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A MODIFIED INHERENT STRAIN METHOD FOR FAST PREDICTION OF RESIDUAL DEFORMATION IN ADDITIVE MANUFACTURING OF METAL PARTS

Xuan Liang*, Qian Chen*, Lin Cheng*, Qingcheng Yang*, and Albert To*

*Department of Mechanical Engineering and Materials Science, University of Pittsburgh,
Pittsburgh, PA 15260

Abstract

Effective prediction of residual deformation is very important to guarantee the quality of metal parts produced by additive manufacturing (AM). However, analytical or numerical models for the AM process are complicated and time consuming thus far. The conventional inherent strain theory can predict residual distortion of the regular metal welding problem. Typically, it is not applicable to the complicated layer-by-layer laser-sintering depositing process. In this paper, a modified inherent strain method is presented to predict the residual deformation much more efficiently. Calculation of the modified inherent strain is based on small-scale thermomechanical simulation of the AM process. Next, the strain values are assigned to the heat-affected zone as material thermal property and a one-time static mechanical analysis is performed. Residual deformation obtained by the new method and the thermomechanical simulation indicates good accuracy and efficiency of the proposed method.

Key words: Additive manufacturing, Inherent strain, Modified method, Residual distortion

Introduction

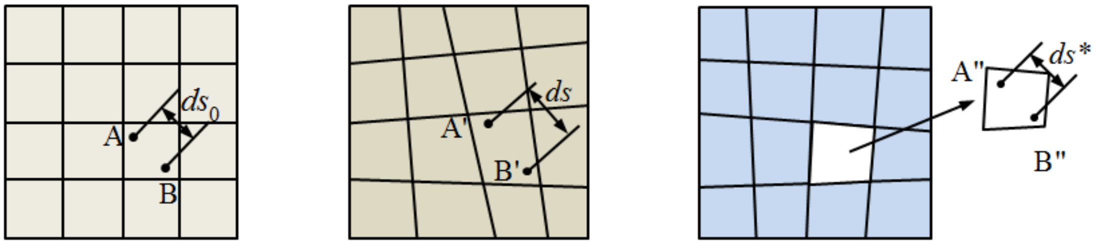
Restricted by its subtractive nature, it is difficult to produce metal parts with complex geometries via conventional machining, and the entire manufacturing process is very energy-consuming. However, the demand for manufacturing complex parts has been rapidly increasing, because metal parts composed of localized microscopic structures, such as cellular structure, are proved to have excellent mechanical performance under certain conditions [1, 2]. For the AM process, a metal part is built from bottom in a layer-by-layer manner. The geometry of the part will be scanned first then sliced into thousands of layers by the computer system. In this way, any complex geometry can be manufactured by advanced 3D printers, such as the EOS DMLS system. For the excellent capability of printing complex geometries, AM technologies have been employed in the manufacturing of complex metal parts constructed by cellular structure [2].

Some of the AM technologies, such as selective laser melting (SLM), are powder based. A metal part is manufactured from a powder bed via a micro-welding process [3]. Generally, the AM technologies are based on an identical physical process similar to the melting and solidification of metal materials [4]. In this process, a large temperature gradient and high cooling rate are involved due to intensive heat input that is moving in space. This complex thermal process can lead to large thermal strain and consequently large residual stress/distortion in the manufactured metal parts. In order to predict the residual distortion and stress in the AM process, there have been numerous research works utilizing the finite element method (FEM) to model the AM process [5, 6]. Since the physical process of AM has something in common with the metal welding phenomenon, some simulation methods on the welding problem have been extended to incorporate the AM simulation [7, 8]. Based on the simulation, optimization of laser scanning strategies, build-up directions or the design of support structures could be further studied [9, 10].

Usually in the thermomechanical simulation, a thermal analysis will be implemented first to acquire the temperature distribution. Then in the mechanical analysis, the temperature history will be assigned as the thermal load [11, 12]. The entire simulation will be time-consuming due to a large number of time steps. To better handle this challenge, the inherent strain method [13-15] was first established to enable fast approximation of the residual distortion and stress in the metal welding problem. Another efficient model inspired by the inherent strain theory is the applied plastic strain method [16-18]. However, the attempt to directly apply the two methods to the AM process fails to obtain an accurate prediction of the residual distortion and stress of the metal parts with multiple deposition layers. For complex parts, it is also challenging to apply the above methods. In addition, there are no further reports on novel approaches of predicting the residual distortion or stress in the metal AM process.

