Theoretical Modeling and Experimental Characterization of Stress Development in Parts Manufactured Through Large Area Maskless Photopolymerization

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Abstract

This paper aims at investigating the evolution of stresses in parts manufactured through large area maskless photopolymerization (LAMP). A theoretical model was established to understand the curing process for LAMP and a finite element analysis was performed to model the dynamic evolution of stresses during the layer-by-layer fabrication process using Abaqus software. This model serves to suggest strategies for reducing stresses, part warpage, and crack development in parts made through LAMP.

Introduction

Large Area Maskless Photopolymerization (LAMP) is a direct digital manufacturing technology that can build three-dimensional objects layer-by-layer with both high speed and fine feature resolution. A schematic of the LAMP process being developed by the direct digital manufacturing (DDM) laboratory at Georgia Tech is shown in Fig 1. The core part of LAMP system is the spatial light modulator (SLM), which is a digital micro-mirror device (DMD). It is an optical chip with more than 1.3 million mirrors which can be turned on or off according to the color (white or black) of pixels in the corresponding bitmap image. A UV source is used to project light onto the DMD and expose the photocurable ceramic-loaded liquid resin according to the input image. The optical imaging head, with the UV lamp and DMD inside, moves in a serpentine path and raster scans the building area. Using this exposure mechanism, LAMP can realize massively parallel scanning lithography and its patterning speed is much higher than the traditional stereolithography in which only one single beam is used for scanning.

In industry, extremely complex interior cooling passages of turbine airfoils are usually produced by investment casting, which typically involves the creation of over thousand tools needed for fabricating the cores, patterns, mold, and setters. The DDM group in Georgia Tech is using the LAMP process to produce the integral ceramic cored molds directly. In this way, the production rate, casting yield and costs can be improved dramatically when compared to the conventional investment casting procedures. Therefore, direct digital manufacturing using LAMP is a kind of technology that disrupts the current state-of-the-art investment casting process, not only in the manufacturing of airfoils but also in many other applications involved complex components.

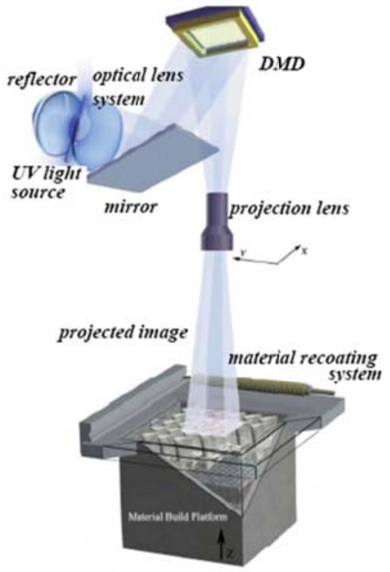


Figure 1. Schematic illustration of Georgia Tech's LAMP machine

This paper mainly focuses on studying the evolution of internal stresses in a part being built through the LAMP process. Residual stress can have a significant effect on the quality of the parts by inducing defects such as distortion, cracks and delamination. The residual stress in LAMP parts results from the different degrees of shrinkage in sequentially exposed layers. The contraction of each currently exposed layer will tend to produce flexure of the layers solidified previously. Thus, the development of residual stress is strongly influenced by processing history. The objective of this paper is to investigate the mechanisms of stress evolution so as to infer strategies for alleviating residual stresses and improving part quality.

Theoretical model

1. Cure Dependent Modulus

This model describes the relation between the conversion degree (α) of resin and its mechanical properties. For the ceramic particle loaded resin composite used in LAMP, the mechanical properties are isotropic and the Poisson's ratio (ν) is assumed constant. The resin modulus is strongly cure dependent, influenced by the kinetic-viscoelastic interactions successfully modeled by Dillman and Seferis [1]. However, this rigorous model requires the evaluation of several kinetic and viscoelastic parameters which need extensive experimental data. Therefore, a more convenient α-mixing rule model [2, 3] is used here to get the cure dependent modulus. The instantaneous isotropic resin modulus, denoted E_0, is expressed explicitly as a function of conversion degree α:

$$E_0 = (1-\alpha)E_{\alpha 0} + \alpha E_{\alpha 1} + \gamma\alpha(1-\alpha)(E_{\alpha 1} - E_{\alpha 0}) \tag{1}$$

Where $E_{\alpha 0}$ and $E_{\alpha 1}$ are the Young's moduli of uncured (α=0) and fully cured (α=1) resin, respectively. The parameter γ (-1<γ<1) is introduced to quantify the competing mechanisms between stress relaxation and chemical hardening [1]. Increasing γ will make the modulus increase more rapidly at a relatively low conversion degree. In this paper, a zero value of γ is used for the simulation.

