DESIGN AND ANALYSIS OF A TOPOLOGY OPTIMIZED TRANSTIBIAL PROSTHETIC SOCKET USING COMBINED STATIC GAIT ANALYSIS

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Abstract

This paper presents the design and analysis of an optimized transtibial prosthetic socket developed using the ground structure method of topology optimization (GSM). The socket wall between the distal 25% of the original socket and a proximal brim is replaced with an optimized truss geometry and a thin wall (1 mm). Separate trusses are developed for the loading conditions of three critical stances: heel strike, vertical (standing), and toe-off. The truss models are combined with critical components to create the final design. The proposed socket is 81.58% of the original socket volume and is designed for manufacturing using Selective Laser Sintering (SLS) and nylon-12. The socket design is analyzed, with the material properties for sintered nylon-12, at 10% increments between heel strike and toe-off to determine the viability of both the socket and the corresponding methodology. Simulation results indicate that the design exceeds requirements for all tested stances.

Introduction

Transtibial prosthetic limbs are used as a means of ambulation by an estimated 380,000 people in the United States alone (Stevens et al., 2019). Transtibial (below the knee) amputations have increased over the past thirty years primarily due to type two diabetes and motorcycle accidents (Sanders & Fatone, 2011). Regular use of existing, commercially available transtibial prostheses is complicated by users’ experience of pain, injury, and psychological distress. Current research shows that the prosthetic socket, the interface between the residual limb and the prosthesis, is the component related to the majority of user complaints (Paternò et al., 2018).

This research proposes a transtibial prosthetic socket developed using the ground structure method (GSM) of topology optimization. Critical components, both a brim at the proximal end and the distal 25% of the socket comprising a socket base, are integrated with a 1 mm thin wall and an optimized truss structure based on the numerical approximation from (Dorn et al., 1964) of the truss optimization methods originally described in (Michell, 1904). The socket is designed for fabrication with Selective Laser Sintering (SLS). This process is chosen due to its capabilities to fabricate long overhangs and thin walls found in this design, and prior vetting as a viable construction method for lower limb prosthetic sockets, specifically for sockets with adaptive geometric features built into their structure (Faustini, 2004; Pham et al., 1999; Rogers et al., 2001). This socket is not user ready, but instead serves as a proof-of-concept meant for bench testing. The purpose is to assess the viability of truss-based optimization for complex loading problems and determine its potential for modifying prosthetic socket topologies. Further work is needed to modify this design for clinical use.
In a review of socket technologies, (Paternò et al., 2018) identified four primary socket failure modes: volumetric fluctuation of the residual limb, including both diurnal fluctuations and semi-permanent changes, which leads to discomfort and shifts in limb position relative to the socket; overheating and the resultant perspiration-related skin conditions; pressure or normal forces, which cause discomfort and injury; and shear stress between the limb and socket (or liner) causing damage to the skin (Paternò et al., 2018).

Historically, little has changed about the topology of transtibial prosthetic sockets. Since the first transtibial socket was developed by Pieter Verduyn in 1696 (Thurston, 2007), the primary changes have been either the material of the socket or the method of suspension. In the 20th century, three structural designs of socket with similar shell models emerged as the primary transtibial prosthetic sockets. These sockets vary in topology to adjust the locations of pressure on the residual limb (Bowker, 1992; Moo et al., 2009). The first is the total surface bearing (TSB) socket, fitted tightly to the body in an attempt to evenly distribute pressure on the residual limb. Next is the patellar tendon bearing (PTB) socket, developed in 1950 at the University of California at Berkeley. The PTB socket includes pressure tolerant regions in the area of the patellar tendon and the posterior of the socket. These are adjusted to bear more of the load within the socket, relieving pressure on the anterior of the limb and the fibular head. It is common for a socket to be a hybrid of these two designs in an attempt to balance the benefits of each (Stevens et al., 2019). Least common of the three, but still widely accessible, are hydrostatic (HTS), or pressure cast, sockets that have consistent pressure on all parts of the residual limb but incorporate a pressure chamber to keep constant surface pressure as the limb adjusts due to soft tissue deformation. The HTS socket relies on the assumption that changing pressure points leads to even pressure absorption as the user changes positions; however, dynamics are not accounted for in the design.

