

INVESTIGATING THE IMPACT OF FUNCTIONALLY GRADED MATERIALS ON FATIGUE LIFE OF MATERIAL JETTED SPECIMENS

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ABSTRACT

The capability of Additive Manufacturing (AM) to manufacture multi-materials allows the fabrication of complex and multifunctional parts with varying mechanical properties. Multi-material AM involves the fabrication of 3D printed objects with multiple heterogeneous material compositions. The material jetting AM process specifically has the capability to manufacture multi-material structures with both rigid and flexible material properties. Existing research has investigated the fatigue properties of 3D printed multi-material specimens and shows that there is a weakness at the multi-material interface. This paper seeks to investigate the effects of gradual material transitions on the fatigue life of 3D printed multi-material specimens, given a constant volume of flexible material. In order to examine the fatigue life at the multi-material interface, discrete digital-material gradient steps are compared against the true functional gradients created through voxel-level design. Results demonstrate the negative effects of material gradient transitions on fatigue life as well as the qualitative material properties of true versus discrete gradients.

1. INTRODUCTION

Additive Manufacturing (AM) is a process that involves producing three dimensional parts by joining materials in a layer wise manner. AM is more advantageous over traditional manufacturing processes [1] because of the design freedom and material complexity offered to designers. Engineers and designers can incorporate different material profiles in their digital models which are then reproduced through compatible AM processes [2]. Currently, the only three commercial AM processes capable of processing multiple materials in a single build are material extrusion, directed energy deposition (DED), and material jetting. The scope of this research focuses on the multi-material capabilities offered by the material jetting process. The material jetting process, specifically the PolyJet process, involves the selective jetting of liquid-based droplets of photopolymer material alongside support material, which are cured by an ultraviolet light [6]. With the simultaneous jetting of multiple materials, such as TangoBlackPlus (TB+) and VeroWhitePlus (VW+), the combined flexible and rigid material properties provide diverse possibilities to develop designs with heterogeneous material properties and different functionalities.

The PolyJet process finely controls the compositions of composite materials through voxel-based deposition by a design approach called dithering, whereby material is gradually suspended within the matrix of another material across a structure's volume. By controlling the material compositions at the voxel-level, functionally graded materials (FGMs) are created. FGMs are

heterogeneous materials consisting of two or more constituent materials that spatially vary in their material composition across the volume of a structure [3, 4]. Using the material complexity and design freedom provided by AM, FGMs can be manipulated to control material properties such as strength, stiffness, and flexibility within a multi-material structure. As illustrated in the figure below, Phase-A and Phase-B particles, also denoted as material A and material B, are varied across the volume of the structure from one end to another forming a material gradient with the transition region [5].

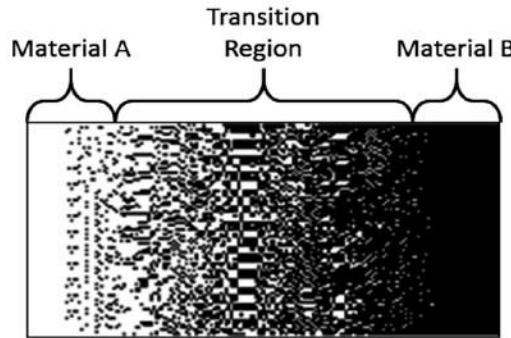


Figure 1. Example of functionally graded structure with gradient transitions of material A and material B

The nature of living hinges offers one of the most intriguing uses of combined rigid/flexible FGM structures. Living hinges eliminate traditional rotational joints in moving structures, and instead use flexible regions of material to allow for motion. With the capabilities available through the PolyJet process, the authors hypothesize that living hinge designs can be improved. Different compositions of materials in forms of gradients can be selectively combined to create structures with regions of specified range of motion in order to attain a desired performance. In order to further understand how these gradient-based materials operate, this work seeks to determine an efficient way to distribute multi-material FGM phases in order to maximize fatigue life and reduce premature failure. Testing two gradient-based designs with varying lengths allows the analysis and visualization of interfacial behavior between different material compositions. The outcome of this work provides designers' information to improve the predictability of fatigue life in living hinges.

