

# INTEGRATION OF TOPOLOGY OPTIMIZATION WITH EFFICIENT DESIGN OF ADDITIVE MANUFACTURED CELLULAR STRUCTURES

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**Abstract:** Cellular structures are promising candidates for additive manufacturing to design lightweight and complex parts to reduce material cost and enhance sustainability. In the paper, we focus on the integration of the topology optimization with the additive manufactured cellular structures. In order to take advantage of these two technologies for lightweight manufacturing, a totally new design and CAD method is developed to build up the bridge between the optimal density distribution and the cellular structure. First, a systematic theoretical and experimental framework is provided to obtain the mechanical properties of cellular structures with variable density profile. Second, a revised topology optimization algorithm is introduced to optimize arbitrary 3D models with given boundary conditions. In this process, the minimum compliance problem and allowable stress problem are considered to get the relative density distribution. Third, CAD methods are developed to obtain the function between the local relative density and the variable density of cellular structure. With the aid of the function, one can convert the density distribution to the cellular vertex radius distribution and build variable density cellular structures in the given parts. Finally, a real part named pillow bracket is designed by this process to illustrate the efficiency and reliability of the new method.

**Keywords:** Additive Manufacturing, Topology Optimization, Cellular Structure, Reconstruction

## **1. Introduction**

Additive manufacturing (AM) makes it possible to manufacture parts and devices with complex geometry and multifunctional performance <sup>[1]</sup>. However, the design for manufacturing (DFM) methodology such as design rules and guidelines for traditional subtractive manufacturing cannot be utilized for AM. Hence new CAD and DFM algorithms are needed in order to take advantage of AM technologies <sup>[2]</sup>. Optimal design with complex surface topology and internal structure through new DFM algorithm becomes a good choice for AM technologies.

Modern topology optimization, a powerful approach to design lightweight parts, was first popularized by Bendsøe and Kikuchi <sup>[3]</sup>. It has been applied to many different fields, such as bone remodeling, automotive industry, aerospace <sup>[4-9]</sup>, et al. More often than not, the topology optimized parts are too complex to be fabricated. However, AM provides a great opportunity to manufacture the designed parts resulted from topology optimization. However, challenging problems still exist when implementing topology optimization for AM technologies. First, topology optimization is a computational process of generating new boundaries and change the original shape of the component. Second, the result of topology optimization does not represent the specific microstructure the penalization that the algorithm employs. Third, the topology optimization design includes some intermediate densities even after penalization, which is

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difficult to fabricate even with AM. To address these issues, continuous (vs discrete 0/1) variable density cellular structures will be employed in the cellular structure optimization algorithm proposed in this paper. Cellular solid structure is promising for engineering structure design due to their lower material and energy consumption<sup>[11]</sup>. Natural cellular solids, e.g, wood and bone,<sup>[12, 13]</sup> have inspired the design of engineering cellular structures such as lightweight structures, wave absorption materials and the cores of sandwich panels<sup>[14-21]</sup>. However, typical applications of cellular structures in industry are uniform density rather than variable density like in natural materials.

In this paper, a novel algorithm is proposed to integrate topology optimization with efficient design of micro cellular solid structure for AM technologies. First, the mechanical properties of cubic cellular structure are obtained using homogenization techniques. Secondly, the mechanical properties of cubic cellular structure are used to perform minimum compliance topology optimization for pillow bracket and obtain the optimal density distribution. Finally, the reconstruction algorithm converts the optimal density distribution into variable density cellular structure and yields the given component. The utilization of the variable density cellular structure makes it possible to deal with intermediate density of topology optimization. Experiment on the pillow bracket filled with cubic cellular structure is conducted to test the efficiency of the algorithm. The experiment agrees well with the optimization result.