The increase in transtibial prosthesis users has led to an increased interest in socket improvements both in academia and industry. Ongoing, widespread user dissatisfaction with transtibial sockets bolsters continued research and calls for explorations of structural innovation in this field (Pirouzi et al., 2014). In literature proposing new topologies for transtibial sockets, additive manufacturing (AM), along with computer-aided design and manufacturing (CAD & CAM) are often critical to the realization of novel designs. SLS and Stereolithography are the most commonly used AM techniques in prosthetic socket development. Double walled sockets, solid sockets with varying wall thicknesses, and sockets with geometrically complex features are possible through AM and account for a large portion of proposed topology changes in transtibial socket research (Faustini, 2004; Faustini et al., 2006; Rogers et al., 2008; Webber & Davis, 2015).

Each socket is customized due to variation in the shape of the residual limb. Prior to any modification, the topologies are complex. Current methods of fabrication include a prosthetist casting or scanning the residual limb geometry and creating a positive mold for forming or laminating the socket materials. These molds are typically made of plaster and carved either by hand or using a specialty computer numerical control (CNC) lathe. Some molds are created additively from computed tomography (CT) scan data of residual limbs. Thus far, this has not been found to improve the fit of sockets, but it does reduce labor and allow molds to be created more quickly. Customization, complex geometry, and high costs, due in part to labor, make this area of study an opportunity for AM sockets of practically any design to be created at
comparable prices more easily (Hsu et al., 2018; Rogers et al., 2007; Sewell et al., 2012; Trower, 2006).

In this research, there is particular interest in prosthetic socket applications that utilize AM to change socket topology or employ topology optimization in the design process. However, it should be noted that there are examples of innovative prosthetic socket structures, both in academia and industry, that are not fabricated using AM ("Martin Bionics," 2022; Quezada, 2017; Vaughan, 2014).

SLS sockets with compliant features in regions of high pressure are described in (Faustini et al., 2006) and (Rogers et al., 2001). The first version developed by (Rogers et al., 2001) used thin walls in high pressure regions. Building on this work, (Faustini et al. 2006) implemented an optimized spiral topology with bracing that served as a spring in these same regions of high pressure. This feature further decreased stiffness in these areas while still maintaining contact with the residual limb. A comparison with the thin wall design was conducted in (Rogers et al., 2008) with a single user. It was found that the user experienced increased comfort during ambulation and was able to increase their walking speed with the optimized pressure relief spring model (Faustini et al., 2006; Rogers et al., 2008; Rogers et al., 2001).

Temperature change is the failure mode of transtibial prosthetic sockets that is addressed the least in current literature. This issue can lead to sweating, skin maceration, and slipping in the socket, making temperature shifts highly uncomfortable and dangerous to the user (Ghoseiri & Safari, 2014; Paternò et al., 2018; Webber & Davis, 2015). An interesting approach to this issue is a design from (Webber & Davis, 2015). Using SLS, they created a socket with a passive helical cooling channel that wound around the residual limb. In bench testing, sensors were placed at the distal and proximal ends of the socket. At 31.1°C, the average delta from the outer wall to the inner wall was 4.5°C at the proximal sensor location and 5.1°C at the distal sensor location.

Recently, examples of topology optimization and predictive design have entered the transtibial prosthesis literature. The first of these was a study described in (Steer et al., 2019, 2020). This study used a multi objective genetic algorithm (MOGA) and surrogate limb models of varying geometry to determine areas appropriate for socket wall removal or reduction. The fitness functions used in this research sought to minimize distal loading and bony prominence loading (loading in areas with the least soft tissue between the residual skeletal structure and the socket). In these models, areas for socket wall removal or reduction were found for each limb geometry for biased optimization towards each fitness function as well as unweighted fitness functions (Steer et al., 2019, 2020).

Another example of a computational design method can be found in (Ballit et al., 2022). This socket generation method was based on CT scan data and soft tissue deformation throughout ambulation. The socket design was constrained by force translation and the pain threshold for soft tissue. Wall thickness was added or removed from areas of the socket model until an optimal socket was obtained. The final model was found to be below the soft tissue pain threshold throughout ambulation (Ballit et al., 2022).
The final relevant example is the light-weighting of a socket base using solid isotropic microstructure (or material) with penalization (SIMP), described in (Sotola et al., 2020). The model of the socket base that was optimized attaches below the socket, providing additional support. However, while this component is preferred by some transtibial amputees, it alters the center of gravity in the leg in a way that causes discomfort and malignant gait adaptation that can pull on the leg, causing a pistoning motion that increases shear stress. Keeping the exterior shell unmodified, the interior of the base was optimized through SIMP using multiple element shapes to compare results. The final design maintained all necessary features for integration with the transtibial socket along with the stiffness necessary to support the limb. Using different element shapes, a maximum reduction in weight of 58% was achieved for the most ambitious design that still met stiffness criteria (Sotola et al., 2020).