2. EXISTING RESEARCH IN FGMS FOR MATERIAL JETTING

Existing research that focuses on different methods of designing and manufacturing structures with FGMs is presented in sections 2.1 and 2.2 respectively. Section 2.3 highlights the relationship between the process, structure, properties, and behavior of material jetted parts.

2.1 Designing of Functionally Graded Materials

The spatial variation in the composition of FGMs is achieved by gradually changing the volume fraction of the constituent materials during the manufacturing process. FGMs can be categorized

in continuous or stepwise graded structures. Continuous graded structures include smooth transitions in the material composition with respect to the position within a structure. Stepwise graded structures have discontinuities in the material compositions, which are due to the stacking of discrete layers while forming multilayered structures with immiscible compositions [3]. Figure 2 provides examples of continuous and stepwise gradients with plots elaborating how functions are used to develop gradients.

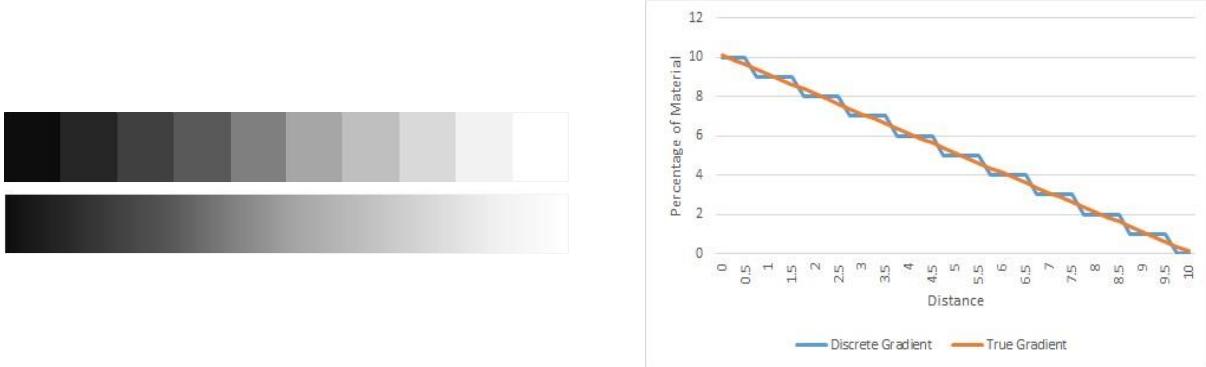


Figure 2. Examples of stepwise and continuous functional gradients (left) with accompanying plots (right)

Several research studies have been performed to develop optimal FGM models with effective material properties and desired performance. Doubrovski [2] used mathematical expressions to create material gradients and voxel-based representation to define the microstructure, material, and structural compositions of continuous gradients to develop functionally graded structures. Kawasaki and Watanabe [7] used function-based representations to define and determine the spatial distribution of material within a specified volume, whereas Tanaka and coauthors [8] designed FGM models based on optimization algorithms to adjust the profiles of compositions that form FGMs. Furthermore, the “inverse design procedure” was introduced by Hirano and Yamada [9] demonstrating sequential steps taken to develop a FGM by identifying the functionally graded structure and specifying the boundary conditions. As noted by Markworth and coauthors [10], developing FGM models based on their microstructures allows the characterization of thermophysical properties because of the wide variety of microstructures that can exist across any graded direction. While various research studies provide possible methods for developing optimal FGM designs, limited information exists regarding other applications such as varying the volume of multi- material structures at the mesostructural level.