## 2. Design Methodology

The purpose of the algorithm is to reduce the weight of the solid component while satisfying certain mechanical performance and shape requirements. Four key techniques are applied in the design process: homogenization, optimization, reconstruction and validation (HORV). Figure 1 shows the flow chart of the design methodology and the detailed procedure of the algorithm are described below:

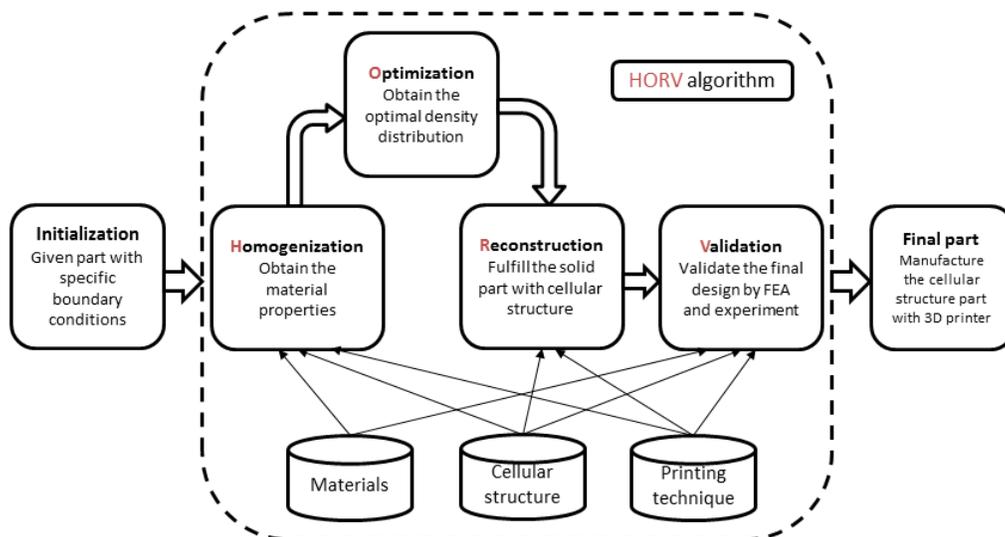


Fig. 1 The flow chart of the HORV algorithm

(i) Initialization: The design component with given boundary conditions are specified in preparation for the cellular structure design, optimization and reconstruction processes.

## (ii) Cellular structure Homogenization, Optimization and Reconstruction (HORV)

In the HORV section, four mini steps are implemented to re-design the specific component with variable density cellular structures

(a) Homogenization. The material type, cellular type, and AM techniques are determined as input parameters for the material model function to the mechanical properties of the cellular structures with specific material type and orientations

(b) Topology Optimization. Once the homogenized material model is obtained from the sub-step (a), the topology optimization of cellular structures can be cast into the revised topology optimization algorithm of the equivalent continuum solid model to do optimization.

(c) Reconstruction. For a given component, the cellular structure reconstruction algorithm automatically generates the variable density mesh according to the density distribution obtained from the topology optimization.

(d) Validation and Assessment. The validation of the re-designed component includes both the finite element analysis (FEA) and experiments. Necessary adjustments and modification should be implemented until the design is finalized.

(iii) Component finalization and fabrication. The component reconstructed with specific cellular structure need to add fillet to fix the corners. The designed cellular structure component should also consider the processing conditions and limitations of the specific AM technique, which has its own processing difficulties such as large overhangs<sup>[22]</sup> and small geometric features.

### **3. Homogenization of Cellular structures**

In this paper, the cubic cellular structure (shown in Fig. 2) is used as the cellular unit for the HORV algorithm to construct the pillow bracket part. According to the Gibson-Ashby model<sup>[11]</sup>, the elastic modulus  $E$ , shear modulus  $G$ , and yield strength  $\sigma_{pl}$  of cellular structures are the functions of relative density,  $\rho = \rho_c / \rho^*$  (i.e. the density of the cellular solid  $\rho_c$  divided by that of the original solid  $\rho^*$ ). The anisotropic constitutive law  $\sigma = C\varepsilon$  is used, where  $\sigma$ ,  $\varepsilon$ , and  $C$  denote stress, strain, and elastic matrix, respectively.