The material used in LAMP is silica particles loaded resin composite. The effective bulk modulus of this composite $\bar{K}$ can be derived according to the self-consistent method [4], shown in equation (2).

$$\bar{K} = K_0 + \frac{c_1(K_1 - K_0)(3\bar{K} + 4\bar{\mu})}{3K_1 + 4\bar{\mu}} \tag{2}$$

Where K_0 and K_1 are the bulk moduli of matrix (resin) and inhomogeneity (particle) respectively. $\bar{\mu}$ is the effective shear modulus of composite and c_1 is the concentration of particles loading.

The particles can be considered rigid compared to resin, thus we obtain:

$$\bar{K} = \frac{3K_0 + 4c_1\bar{\mu}}{3(1 - c_1)} \tag{3}$$

$\bar{\mu}$ can be expressed as a function of $\bar{K}$ and the effective Poisson's ratio ν, which is considered as constant here.

$$\bar{\mu} = \frac{3\bar{K}(1 - 2\nu)}{2(1 + \nu)} \tag{4}$$

Substitute into equation (3), we obtain:

$$\bar{K} = \frac{1 + \nu}{1 + \nu - 3c_1 + 3\nu c_1} K_0 \tag{5}$$

The relation between bulk modulus and Young's modulus can be expressed as:

$$\bar{K} = \frac{\bar{E}}{3(1 - 2\nu)}, K_0 = \frac{E_0}{3(1 - 2\nu)} \tag{6}$$

Make use of equations (5) and (6), we obtain the effective Young's modulus of composite as a function of resin's modulus E_0, effective Poisson's ratio ν and particles concentration c_1.

$$\bar{E} = \frac{1 + \nu}{1 + \nu - 3c_1 + 3\nu c_1} E_0 \tag{7}$$

It can be seen that the effective modulus is proportional to resin's modulus. So, the effective modulus of the composite is also related to the conversion degree α of resin.

$$\bar{E} = (1 - \alpha)\bar{E}_{\alpha 0} + \alpha \bar{E}_{\alpha 1} + \gamma\alpha(1 - \alpha)(\bar{E}_{\alpha 1} - \bar{E}_{\alpha 0}) \tag{8}$$

Equation (8) is similar with (1), but here $\bar{E}, \bar{E}_{\propto 0}, \bar{E}_{\propto 1}$ are moduli of the composite, not resin.

The experimental data obtained from tensile testing according to ASTM D638 is shown in Table 1.

Properties	Value
$\bar{E}_{\alpha 0}$ (MPa)	0.829
$\bar{E}_{\alpha 1}$ (MPa)	829
ν	0.437

Table 1. Mechanical properties of acrylic-based composite

Here the modulus of uncured composite is chosen arbitrarily small due to the negligible stiffness.

2. Cure Dependent Shrinkage

This model proposes a theoretical relationship between the shrinkage strain and degree of conversion. The volumetric shrinkage occurring during LAMP process is related to the photopolymerization mechanism. The linkage of small monomer units produces large polymer chains and the corresponding intermolecular spacing is reduced from Van der Waals distance (~10^4Å) to covalent bond (C=C) lengths (~ 1 Å). This results in density changes and thus bulk contraction in the cured resin, which accumulates as the part is fabricated layer-by-layer. Thus, the volumetric shrinkage is a direct measure of the number of covalent bonds formed (degree of conversion) and an exact semi empirical relationship can be derived. Experimentally, the volume change per mole of acrylate groups (C=C) is 22.5cm^3/mol [5] when acrylate monomer is polymerized. For a general case of multiacrylates with ceramic filler particles, the volumetric shrinkage value can be estimated through the following equation [6]:

$$\frac{\Delta V}{V}(\%) = 22.5 \times \alpha \frac{\Sigma_i (f_i \chi_i)}{\Sigma_i (M_{mi} \chi_i)} \rho_{mix} \times (1 - \frac{FL}{100}) \times 100 \tag{9}$$

Where f_i is the functionality of monomer (i), χ_i is mole fraction of monomer (i), M_{mi} is molecular weight of monomer (i) and ρ_{mix} is the density of the mixture.

For the material system used in LAMP, the values of above parameters are shown in Table 2.