2.2 Manufacturing of Functionally Graded Structures

Traditional manufacturing for FGMs, such as centrifugal casting, slip casting, jet solidification, and surface treating, face significant limitations compared with AM [11]. AM techniques provide better opportunities for manufacturing functionally graded structures by simultaneously changing the composition of the material during deposition. The material extrusion process is capable of

processing multi-materials by simultaneously depositing layers of material extruded from two nozzles. The process, however, has limited capability in automatically spatially varying the material composition during the fabrication process. Garland and Fadel [12] confirmed this aspect by analyzing the printing process of an FDM 3D printer, the Big Builder printer. It was found that during material deposition, the printer flushed out one material before extruding the second material instead of automatically changing the composition of material.

The DED process, as highlighted by Vaezi and coauthors [13], has the capability of varying the compositions of powder during its actual deposition and still maintain full density. Shin and coauthors [14] focused on the LENSTTM process, as part of the DED process, to produce functionally graded metal parts by varying the material composition at the exact point of material deposition. Bruyas and coauthors [15] presented an example of a compliant mechanism with a revolute joint created by the PolyJet process. This concept is similar to designing living hinges that can be produced using FGMs. For all living hinges, the conformal behavior of functionally graded structures is heavily dependent on their structural design. Research involving FGM processing with the PolyJet process is continuously evolving as researchers are seeking appropriate methods and approaches for optimal FGM design, processing, and performance. By doing so, different material types, compositions, geometries, and properties onto which the material gradient is imposed are considered. The overarching goal, then, is to develop relevant design frameworks that allow the manufacturing of functionally graded structures for a desired performance.

2.3 Relating the Process, Structure, Properties, and Behavior of Material Jetted Parts

Developing functionally graded structures not only involves developing optimal FGM designs and manufacturing capabilities but also relating the structures and material properties to the behavior of functionally graded materials. Various researchers have mostly focused on the impact of parameters such as orientation, surface finish, part spacing, material anisotropy, and UV exposure, among others, on the mechanical properties of material jetted parts [16–19]. However, the material property most relevant to the discussion in this paper is the fatigue life of material jetted specimens. Moore and Williams [20, 21] specifically focused on the fatigue characteristics of material jetted specimens with a single elastomeric material, TangoBlackPlus, at the interface as well as in the reduced central region. This particular research sought to predict the expected fatigue life based on their elongation. Results showed that a single elastomeric material interface have much higher fatigue life. In addition, for smaller values of strain rates used during fatigue testing, the fatigue life of the TangoBlackPlus material increased. The variability of the fatigue life in the specimens was due to failure at the material interface. Fatigue failure at the TangoBlackPlus material implied that the material had a more predictable fatigue life based on its material properties. This work showed the lack of predictability of the failure location at the material interface. Additional data will be collected in this research to further study fatigue failure at the material interface for functionally graded parts.

Following this literature review, previous research studies have focused on developing optimum design methods and workflows for producing functionally graded structures in addition to different aspects of FGM designs focused on by AM designers. Most theoretical efforts minimally address the relationship between the mechanical properties and the structures of parts printed using the PolyJet process that are necessary for designing multi-material living hinges. In order to address this aspect of functionally graded structures, this research study primarily focuses on analyzing the effect of different multi-material gradient types with varied lengths of material concentrations on the fatigue life of functionally graded fatigue specimens.

3. DESIGN AND EXPERIMENTAL APPROACH

The objective of this research is to investigate the effects of different material transitions on the fatigue life of material jetted specimens, when the volume of flexible material is held constant. Specimens with discrete material gradients were compared with specimens with true material gradients to demonstrate whether any specific material gradient type has an impact on the expected lifespan of such specimens. In addition, different concentrations of materials were selected to be distributed across a specified region of the specimens so as to analyze the effect of different material transitions on the fatigue life of the specimens. In summary, this research seeks to answer the following questions:

1. How does the length of the material transition within a specified region affect the fatigue life of multi-material parts?
2. Will true material gradients offer a better advantage on the fatigue performance of printed multi-material parts compared to discrete material gradients?

For this research, the authors hypothesize that as the length of the transition region is increased linearly, the distributed material properties of the specimen will improve fatigue life. In addition, the fatigue life would increase as the specimen accurately converges to an ideal linear gradient pattern. The work presented in this research will help determine whether varying material over a specified length improves the fatigue life of specimens with different material gradients.