In this paper, a modified inherent strain method is proposed, which enables fast and accurate prediction of the residual distortion in the metal AM process. The content of the paper is organized as follows: Section 2 is the theory of the modified inherent strain which is demonstrated and compared with the original method. Section 3 briefly introduces the thermal and mechanical governing equations. In Section 4, two numerical examples are shown to verify the accuracy and efficiency of the proposed method; the experimental validation will also be included. Finally in Section 5, some conclusions are drawn and future work is discussed.

Theory of the modified inherent strain



(a) (b) (c)

Figure 1: Definition of the inherent strain (a) standard state, (b) stressed state, and (c) stress-free state

The definition of the inherent strain is elucidated according to the literatures on prediction of the residual distortion in metal welding problems [14]. As shown in Fig. 1, for the concerned two points A and B, the distance between them is ds_0 and ds at the standard and stressed state before and after the thermal process, respectively. Assuming after the residual stress is released, the distance becomes ds^*. Therefore, the residual strain in the element, defined as the inherent strain, is calculated by the following equation:

$$\varepsilon^* = \frac{ds^* - ds_0}{ds_0} \tag{1}$$

The original method estimates the welding distortion by applying the inherent strain to an elastic finite element model. The inherent strain is loaded to the heat-affected zone (HAZ) along the welding path [18,19]. In the actual process, the thermal and plastic strain as well as phase change strain will be generated. However, the strain caused by phase change is relatively small [11], and hence it can be ignored. After the entire thermal process, the metal parts will be cooled to room temperature and thermal strain in the part will vanish. Since our reference temperature is always room temperature, the thermal strain is not a concern. Thus, only mechanical strain is involved. Eq. (1) can be rearranged as follows:

$$\varepsilon^* = \frac{ds-ds_0}{ds_0} - \frac{ds-ds^*}{ds_0} = \varepsilon - \varepsilon_e = \varepsilon_p \tag{2}$$

where ε is the total strain, while ε_e and ε_p are the elastic and plastic strain, respectively. Based on Eq. (2), it is known that for a simple welding problem, the inherent strain equals the residual plastic strain in the metal parts [19,20]. However, for the AM problem, the inherent strain shown in Eq. (2) may not apply since the physical process of AM is much different from simple welding. For example, a new mechanical boundary continuously emerges with the deposition of new materials. It is necessary to take into account the influence of thermal history and evolution of the mechanical boundary. Consequently, the modified inherent strain is proposed to predict the residual distortion.

It is reasonable to assume the inherent strain in the HAZ contributes to the distortion in the rest part of the substrate. In the metal AM physical process, a large temperature gradient occurs in front of the center of the heat source [9]. As the heat source moves away, the temperature of the deposition decreases quickly, and the deformation continues to accumulate. Different from the simple welding problem, the elastic deformation in each deposition layer may be significantly affected by the evolving mechanical boundary. Thus, the residual elastic strain in each layer may reflect the influence of the surrounding layers and the substrate. As a result, two different states in the AM process need to be considered in computing the real inherent strain. Taking the first layer of a two-layer deposition as an instance, the first is the initial state, where the depositing of the upper layer is just finished. The second is the final state, where the entire part is cooled to room temperature. The plastic strain at the initial state is involved since it does not vanish via relaxation. Additionally, difference of the elastic strain between the initial and final state must be considered, since it gives the amount of influence by the surrounding layers. Based on the above discussion, the modified inherent strain ε^{In} is written as follows:

$$\varepsilon^{In} = \varepsilon_{t_1}^{P} + \varepsilon_{t_1}^{E} - \varepsilon_{t_2}^{E} \tag{3}$$

t_1 is the time for the initial state and t_2 for the final state. $\varepsilon_{t_1}^{P}$ and $\varepsilon_{t_1}^{E}$ represent the respective plastic and elastic strain at the initial state, while $\varepsilon_{t_2}^{E}$ is the elastic strain at the final state.