Components	f_i	M_{mi}	ρ(g/cm^3)	v/o (%)	w/o (%)	χ_i	ρ_{mix}(g/cm^3)
Hexanediol diacrylate, HDDA	2	226	1.02	3.4	18.6	.95	1.68
Ethoxylated Penta erythryitol tetra acrylate, EPETA	4	528	1.12	31	2.3	.05	
Filler (ceramic powder)			2.2	FL=55	72		

Table 2. Data of the material system

Assuming a uniform strain contraction for all principle strain components and considering the volumetric strain to be smaller than 10 percent, the value of linear shrinkage strain is approximately one third of the volumetric shrinkage value. Thus the linear shrinkage strain ε_T can be expressed as:

$$\varepsilon_T = \frac{1}{3} \times 22.5 \times \alpha \frac{\Sigma_i (f_i \chi_i)}{\Sigma_i (M_{mi} \chi_i)} \rho_{mix} \times (1 - \frac{FL}{100}) \times 100 \tag{10}$$

3. Print-through Mediated Incremental Curing

Due to the light penetration through upper layers, almost all of the layers, except for the topmost ones, are incrementally cured as subsequent layers above them are being exposed. For each layer, this print-through phenomenon will keep occurring for several times. Fig 2. [6] shows the increasing process of conversion degree versus the layers exposed above it in LAMP process. The experiments were conducted with different photoinitiators concentrations and exposure time. The bottom layer of samples with different layers above it was analyzed through FTIR.

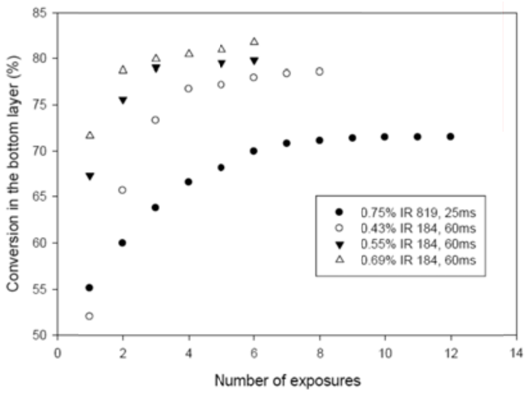


Figure 2. Conversion in the bottom layer vs. number of exposures above the bottom layer

Print-through results in good layer-to-layer bonding. However, additional curing due to light penetration at the bottom layer from several layers above results in accumulation of internal stresses and warping of the fabricated part. It can be seen from the figure above that with 0.43% IR 184, the conversion degree will continuously increase for about 4 print-through exposures, while with 0.75% IR 819, the conversion degree will increase up to 7 exposures. This paper will show how these two different print-through curves would affect the accumulation of internal stress.

4. Depth Profiling of A Single Layer

Print-through discussed above gives the conversion degree of each layer during one exposure, which actually is the average value in one layer. However, due to Lambert-Beer's law of absorption, conversion degree more or less is a function of depth through a single layer. In this study, sample was sliced every 25μm using microtome and the slices were characterized through Fourier Transforms Infrared (FTIR). Results were shown in Fig 3.

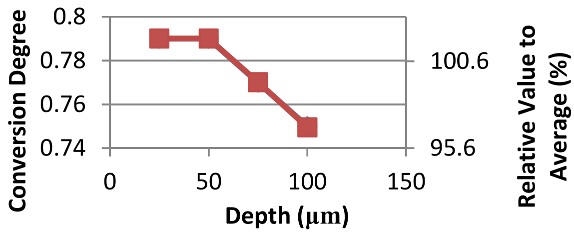


Fig 3. Depth profile of curing conversion in one layer

This figure shows that the conversion degree doesn't change much for our material system. In the simulation, each layer has four elements in the vertical direction and these four elements are assigned 101.9%, 101.9%, 99.4%, 96.8% of the average conversion degree respectively.

Simulation Method

The finite element analysis was conducted using ABQUS. User subroutines UFIELD and USDFLD were used to define the conversion degree field, so as to make the modulus dependent on this field, and realize the print-through process and depth profiling. Another user subroutine UEXPAN was used to make the shrinkage strain as a function of conversion degree. Model change command in ABQUS was used to remove and reactivate the elements of each layer, so as to simulate the layer-by-layer manufacturing process. Element type C3D20 (20-node quadratic brick) was mainly used because of its good precision in simulating stress concentration scenario. However, a few of C3D8 (8-node linear brick) and C3D8R (8-node linear brick, reduced integration) elements are also used in some special positions to avoid serious distortion of element. The meshing seed size along the part building direction is one fourth of layer thickness and the general aspect ratio of element is about three.

Results and Discussion

DDM group in Georgia Tech is using LAMP process to develop a new way for fabricating the airfoils mold. The trailing edge, where many tiny holes exist for cooling purpose, is always the part easy to be cracked or delaminated (Fig 4.). On the part built through LAMP process, the area with dense holes is very likely to be highly stress concentrated. Thereby structures with holes are mainly analyzed in this paper.