3.1 Experimental Design

The fatigue specimens used for this study, shown in Figure 3, were designed according to ASTM D4482-11 [22]. Additionally, beaded edges were designed and length to the grip region was added in order for the specimens to fit securely into a set of special grips during testing.

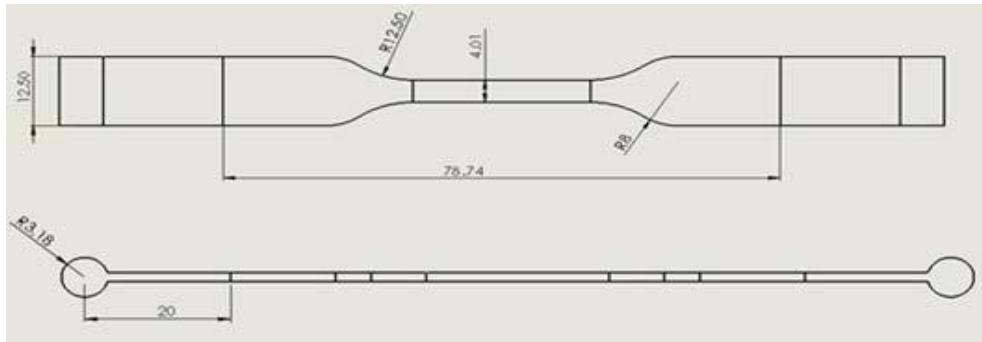


Figure 3. A CAD model of a fatigue specimen. All units are in millimeters

In order to maintain a consistent testing procedure a fixed volume of TB+ (flexible) material was used throughout. Emphasis was instead placed on how the material is distributed through the central, living hinge region. The distribution of material mimics common considerations in the design of living hinges, where allotted material must be distributed optimally for a desired deflection. This distribution was defined as a function of length across the 25mm region. Due to established gradient mixtures from Stratasys, two types of discrete material gradient designs were established to simulate the out-of-the-box capabilities that would be available to a designer. The variables considered in the research study are displayed in Table 1.

Table 1. Variables selected for the research study

Variables	Parameters
Constant	Volume of TB+
Independent	Distribution of TB+ across the central region
	True and discrete material gradients
Dependent	Fatigue life

Using TB+ material, an analogy of a living hinge was used to simulate how the material would be dispersed throughout the central region of the specimen. The control specimens were constructed with TB+ and rigid VeroCyan (VC) materials connected at an interface. The central region of TB+ simulated a simple flex point around which the rigid VC sections pivoted. Control specimens were the base specimens with which the constant volume of TB+ used across all other fatigue specimens was determined. To test the effectiveness of mixing these materials, the transition regions were spread across multiple lengths with linear transition regions. The smaller length, denoted as min-length, minimized the transition region while the maximum length, denoted as max-length, stretched the functionally graded transition regions to the full length available. Each specimen contained a constant volume of TB+ material and was tested under the same conditions.

Based on the maximum and minimum lengths of the regions with 100% TB+, two equations were formulated based on intuition of the simplest form of gradients applied linearly in the central

region of the specimen. The volume of TB+ was dispersed linearly across the surrounding area within the central region to create a smooth transition between the two materials. As a point of comparison, stock material mixtures from Stratasys were used to create discrete transition regions with interfaces every 2.45 millimeters. These regions were then recreated with true gradients stretching across the same length region. Using the fatigue specimen CAD geometry, true and discrete material gradients were designed as described in the following sections.

3.2 Design Process for True Material Gradients

True material gradients were created based on linear equations that describe a systematic gradual transition within a specific region. Figure 4 shows these functions, along with the maximum and minimum lengths over which the true material gradients were applied within the central region of the fatigue specimen. The maximum and minimum lengths of regions of 100% TB+ are plotted along with the control (sharp interface between TB+ and VC) to show how the material gradient varies for each gradient type over a specified length. The plots range from 0 to 1 for the material transitions implying the rigidity of the material for values closer to 1 or the flexibility of the material for values closer to 0. A sharper incline pictured in Figure 4 indicates a narrower transition region.