In order to extract the modified inherent strain values, we have developed an efficient thermomechanical simulation of which the accuracy has been validated through experiments. However, in the future, a database of the modified inherent strain for some typical material-process combinations may be established so that the complex thermomechanical simulation will be unnecessary.

Thermal and mechanical governing equations

Assume a Lagrangian frame Ω and a material point located by $\boldsymbol{r}$ ($\boldsymbol{r} \in \Omega$) as the reference. Given the thermal energy balance at time t, the governing equation is formulated as follows [12]:

$$\rho C_p \frac{dT(\boldsymbol{r},t)}{dt} = -\nabla \cdot \boldsymbol{q}(\boldsymbol{r},t) + Q(\boldsymbol{r},t), \boldsymbol{r} \in \Omega \tag{4}$$

where ρ is the material density; C_p is the specific heat capacity; T is the temperature field; $\boldsymbol{q}$ is the thermal flux vector and Q is the internal heat source. Material properties such as the thermal conductivity coefficient and specific heat capacity are usually temperature dependent. The internal heat source Q exerts a significant influence on the thermal modeling of the AM process. Different mathematical models have been involved in the literatures [21,22], among which the double ellipsoidal model [23] has been widely used [24,25]. Different types of thermal boundary conditions are employed including the Dirichlet, Neumann and Robin boundary condition.

A quasi-static mechanical analysis is generally implemented to calculate the residual distortion and stress of the workpieces in the AM process. The temperature history obtained by the thermal analysis is applied to the model as an external load and boundary constraint. The governing equation corresponding to the quasi-static mechanical analysis can be written as:

$$\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{b} = \boldsymbol{0}, \boldsymbol{r} \in \Omega \tag{5}$$

where $\boldsymbol{\sigma}$ is the stress tensor and $\boldsymbol{b}$ is the body force vector. As for the mechanical boundary conditions, the Dirichlet and Neumann boundary condition will be taken into account.

Numerical validation

Since the values of modified inherent strain cannot be applied directly as an external load to the finite element model, one feasible solution is treating the strain as equivalent thermal expanding coefficients of the material in three dimensions [18]. After the material property of the elements in the HAZ is modified with equivalent thermal property, a static equilibrium analysis is performed to calculate the residual distortion and stress. Since the numerical analysis is done by just a single static analysis, the modeling will save plenty of computational time.

A wall deposition and the laser scanning path which we are simulating are shown in Fig 2. The left end of the substrate consists of two fixtures connected to the platform in the AM process. For depositions with one to three layers, the length and width of the wall are 44.57 mm and 2.0 mm, respectively. For the one-layer deposition, the height is 0.99 mm while 1.8 and 2.7 mm for the two and three-layer deposition. For the five-layer deposition, the length, width and height are 44.57, 2.7 and 4.75 mm respectively. The size of the substrate is 102.0×102.0×3.22 mm. Both the deposition and the substrate are made of Ti6Al4V. Material property of the Ti6Al4V is temperature dependent and can be accessed in the reference [24].