Fig 4. a. CAD model of airfoils mold b. ailfoils mold built by LAMP c. cracked trailing edge

1. **Comparison of different print-through effect**
   In order to compare the effect of different print-through curves (0.43% IR184, 60ms and 0.75% IR819, 25ms in Fig 2.) on the evolution of residual stress during the LAMP process. A

simplified unit model was proposed for simulation, as shown in Fig 5. It is a square board with a hole in the center and two side surfaces of the board are fixed to surroundings.

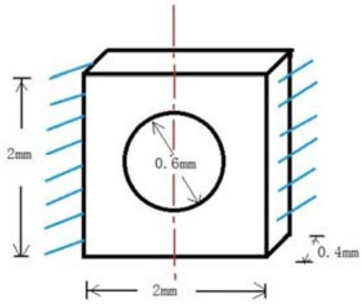


Fig 5. Center-hole unit model

Due to the symmetric geometry, half of the model was analyzed. Two print-through curves were used for simulation and the corresponding results are shown in Fig. 6 ~ 8.

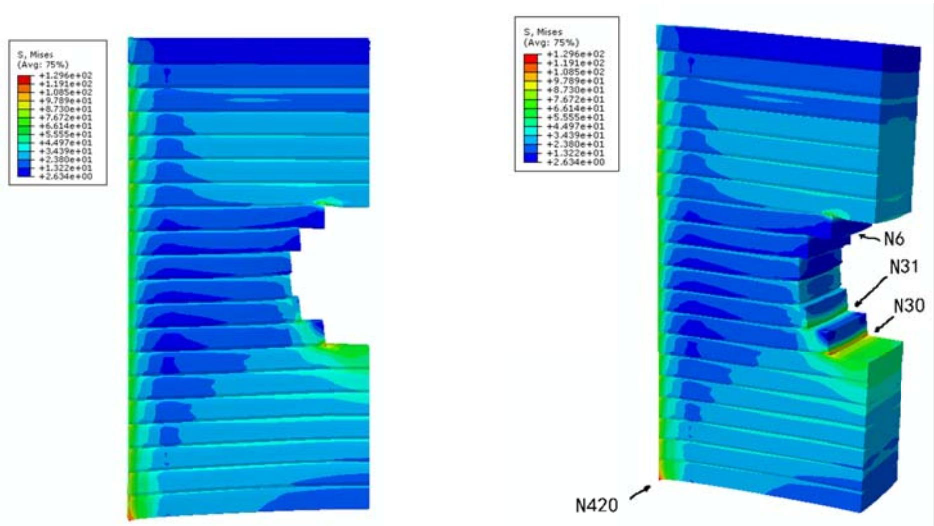


Fig 6. Residual Mises stress after fabrication, LAMP process modeling with print-through of 4 exposures (0.43% IR184, 60ms)

It can be seen from the figure above that corner is easy to become stress concentration area in LAMP built part. In particular, the most bottom corner (corresponding node number N30) around the hole and the bottom corner (N420) where boundary conditions applied possess the largest stresses (130MPa). These positions are where crack and delamination tend to initiate. The initiated cracks will always propagate along the interface between two layers because the interlaminar stresses are relatively large while the mechanical strength is weak.

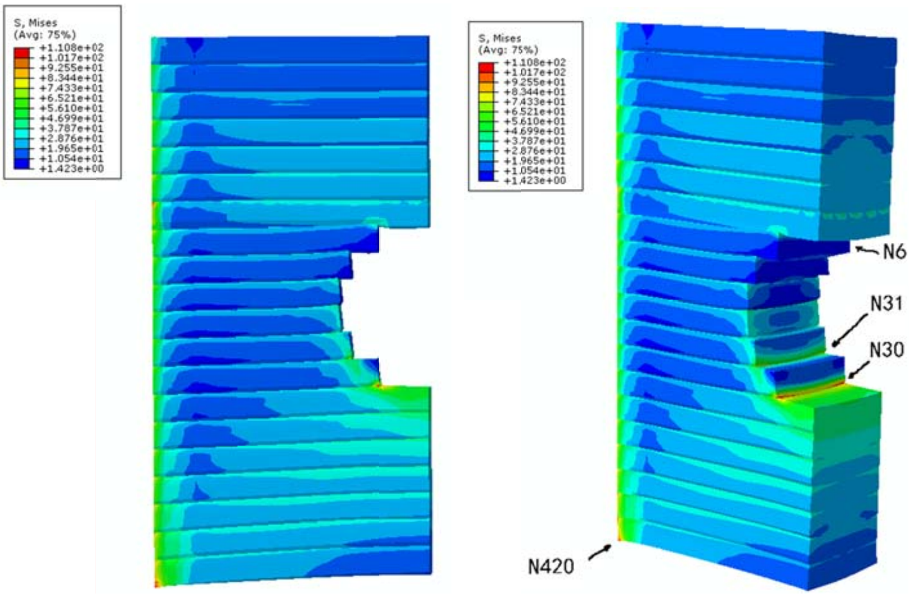


Fig 7. Residual Mises stress after fabrication, LAMP process modeling with print-through of 7 exposures (0.75% IR819, 25ms)

In the other case where print-through lasts for 7 exposures, Fig 7. presents similar residual stress distribution as Fig 6. However, the largest value of von Mises stress is smaller (111MPa).