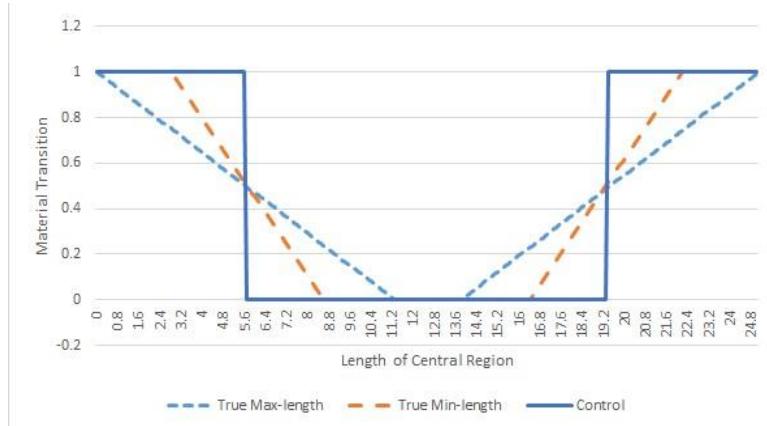


Figure 4. Plot showing the maximum and minimum lengths of the true material gradients in relation to material transition for control specimens

In order to support the voxel-by-voxel deposition needed to create the true FGMs, Stratasys' Voxel Print software was used. This allows for material composition to be specified at the voxel level; when combined with a dithering approach, FGMs can be created. The following flowchart details the general steps taken to obtain true material gradient specimens.

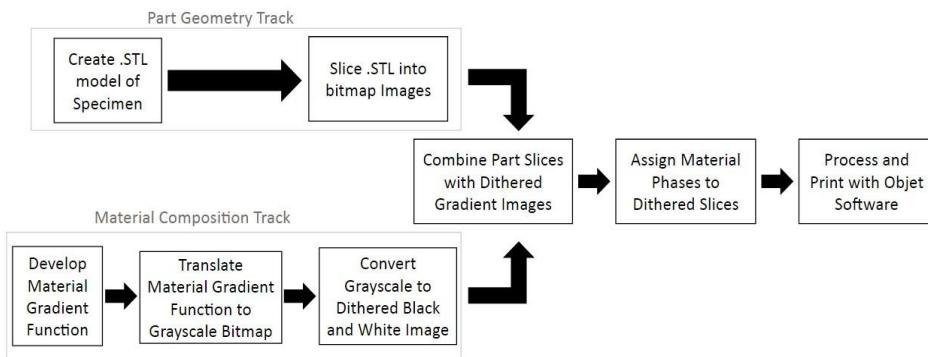


Figure 5. Flowchart showing the design process of the true material gradient specimens

Regarding the material composition track shown in Figure 5, an in-house MATLAB code was created in order to take the linear equations and convert them into RGM gradient images. The RGB gradient images were transformed into binary black and white image using dithering. For the part geometry track, Autodesk Netfabb software was used to create a series of .BMP slices from a fatigue specimen .STL file. These slices act as direct toolpath instructions for voxel-based design. The .BMP slices from Netfabb were incorporated in the MATLAB code and used as masks to be overlaid onto the gradient binary images. The binary images were saved in the same pixel size as the .BMP images. The final result, a masked image, consisted of a rendering of the .BMP slices with the gradient binary images. For each material assignment, a total of 245 bitmap slices were generated with a uniform layer thickness of 0.030 mm. Figure 6 shows the true gradient specimens that resulted from this design approach.



Figure 6. Specimens showing true material gradients: min-length (top) and max-length (bottom)

3.3 Design Process of Discrete Material Gradients

The concept of plotting the maximum and minimum lengths for the discrete material gradients is similar to that in Figure 4, however, instead of shaper inclines, the vertical transitions pictured in Figure 7 indicate shear transitions between two pre-set material mixtures.

