k \frac{\partial T}{\partial z}\right]. \tag{2.4}$$

The density ρ, the heat capacity C and the thermal conductivity k have constant values in Ω_m and in Ω_p separately. For a typical injection experiment of polyethylene in a SLS-mold produced with the EOSINT M DirectMetal 50-V2 powder and infiltrated with epoxy, the values for ρ, C and k are given in table 1.

	in Ω_m	in Ω_p	unit
ρ	7850.0	923.0	$[kg/m^3]$
C	452.0	2200.0	$[J/kg/K]$
k	46.6	0.3	$[W/m/K]$

Table 1. Typical values of the thermal parameters

The steady cycle. There is a linear relationship between the temperature distribution T_{ini}, which is observed in the beginning of the steady cycle, just after the filling and packing stages, and the temperature distribution T_{ej} reached by the system after the cooling cycle time τ. If T_c^0 is the inlet temperature (see (2.3)), this relationship reads

$$T_{ej} = T_c^0 + A_\tau (T_{ini} - T_c^0) \tag{2.5}$$

where A_τ is the semi-group of evolution operators corresponding to the heat equation (2.4) and the boundary conditions (2.1)-(2.2) [6]. To use equation (2.5) for computing T_{ej}, we still have to determine the temperature distribution T_{ini}. The evaluation of T_{ini} is based on the assumption that the temperature cannot evolve significantly during the filling and packing stages. Therefore T_{ini} is close to the injection temperature T^{inj} of the polymer into the cavity and close to the ejection temperature T^{ej} in Ω_m (because the mold returns to its initial state at the end of each steady cycle):

$$T_{ini} = \begin{cases} T^{inj}, & \text{in } \Omega_p, \\ T_{ej}, & \text{in } \Omega_m. \end{cases} \tag{2.6}$$

The set of equations (2.5)-(2.6) is a closed system for the ejection temperature T_{ej} which can be solved by FEM [7].The temperature T_{ej} at the end of the steady cycle is not the only information we can get from these computations. The temperature distributions T_{steady} at each time steps can also be recovered.

Remark 2.1. *For accuracy reasons, the size of finite elements in the neighbourhood of Ω_p should be comparable to the thickness of the part. To avoid meshing difficulties when the part has thin walls, we change the problem formulation to eliminate the domain Ω_p and to replace it by an average surface Σ_p. In that case, suitable jump conditions for the heat fluxes across Σ_p allow for the thermal inertia of the part* [8].

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The optimization. To illustrate the optimization procedure, we design a cooling system for a mold producing cups (see Figure 2). We assume that the radius of the cooling channel is given ($r = 4mm$) and its axis is the only thing to be determined. We expect that the optimal channel will coil up around the cup and we choose the design parameters as the polar coordinates of the intersections of its axis with a family of horizontal planes. We denote those parameters as $\xi_1 \dots \xi_N$.

In this example, productivity enhancement is the only goal to be optimized, with no regard for part quality. An appropriate cost function to be used is the total heat taken away from the cavity during the cooling cycle,

$$Q = \int_{\Omega_p} \rho C(T^{inj} - T_{ej}(x, y, z))\, dxdydz \tag{2.7}$$

(see (2.4) and (2.6) for the meanings of ρ, C and T^{inj}). For given values of the design parameters, the cost function Q is evaluated by a FE analysis. The design parameters are then updated in the direction given by the gradient of Q:

$$\xi_i^{new} = \xi_i^{old} + \varepsilon \frac{\partial Q}{\partial \xi_i} \tag{2.8}$$

where $\varepsilon > 0$ is a relaxation parameter.