For the four stress concentration positions marked in Fig 6. and 7, the evolutions of the von Mises stress versus number of layers exposed are plotted in Fig 8.

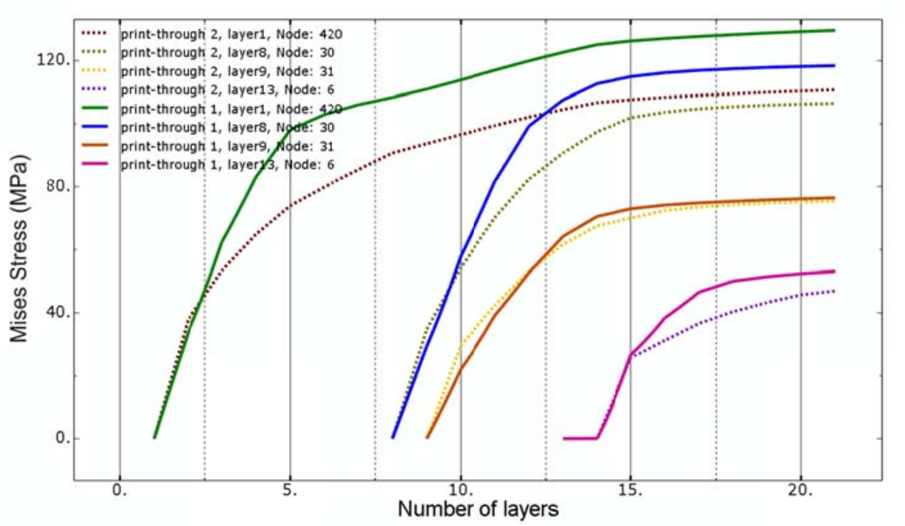


Fig 8. Evolution of the nodal value of von Mises stress at 4 stress concentration positions for two different print-through processes (print-through1: 4 exposures, print-through2: 7 exposures)

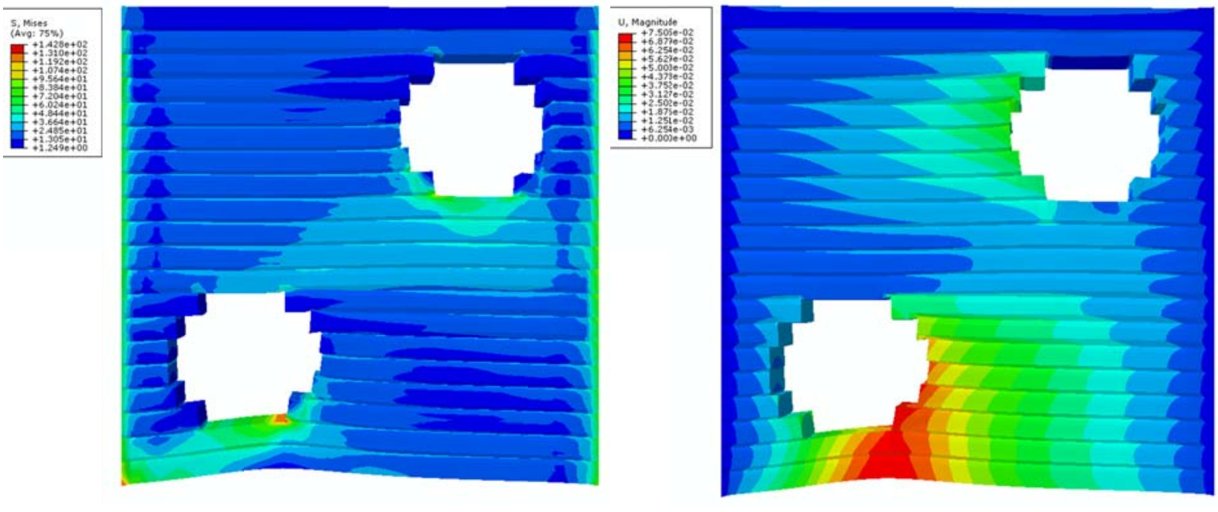
Although the print–through 1 phenomenon is that each layer can continuously cure and contract for up to 4 exposures, it can be seen that in both print-through cases, the residual stresses increase for about 7 exposures and then approximate a constant value. Thus, for each layer the cumulative internal stress will not change after 6 layers are built above. Fig 8. also shows that print-through 1 with IR184 causes the internal stress to increase faster than the case of print-through 2 with IR819. This is due to the fact that for print-through 1 the two neighboring layers have larger difference of conversion degree and thus can accumulate more stress. The lower non-uniformity in the conversion as a result of UV light penetration in the case of IR 819 is also an indicative of its deeper curing capability and lower absorption coefficient.