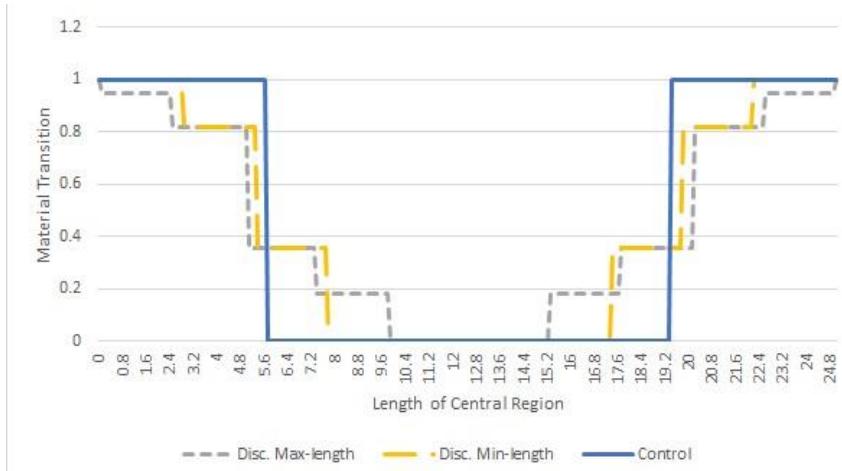


Figure 7. Plot showing the maximum and minimum length of the discrete material gradients in relation to material transition for control specimens

Based on the plots above, the reduced central region of the CAD model shown in Figure 3 was sectioned into different lengths onto which different material gradient concentrations were applied. The material concentrations were selected from a compilation of percentages as represented below.

Table 2. Material concentration percentages used for the discrete material gradients at maximum and medium lengths

Lengths	Percentages of TB+
Max-Length	10%, 25%, 65%, 80%
Min-Length	25%, 65%

3.4 Printing Specifications and Fatigue Testing Procedure

All fatigue specimens were printed using the Objet350 Connex3 with VC and TB+ as the model materials as well as FullCure SUP705 as the support material. The ideal orientation for the specimens on the build platform was the XY orientation [6] because it ensures a single pass of the print head per layer of material deposited. It is also important because the specimens receive uniform curing of material from the uniform exposure of UV light during printing. Objet Studio, a server software, was used to prepare all specimens for printing. During preparation of the build tray for the specimens with discrete material gradients, the material gradient concentrations were selected manually from a drop-down menu of a variety of materials based on the corresponding gradient percentages. Figure 8 shows an example of a fully printed fatigue specimen with discrete material gradients.



Figure 8. Printed specimens with discrete material gradients

For the true material gradients discussed in Section 3.2.1, the specimens were manufactured using the VoxelPrint software supplied by Stratasys. Individual bitmap slices were printed in the digital materials (DM) mode with a print resolution of 600 dpi by 300 dpi. An example of a fully printed fatigue specimen with a true material gradient is shown in Figure 9 below.



Figure 9. Voxel printed specimens with true material gradients

Regarding fatigue testing, the test method provided in the ASTM standard [22] was used. The fatigue testing procedure involves measures the number of cycles to failure for specimens undergoing repeated cyclic loads under a fixed displacement control. The fatigue specimens were mounted and tested for fatigue on a MTS 880 Servohydraulic material test system (Figure 10).