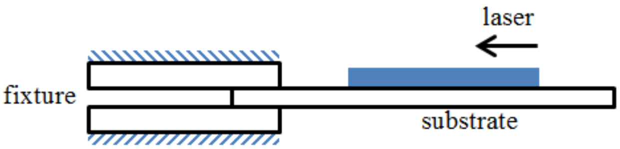


Figure 2: The wall deposition and the laser scanning path

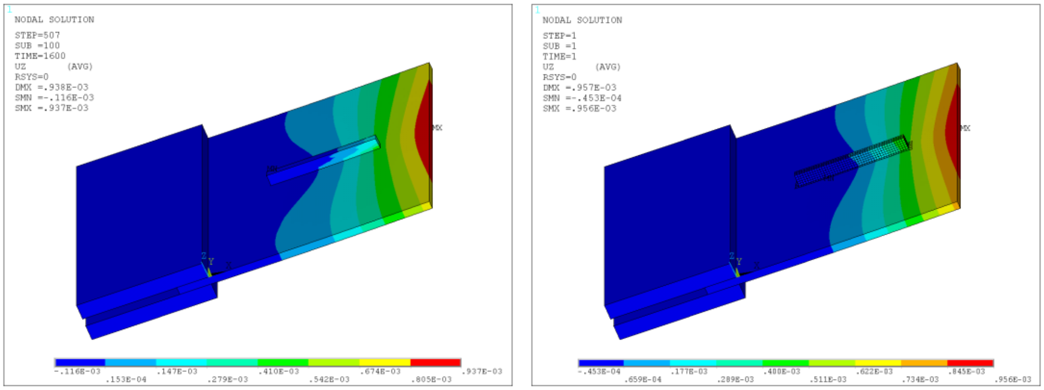
It could take 2 to 6 hours to finish each thermomechanical simulation in this section. Usually, the maximum residual distortion is the highest concern. As shown in Fig. 3(a), the maximum distortion for the five-layer wall is 0.937 mm and occurs at the free end of the substrate in the vertical direction. In addition, many elements in the deposition are at the yield state. The maximum Von Mises stress is about 765 MPa, which matches the yield criterion set for Ti6Al4V at room temperature.

First, we show the conventional inherent strain method is not applicable to the AM process. The inherent strain computed based on Eq. 2 is utilized to make a prediction of the residual distortion. The results are listed in Table 1 and compared with those obtained via the detailed process simulation. The large errors indicate that it is inaccurate to employ the conventional inherent strain method to predict the residual distortion. Thus, the modified theory is of great importance to fix the conventional method.

Table 1: The vertical residual distortion of the depositions by simulation and conventional inherent strain method

Number of layers	Maximum vertical distortion (mm) by simulation	Maximum vertical distortion (mm) by conventional inherent strain method	Error (%)
1	0.326	0.170	47.9
2	0.502	0.299	40.4
3	0.584	0.412	29.8
5	0.937	0.672	28.3

Through the modified inherent strain method, the residual distortion of the part can be calculated within a couple of minutes. Especially for the five-layer wall deposition, the pattern of the vertical residual distortion is shown in Fig. 3(b) with a maximum value of 0.956 mm. Essentially, the distribution of the vertical distortion is nearly symmetric since the laser scanning path and the mechanical constraint are both symmetric in this problem. Clearly good agreement is seen between the results calculated by the thermomechanical simulation and the modified inherent strain method.



(a) (b)

Figure 3: The vertical residual distortion (a) by simulation (b) by the modified inherent strain method

Additionally, the vertical residual distortion computed using our thermomechanical simulation and the modified inherent strain method are compared in Table 2. Clearly, much better accuracy is obtained by the modified inherent strain method as can be seen from the smaller errors.

Table 2: The vertical residual distortion of the wall depositions computed in two ways

Number of layers	Maximum vertical distortion (mm) by simulation	Maximum vertical distortion (mm) by modified inherent strain method	Error (%)
1	0.326	0.348	6.7
2	0.502	0.557	10.9
3	0.584	0.606	3.8
5	0.937	0.956	2.0

Conclusions and discussions

Predicting of the residual deformation in the AM process is very important to guarantee the quality of the printed metal parts. With respect to current prediction methods, the widely used thermomechanical simulation is very complicated and time-consuming, while the conventional inherent strain method is fast but not accurate. In this paper, the modified inherent strain method is established. The AM process of the wall depositions is investigated. The results and contrasts indicate that the proposed method is accurate and effective since the residual distortion of the depositions can be precisely predicted in a short time.

In addition, when the proposed method is applied to some larger models, the procedure may consume more time if the modified inherent strain in the HAZ is varied from element to element. As a further work, an averaged modified inherent strain may be investigated. Another concern is the extraction of the modified inherent strain is based on our thermomechanical simulation in this paper. Once a database of the modified inherent strain for different kinds of process-material combinations is established in the future, the detailed process simulation will no longer be necessary. More advanced applications of the proposed method hereby will be seen in the metal AM industry.

Acknowledgements

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