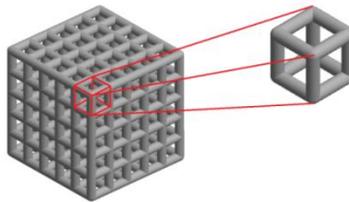


Fig. 2 Assembly of cubic structure and its unit cubic cell

In the case of relative density,  $\rho(x)$ , the elastic matrix of cellular structures can be written as a polynomial of the density:

$$C(\rho) = C_0 + C_1\rho + C_2\rho^2 + \dots \quad (1)$$

where  $C_i$  ( $i = 0, 1, 2, \dots$ ) are the constant symmetric matrices to be determined by experiment or computation. In our case, the elastic scaling laws for the cubic cellular structure are fitted from the finite element analysis (FEA) simulation results as a function of relative density. Take the Poisson's ratio of the material to be 0.3, the three independent material constants of the cubic cellular structure have been obtained and shown in Fig. 3.

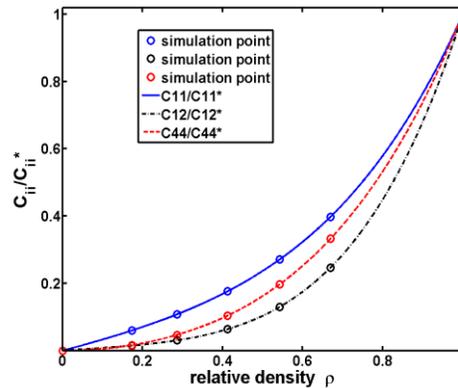


Fig. 3 Elastic scaling laws of cubic cellular structure

#### **4. Topology optimization**

The proportional topology optimization (PTO) [23] is used as the optimization algorithm to solve the minimum compliance problem in this paper. PTO algorithm is a non-gradient method, which is easy to implement. In this work, the conventional minimum compliance topology optimization problem is used to optimize the pillow bracket component. In the optimization, the relative density range is 0.2-0.95.

#### **Numerical model and optimization result**

In this work, the pillow bracket, a conventional component used in industry, is selected as the object to test and validate the HORV algorithm. As illustrated in Fig. 4, the total dimension of pillow bracket is 130mm x 52mm x 52mm. There are four holes in the bottom plate and two holes in the ears. The six holes are the same size, each of diameter 13 mm.



Fig. 4 Illustration for the dimensions and boundary conditions of a pillow bracket

The total number of elements for the pillow bracket is 45,070 and the element edge length is 1.2 mm. ANSYS software package (ANSYS® Academic Research, Release 15.0) is used as the tool to conduct FEA. The material properties obtained from the scaling laws are inserted into the FEA by APDL program. Figure 5 shows the result of pillow bracket with volume fraction equal to 0.5. The high compliance values mainly distribute along the two ears areas. And the compliance concentration is in the corner between the ear and the bottom plate and the four bottom holes. Figure 5b shows the optimal density distribution of pillow bracket, which is nearly the same as the compliance distribution.

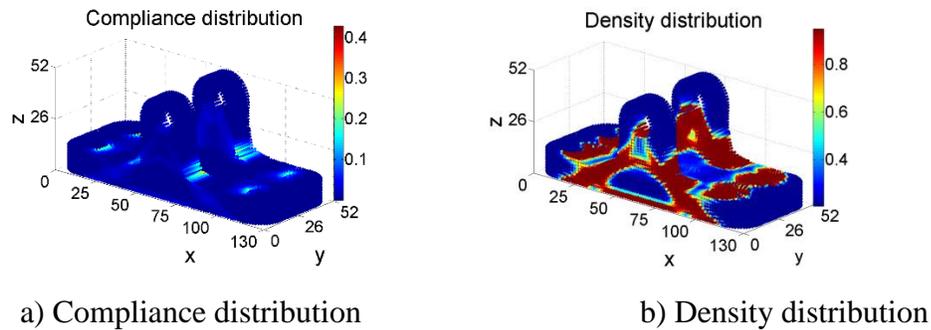


Fig. 5 Topology optimization results of a pillow bracket part with volume fraction as 0.5

## 5. Reconstruction

In this section, the way to reconstruct the given component with specific cellular structure is discussed. Take the cubic cellular structure as an example, the cubic structure shown in Figure 2 is consisted of eight ligaments. Each ligament can be divided into two balls and one cylinder. By setting different radius for the vertex of the repeating unit, we can obtain the function of the volume fraction with respect to the radius. And the radius distribution is used to reconstruct the given component. Figure 6 shows the reconstruction result of the pillow bracket with variable density cubic cellular structure.