Figure 10. The MTS 880 Servohydraulic Material Testing System (left) and the non-compression grips and a sample specimen (right)

The specimens were tested for fatigue in a loading and relaxation cycle using displacement control until complete failure (or until the maximum cycles were exceeded) to simulate a limited

range of motion as seen in common examples of organic living hinges. Failure was determined when the specimens were completely ruptured. At a fixed frequency of 1.7Hz for each testing cycle, the fatigue specimens were tested at a mid-range extension of 40% elongation of the total volume of TangoBlackPlus material in the control specimens. One cycle was measured starting from the rest position at zero extension to the position at maximum extension. Specimens were tested with four phases of equal length: i) hold at zero extension, ii) ramp linearly upwards to maximum extension, iii) hold at the maximum extension, and iv) ramp downwards to zero extension. Based on the design of the specimens, mechanical grips were deemed necessary to fasten the specimens onto the load frame. For this, two special grips were machined out of aluminum blocks with M6 x 1 bolt taps drilled and slots cut through. For each grip, the slots were cut to snugly fit the thickness on either end of the specimens.

4. DATA ANALYSIS AND DISCUSSION

Results obtained from the fatigue testing procedure of all printed specimens are presented in Table 3 below. Following in this section is the data analysis and detailed discussion of the results obtained.

Table 3. Data obtained from fatigue testing procedure

<i>Material Gradient Types</i>	<i>Lengths</i>	<i>No. of Specimens</i>	<i>Cycles to Failure</i>	<i>Failure Location</i>
True Gradients	Max-length	Specimen - 1	10	TB+ and Transition Gradient Interface
		Specimen - 2	11	TB+ Region
		Specimen - 3	35	TB+ Region
	Min-length	Specimen - 1	1077	TB+ and Transition Gradient Interface
		Specimen - 2	895	TB+ Region
		Specimen - 3	739	TB+ Region
Discrete Gradients	Max-length	Specimen - 1	292	TB+ Region
		Specimen - 2	693	TB+ Region
		Specimen - 3	157	TB+ Region
	Min-length	Specimen - 1	627	TB+ Region
		Specimen - 2	2001	No Failure
		Specimen - 3	2001	No Failure
Control Specimens		Specimen - 1	2001	No Failure
		Specimen - 2	2001	No Failure
		Specimen - 3	2001	No Failure

The data was analyzed for statistical significance using two-way analysis of variance (ANOVA) to determine the main and interaction effects of material gradient type and gradient length with respect to the number of cycles to failure. All the assumptions for this test were met accordingly except for the homogeneity of variance test, which was not met for $p>0.05$; however, as ANOVA is generally robust against violations of homogeneity of variance when sample sizes are equal, statistical analysis was still conducted. The two-way ANOVA tests showed a statistically significant difference between the true and discrete material gradients with a p-value of 0.047 as

well as the maximum and minimum lengths with a p-value of 0.001. The interaction, however, between the gradient types and the maximum and minimum lengths showed no statistical significance.

Furthermore, the average number of cycles to failure was compared for each specimen group as shown in the plot in Figure 11. According to the plot, the control specimens out-performed the specimens with true and discrete gradients. When compared for significance amongst one another, the Tukey test shows a statistically significant difference between both gradient types and control specimens. The maximum and minimum lengths also showed a statistically significant difference from the control specimens. Based on the maximum and minimum lengths of the transition region, with respect to a constant volume of TB+, it was seen that the specimens did not perform as expected. The authors hypothesized that as material is distributed into the transition regions, the gradient specimens would offer more reliable locations of failure.