These optimized sockets and socket components have yet to be manufactured as of this writing. However, it is likely these designs will utilize an additive process to vary wall thickness. Due to the complex geometry generated through SIMP, and the desire to maintain the exterior shell, (Sotola et al., 2020) specifies plans to use SLS to fabricate their socket base in future work.

**Methods**

The methods for creating this proof-of-concept for GSM use in prosthetic socket development began with CT scan data from (Faustini et al., 2006). The original data is no longer available, but the dimensions and subsequent socket design were used to create a surrogate patient with similar metrics to the subject in this research. Images of the original meshed CT scan elements and their surrogate counterparts are shown in Figure 1.

The original patient from (Faustini et al., 2006), was 70 years old, male, 1.82 m tall, and ~90 kg. These same metrics are used in the portion of this research that analyzes socket performance under “patient metrics” with an 800 N force. A noteworthy difference between the CT scan and this surrogate model is the simplification of the residual skeletal structure as one solid piece. This results in direct translation of forces through the skeletal structure rather than through a layer of soft tissue. While not as accurate to the residual limb anatomy, this decision reduces the problem size, as is required by software used in this research. The lack of soft tissue may decrease the accuracy of the stress locations for some stances; however, it appears to increase the overall magnitude and make the stress in stances with large angles more extreme. With this in mind, the decision is made that this is an acceptable model for this proof-of-concept.

A surrogate TSB socket based on the original geometry found in (Faustini et al. 2006) is developed for the surrogate limb model and serves as a method for validating that a simple socket model based on prior SLS sockets is sufficient for both the patient and a chosen load from the International Organization for
Standardization Standard 10328: Prosthetics – Structural testing of lower limb Prostheses (ISO 10328) static proof load 2240 N for case P5 condition 1. Case P5 refers to a heavier patient of up to 125 kg, and condition 1 refers to loading in the portion of the gait where the reaction force is through the heel rather than the toe. In the simulations performed on the socket in this research, the socket is fixed at the distal end and moments from the ground force on the foot of the prosthesis are not considered. Therefore, the required load for a reaction force more aligned with the distal end of the socket better suits this model. It should be noted that in ISO 10328 an ultimate static test load of 4480 N is included for case P5 condition 1 ("International Standard (ISO 10328)", 2016). Due to the nature of this design as a proof-of-concept for GSM in prosthetic socket applications, only the static proof load was targeted for this iteration. The domain for the socket optimization comes from an earlier unsmoothed version of the surrogate socket. Images of both the smooth socket, used for validation of the model, and the domain, used for truss development, are shown in Figure 2. The domain is larger than the surrogate model and lacks curvature. This reduces complexity to meet the available problem size for the initial analysis, while increasing the area for the truss model to create novel support geometry.

Simulations run on both sockets use the following procedure and boundary conditions. All materials are treated as linear. The distal face of the socket is considered fixed, and all forces are applied to the proximal face of the residual skeletal structure, with the force angle determining the stance. Materials are treated as isotropic; in the case of SLS nylon-12, this means using the material properties in the weakest direction for the entirety of the model. Mesh resolution is maximized for problem size constraints in the available software. The surrogate socket is statically analyzed with both linear and quadratic order tetrahedral meshing for comparison. Linear tetrahedral elements are necessary for the optimization process. However, due to unaccounted for bending of elements, linear order elements tend to raise the safety factor in finite element analysis (FEA). Therefore, a comparison between the two was considered to be a necessary study. The domain is analyzed with linear order tetrahedral elements. Mechanical properties for each material in the model are found in Table 1.