Remark 2.2. *To ensure the mechanical strength of the mold, the relaxation parameter ε in (2.8) has to be chosen small enough. We actually have to prevent the cooling channel to come too close to the part and to make sure that the curvature radius of the axis does not go under a specified limit.*

The last problem to address in this paragraph is the computation of the N partial derivatives $\partial Q/\partial \xi_i$, $i = 1 \dots N$. Standard sensitivity analyses are based on the finite difference formula

$$\frac{\partial Q}{\partial \xi_i} \simeq \frac{Q_i - Q}{\Delta \xi} \tag{2.9}$$

where Q_i denotes a new evaluation of the cost function with the same design parameters as before except ξ_i, which has to be shifted of a small quantity $\Delta\xi \neq 0$. Since the number of design parameters is large (several hundred) and since each evaluation of the cost function is expensive (several hours of CPU-time) the standard sensitivity technique (2.9), cannot be applied to our situation.

We actually had to design a new approach based on the *dual problem*. In [8] it is proved that $\partial Q/\partial \xi_i$ can be obtained by integrating the normal derivative of the temperature distribution T_{steady} along the surface Γ_{cool} of the cooling channels against some appropriate weight w_i:

$$\frac{\partial Q}{\partial \xi_i} = -\int_0^{\tau} dt \int_{\Gamma_{cool}} d\sigma \frac{\partial T_{steady}}{\partial n} w_i. \tag{2.10}$$

The computation of <u>all</u> the weights $w_1 \dots w_N$ requires only <u>one</u> FE-analysis of a problem similar to the problem (2.5)-(2.6) for T_{ej}. Since this problem is also posed in the domain Ω, no remeshing is needed. Observe finally that the integration in (2.10) can be done numerically and that it is unexpensive.

3. The mold realisation

The Figure 2(a) represents a conventional injection mold for the production of cups. We applied our optimization algorithm to design a cooling system allowing a shorter cycle time. The length of the cooling line was imposed as well as the minimal distance between the channels and the part. It was not surprising that the optimization algorithm proposed to position the channels as close as possible to the part. It was more interesting to observe that the cooling spirals were concentrated near the handle. Since we had no possibility to

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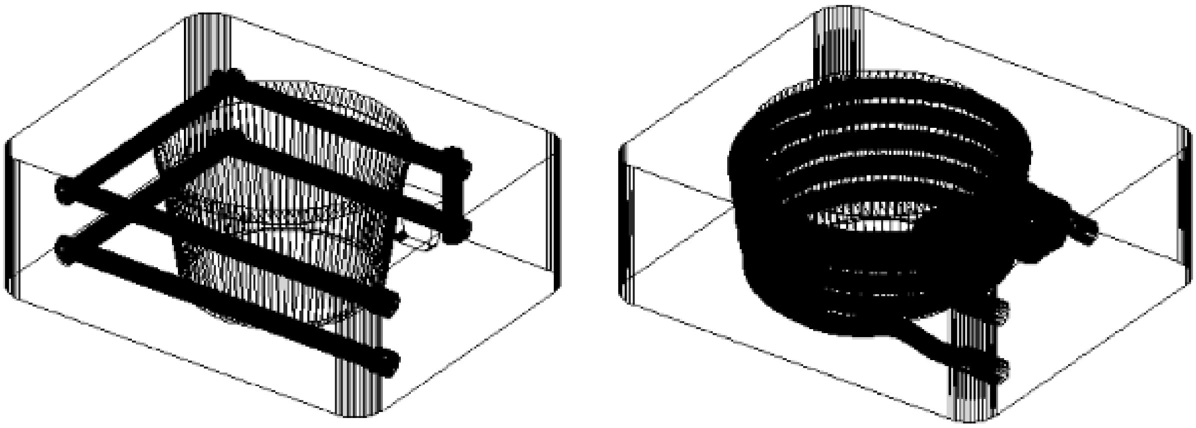
put any cooling channel below the cup (The total height of the mold was imposed), the optimization algorithm also concentrated the first spirals near the bottom of the cup. The optimized cooling channels is represented on Figure 2(b)



(a) Conv. cooling. (b) Opt. cooling.