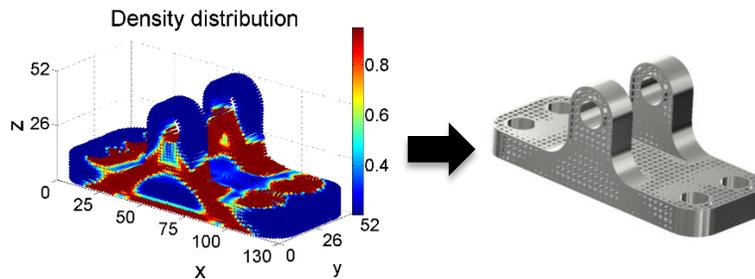


Fig. 6 The reconstruction process for pillow bracket with cubic cellular structure

## 6. Experiment

Model validation in the HORV method is carried out by experimental testing. Figure 7 shows the experimental setup for 3-point bending test. The experiment is conducted on the MTS 880 material testing system.



Fig. 7 Experimental setup of the 3-point bending test

The samples used for 3-point bending testing were printed by the Stratasys Object260 Connex 3D printer. Figure 8 shows the experimental results of the uniform and variable-density cellular structured pillow brackets printed. The strength of the variable density component increases 75.4% on average compared with the uniform component. And the average flexural stiffness increases 133.5% on average. The energy absorption increases 33.1%. It is obvious that the HORV algorithm is a powerful tool to design lightweight components while enhancing their stiffness, strength, and energy absorption.

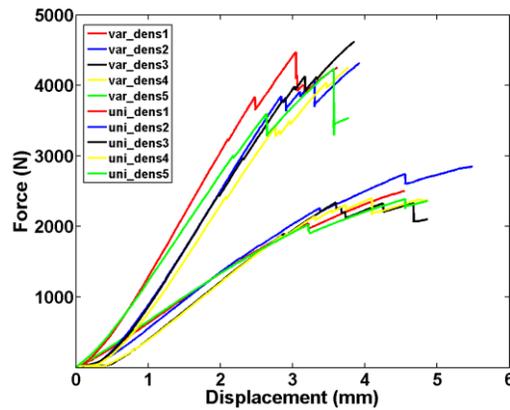


Fig. 8 Testing curves of cubic cellular structure pillow brackets

Table 1 Experimental comparison of the uniform component and optimized component

| Item                          | Uniform component | Optimized component | Increment (%) |
|-------------------------------|-------------------|---------------------|---------------|
| Average Stiffness (N/mm)      | 675.7             | 1578                | 133.5         |
| Average Strength (N)          | 2497              | 4379                | 75.4          |
| Average Energy absorption (J) | 6.65              | 8.85                | 33.1          |

## 7. Conclusion

An algorithm that can enhance the stiffness of designed component while reducing the mass of solid structure has been proposed by integrating topology optimization and cellular solid structures. Due to the utilization of real material properties, the topology optimization procedure is more reasonable and reliable. In fact, there are many intermediate densities even when severe penalization is applied to topology optimization. The cellular structure reconstruction not only

employs the specific cellular structure to reconstruct the given component, but also solves the intermediate density problem. The effectiveness of the algorithm has been demonstrated by mechanical testing. The resulting stiffness of the optimal component by HORV algorithm is 2.3 times better than the uniform density part. The strength of the optimal parts increases 75.4%. And the energy absorption increases 33.1%. In brief, the experiments indicate that the algorithm is quite efficient and reliability for the component design.

## **8. Acknowledgments**

Financial support for this work has been provided by American Makes and is gratefully acknowledged.

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