2. Effect of the structure configuration

LAMP is a layer-by-layer manufacturing process in which the internal stresses of a part evolve continuously. Since the building direction of part is vertical to each layer and constrained shrinkage occurs parallel to the layer surface, the evolution of stresses should also in some sense strongly depend on orientations and directions, such as the relative position among holes and constrained boundaries.

To study the configuration effect for LAMP built part, simulations were still conducted based on a square board of which two side surfaces are fixed, just like the trailing edge inside the airfoil mold. Now we need to add two holes on this board, the configuration of these two holes should be carefully designed to minimize the internal stress produced during the layer-by-layer curing process. The diameter of each hole is 0.6mm and the square board size is 2mm×2mm×0.4mm.

The first configuration is to set these two holes on the diagonal of the square. The modeling results are shown in Fig 9.

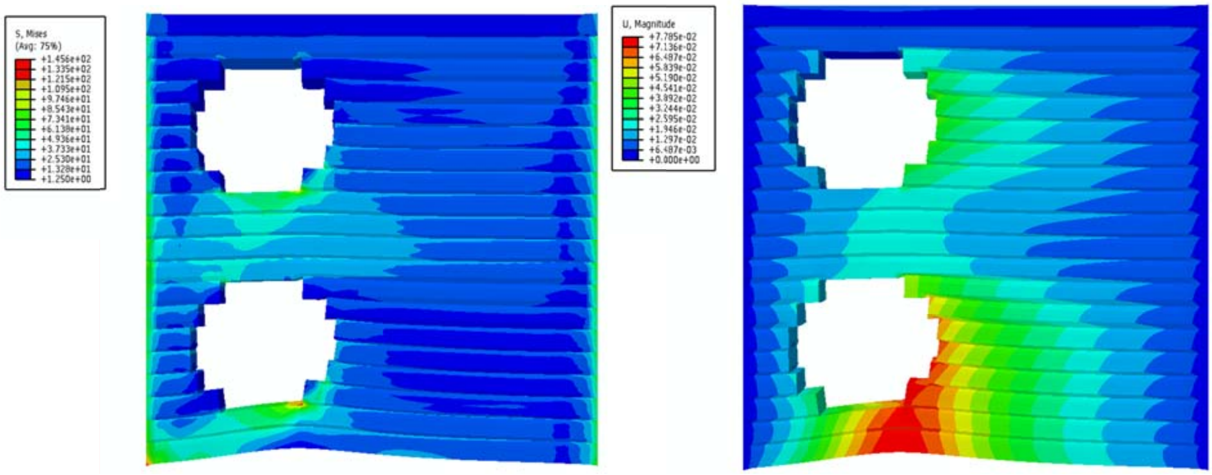


a. Mises Stress b. Displacement Magnitude

Fig 9. 3-D Modeling result of square board with diagonal holes

It can be seen that both the stress and distortion around the bottom-left hole are large while values of top-right hole are pretty small. For the bottom-left hole, the most stress concentrated and distorted positions are the bottom-right corner, where close to the bottom bound of the square and the right solid area. For the top hole, the stress concentrated position is the bottom-left corner and the distorted area is the left border of the hole.