FIGURE 2. The two dies

The cooling performances of both molds have been analysed thanks to two thermocouples. The first one was located near the injection point and the other one at the extremity of the handle (see Figure 3)

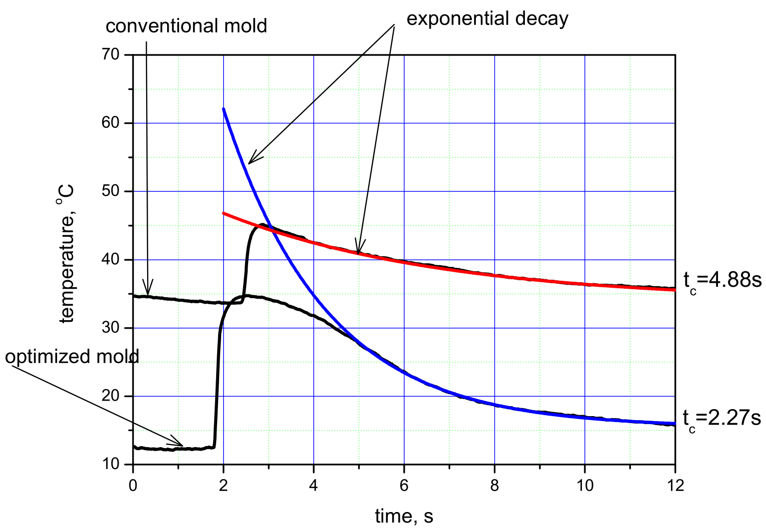


FIGURE 3. The handle thermocouple

The temperature curves first grow (during the thermocouple thermalisation stage) and then decay exponentially for sufficiently large time t (see Figure 3):

$$T_{thermo}(t) \simeq T_{\infty} + \Delta T \times \exp - \frac{t}{t_c} \quad (3.11)$$

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The decay parameter t_c corresponds to the characteristic cooling time of the mold in the region analysed by the thermocouple. Determining its precise value is however a difficult task because of the following reasons:

i) the signal $T_{thermo}(t)$ is in principle noisy,
ii) the slow decaying exponential term $\exp -t/t_c$ is covered by other modes: $\exp -t/t_c^i$, $i = 0, 1, 2 \ldots$, with smaller characteristic times $t_c^i < t_c$.

In [9], we expose an original way of processing the signal $T_{thermo}(t)$ to obtain an accurate approximation for t_c. We use a shift technique to filtrate the constant T_∞ and we integrate the signal to amplify the slow decaying exponential. This *"shift-integration"* process has to be repeated a certain number of time. The optimal number of repetitions can be computed under some assumptions. This procedure has been proved to be robust with respect to the noise. The obtained approximation of t_c are given in table 2.

thermocouples	optimized mold	conventional mold
injection point	$7.93\,s$	$16.97\,s$
handle	$2.27\,s$	$4.88\,s$

Table 2. The decay times

At corresponding locations, the characteristic decay time is systematically twice lower in the optimized mold. The conclusion one can draw from this observation is clear: For comparable performances, one can reduce the cooling cycle time by a factor 2 if the injection die is equipped with optimized channels. It should actually be pointed out that the EOSINT M material infiltrated with epoxy has a much lower thermal conductivity then steel (see table 1). One could therefore expect that the decay times for a conventional mold made out of steel would be shorter than those indicated in the last column of table 2 for the conventional EOSINT M mold.

4. Conclusions

The results of section 3 demonstrate that the proposed optimization methodology helps us to design efficient molds which can be produced by layered manufacturing processes. However, the optimization algortihm still suffers from two major drawbacks.

i) The results of the optimization process strongly depend on the design parameters used to characterize the channels. For the moment, the choice of these parameters is not automated at all. It is proposed by the operator himself. This is a hard task when the part to be injected is complicated and requires additional coding efforts. In particular, the expressions of the weights w_i, used to differentiate the cost function with respect to the design parameters (see (2.10)), have to be programmed manually.
ii) Reducing the relaxation parameter in the gradient algortihm (2.8) is probably not an optimal way to fulfill the geometrical constraints. Alternative solutions should thus be proposed.

These two major issues should be addressed in a future work.

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