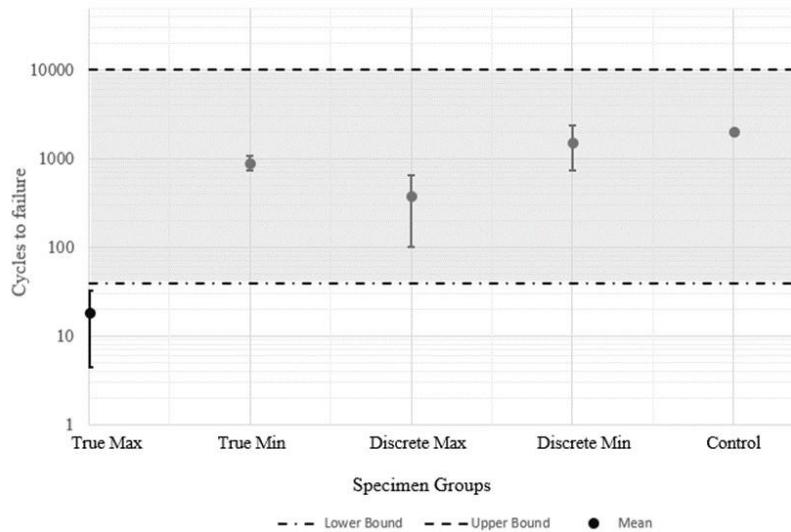


Figure 11. Showing the average cycles to failure with standard deviation as error bars for all the specimen groups

As previously discussed, one of the key questions targeted by this study was whether the use of gradient transitions can avoid the unpredictable interface failure observed in previous studies by Moore and Williams [21]. However, this premature interfacial failure was not observed in initial testing, potentially due to the small number of replications for each condition. As such, the shaded region in Figure 11 has been provided to denote the 95% confidence interval established by Moore and Williams at 40% elongation, which allows for this new work to be framed within the potential unpredictability of interfacial failure. Moore and Williams also provided initial insight about locations of failure which lead us to believe that failure would occur at the material interface. However, most tested specimens failed within the TB+ region with an exception of two specimens

that experienced failure at the interface. Unfortunately, insufficient data failed to provide statistical significance to support a conclusion on the effect the use of FGMs has on the failure location in fatigue specimens.

In order to understand the trends seen in Figure 11, the authors turned to the failed fatigue specimens. By observing the failed fatigue specimens (as shown in Figure 12), unexpected elastomeric behavior was noted. It was assumed that when elongated during fatigue testing, specimens would behave elastomerically in regions where TB+ material outnumbers the concentration of VW+ material. However, contrary to this assumption, qualitative observation of the final specimens shows that there is a visible step discontinuity in flexibility as the gradient material transition region approaches the flexible material region. This implies that with the designed material gradients, there is no smooth transition in the materials and hence the material properties. Noting this, the authors are led to hypothesize that a linear change in material distribution does not result in a linear change in properties; the elastomeric region ends well before the 50/50 concentration point is reached. Therefore, the deformable central region is reduced in length for both true and discrete gradient specimen in comparison to the control specimens. This results in a larger stress being experienced in the flexible regions of the graded specimens than the control specimens, which will cause decreased fatigue life. This qualitative observation is confirmed by the fatigue life data shown in Figure 11; the gradient patterns that result in the lowest fatigue life are the ones that also have in the shortest elastomeric regions.



Figure 12. Tested fatigue specimens with true (left) and discrete (right) material gradients

5. CONCLUSIONS AND FUTURE WORK

The goal of this research study was to investigate the fatigue properties of material jetted multi-material specimens. This study particularly focused on determining whether fatigue life is improved by comparing different material gradient types, specifically true and discrete material gradients as well as comparing different lengths of material transitions in the center region of the printed specimens. Based on the results obtained from the fatigue tests and the qualitative analysis of the data, the authors can conclude that:

- Specimens with true and discrete gradients had decreased fatigue life as the length of the transition region increased.
- The linear gradient used for the true material gradient specimens did not increase the elasticity of the tested specimens.
- The control specimens are more flexible than the discrete gradient specimens.
- The discrete gradient specimens are more flexible than the true gradient specimens.

- Specimens with discrete gradients survived a larger number of cycles to failure compared to specimens with true gradients.

With these conclusions in mind, designers could potentially focus on tailoring material gradient patterns such that failure occurs in certain locations and fatigue life is maximized. While this research showed significant data relating material distribution to fatigue life, there was limited data to support additional conclusions. In order to expand the breadth of this research, additional experimentation will be conducted in the future to:

- Assess the locations of failure across gradient transition regions.
- Analyze the fatigue life of specimens with varied true gradient patterns.
- Determine the effect of material distributions as the central TB+ region is preserved.
- Study the effect of different levels of elongations on the number of cycles to failure.

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