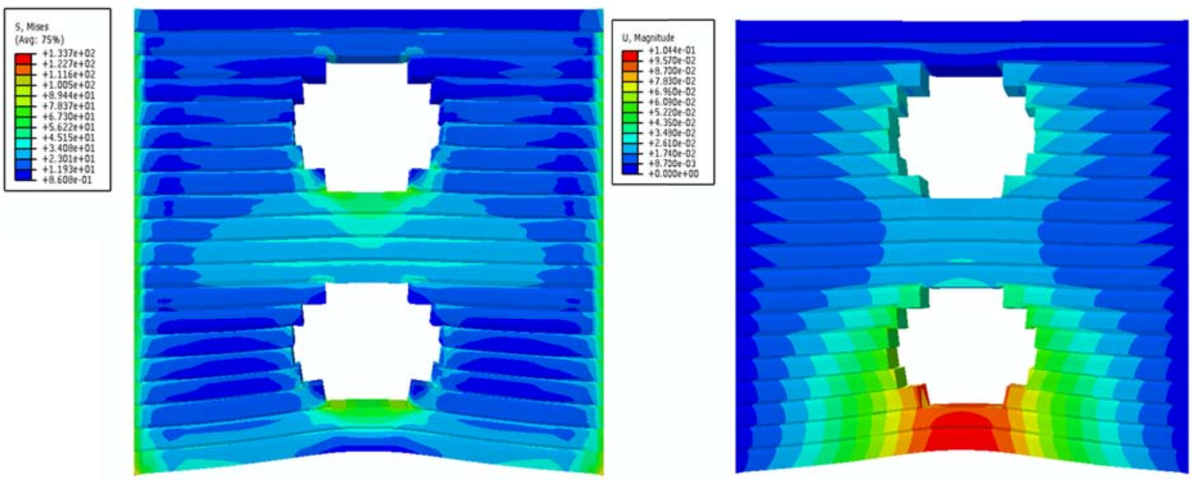
In order to see the effect of the relative position of these two holes, we fix the position of the bottom-left hole and move the top hole to be just vertical aligned to the bottom one. The modeling results are shown in Fig 10.



a. Mises Stress b. Displacement Magnitude

Fig 10. 3-D Modeling result of square board with vertical alligned holes at the left

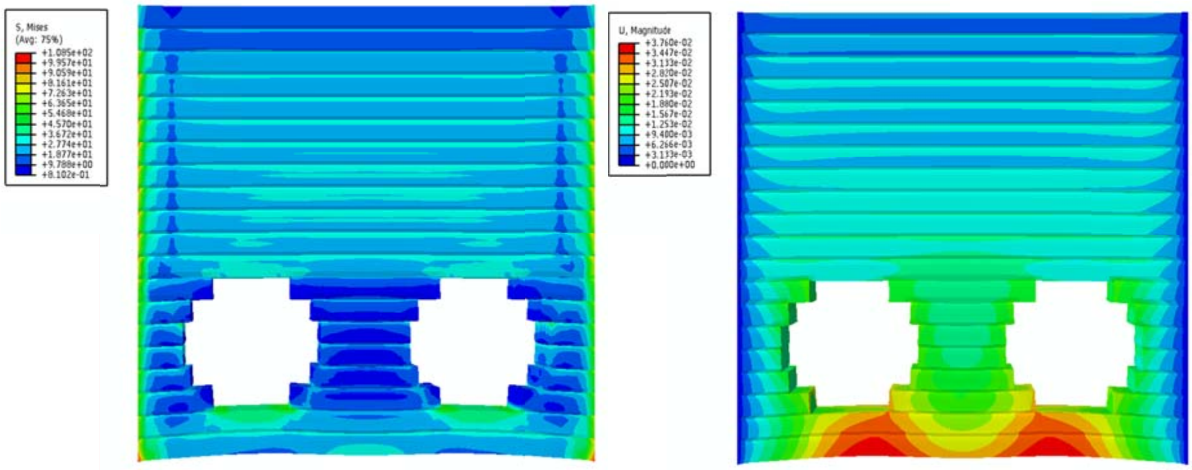
It is found that both the values of the largest stress and the displacement are a little bigger than those in the diagonal aligned case. The stress concentrated and distorted positions of the bottom hole are not changed while the positions of top hole move from its left to right. The figure also shows that stress concentration area of the top hole is very close even get touch with the bottom hole. So, in the vertical direction, holes should not be too close to each other and holes are desired not to be aligned vertically. Also, the sizes of solid areas around the hole should be kept similar at different directions.



a. Mises Stress b. Displacement Magnitude

Fig 11. 3-D Modeling result of square board with vertical alligned holes at the center

Fig 11. shows the modeling result of the case where vertical aligned holes are moved to the center of the square. The stresses are smaller than case 2, but the distortion at the bottom becomes even worse.



a. Mises Stress b. Displacement Magnitude

Fig 12. 3-D Modeling result of square board with horizontal alligned holes

Fig 12 shows the modeling result of the case where two holes are aligned horizontally. Although the positions of stress concentration and distortion are not changed, the values become much less than the front cases. The comparison results for the evolutions of stress and displacement at critical positions (bottom corner of hole for stress and largest value position for displacement) are plot in Fig 13. and 14.

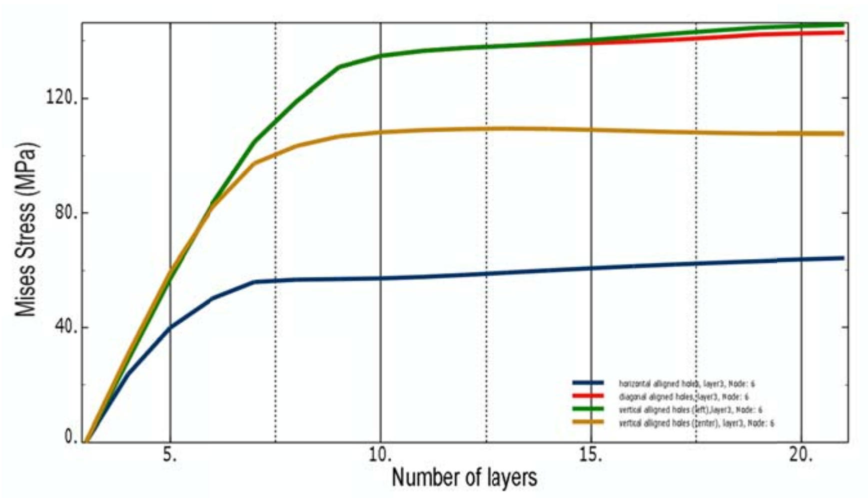


Fig 13. Comparison for the evolutions of stress at bottom corner (layer3) in four cases

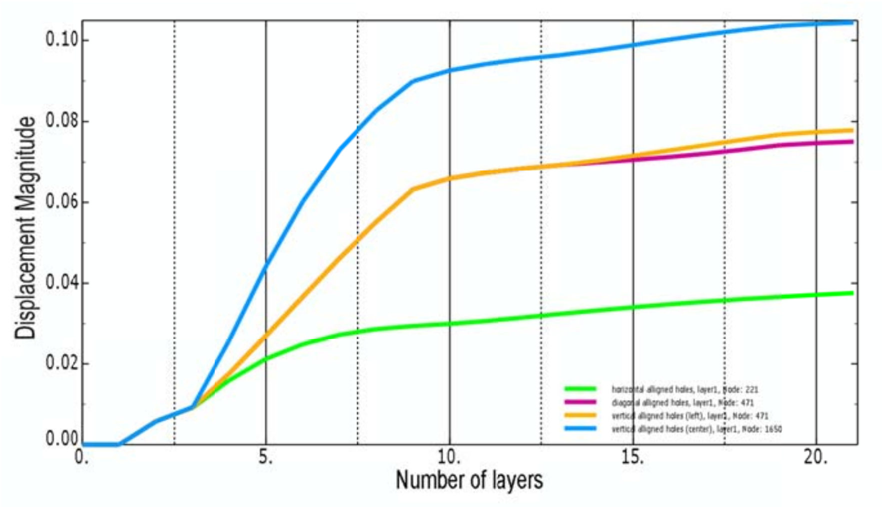


Fig 14. Comparison for the evolutions of largest displacement (layer1) in four cases

Obviously, for both stress and distortion, the configuration where holes are aligned horizontally has much smaller value than other cases. For stress, cases in which holes are aligned diagonally and vertically (left) are the worst. For distortion, the case with vertically aligned holes (center) is the worst. So we can conclude that for decreasing the stress concentration and distortion, horizontal alignment of holes is better when the spacing is small and the areas around the hole should be as uniform as possible. The position of hole should be far away from the free boundaries, especially the horizontal ones, to minimize distortion. Also, values of stress and distortion increase rapidly in 7 layers and become stable after about 10 layers. So it is better to keep the holes away from each other for at least 7 layers vertically.

Summary and Conclusions

This paper presents theoretical concepts and a mechanical model for the layer-by-layer LAMP process. Simulations are conducted to study the effect of different print-through phenomena. For both 4 layers and 7 layers deep curing, the stress continues to increase for 7 layers and then becomes constant. The most serious stress concentration occurs at the bottom corner of hole and possibly causes the initiation of crack and delamination. The effects of holes configuration on stress and distortion are studied through simulation of four cases. It is found that the case in which holes are aligned horizontally has the smallest value of stress and distortion. And the vertical alignment of holes should be avoided when spacing is small. To minimize the distortion, distribution of holes should be as uniform as possible so as to keep the cure of solid area around the hole in a balanced state in some sense. Also, the hollow structure is better to be far away from the free boundaries to avoid curl distortion. This work is a fundamental study for understanding the LAMP process and for providing strategies to reduce defects such as cracks, delamination and dimensional discrepancy caused by stress concentrations and distortion. The trailing edge in the airfoil mold built by LAMP is one of the parts with most serious defects. Future work will be conducted to model the stress evolution in the trailing edge.

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