Table 1: Material properties for analysis

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s Modulus (Pa)</th>
<th>Poisson’s Ratio</th>
<th>Density (kg/m³)</th>
<th>Source(s) and Relevant Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trabecular Bone</td>
<td>1.48 e+10</td>
<td>0.30</td>
<td>1764</td>
<td>(Morgan et al., 2018; Rho et al., 1993)</td>
</tr>
<tr>
<td>Limb (soft tissue)</td>
<td>2.5 e+05</td>
<td>0.49</td>
<td>1096</td>
<td>(Faustini et al., 2006; Heymsfield et al., 1989)</td>
</tr>
<tr>
<td>Pe-Lite™ Liner</td>
<td>3.8 e+05</td>
<td>0.39</td>
<td>464.54</td>
<td>(Coleman et al., 2004; Rothman, 1962)</td>
</tr>
<tr>
<td>SLS Nylon-12</td>
<td>1.9 e+09</td>
<td>0.39</td>
<td>1020</td>
<td>(ProtoLabs, 2019)</td>
</tr>
</tbody>
</table>

The compressive and tensile yield strengths are assigned as 5 e+07 Pa and 7.41 e+08 Pa, respectively, and are found in the datasheet for sintered nylon-12 PA650 supplied by ProtoLabs® (Maple Plain, MN, USA). These yield strengths are typical of the weakest print direction, perpendicular to the bed of the machine (ProtoLabs, 2019). Data from (Hooreweder & Kruth, 2014)
are used to obtain an approximate S-N curve for sintered nylon-12 in this same direction. Analysis finds both sockets sufficient for supporting residual limb model for the patient load and the ISO 10328 static proof load. The results of the analysis conducted in ANSYS® Mechanical, Release 21.2 (ANSYS, Inc., Canonsburg, PA, USA) are discussed in the findings.

In the development of this prosthetic socket, unnecessary material is removed using the ground structure method of topology optimization (GSM). This method is a numerical approximation of an optimized Michell Structure, or truss, where an optimal truss is one with both minimized volume and compliance. While the truss originally described by Michell exists on a continuum, this approximation results in a finite number of trusses. Topology optimization using GSM treats an existing geometry, referred to as the domain, or ground structure, as an overconnected truss with members connecting the nodes of the meshed domain space. Members deemed unnecessary are removed to create the least volume, stiffest structure for the given load case (Dorn et al., 1964; Hemp, 1973; Michell, 1904; Prager, 1970).

This method is chosen for two primary reasons. First, due to the lack of topological changes to the socket structure over time, there is interest in approaching the loading of a transtibial prosthesis as a problem with structural change inherent to the output. By considering the socket geometry as a redundant truss, the output has already taken a step to further remove itself from increases and decreases in shell wall thickness. Furthermore, it is theorized that if this method is successful, domain spaces further removed from the conventional socket space can be examined to build trusses around the residual limb that more optimally support ambulation or maximize other socket attributes. The second reason for using GSM in this application can be attributed to the extensive body of research reformulating GSM as a linear programming problem, developing closed form solutions, and applying these discoveries to complex domains (Hagishita & Ohsaki, 2009; Hemp, 1973; Rozvany, 1996; Rozvany et al., 1995; Sokół & Rozvany, 2013; Zegard & Paulino, 2014, 2015). These developments have made GSM an excellent candidate for optimization of a complex loading case on a non-orthogonal structure.

Software developed to approach these problems is described in (Zegard & Paulino, 2015). *Ground Structure Based Topology Optimization for Arbitrary 3D Domains*, or GRAND-3 is a MATLAB® implementation of GSM for domains that have holes and concave regions. This implementation expands on previous work for 2D domains that used a similar linear algebraic approach and video-game development techniques to avoid collision with designated holes that engineers often require for hardware and other components (Zegard & Paulino, 2014). GRAND-3 is based on the formulation of the optimal truss problem as a linear programming problem in (Hemp, 1973). The stiffest truss formulation from (Bendsøe et al., 1994), followed by Hemp’s formulation, is summarized below. Note that in Hemp’s formulation, (Bendsøe et al., 1994)’s notation is still used for continuity.

First, examine the loading. Let $\mathbf{q}$ and $\mathbf{p}$ be vectors of all the nodal forces and the member (axial) forces of a truss, respectively. The nodal equilibrium matrix is defined as $\mathbf{B}^T$ and is built from the directional cosines of all the members so that:

$$\mathbf{B}^T \mathbf{q} = \mathbf{p}$$
If the volume of each individual bar is represented by $t_i$, the stiffness matrix $K$ of the truss is given by the equation:

$$ K = \sum_{i=1}^{m} t_i K_i $$

where $K_i$ is the specific stiffness matrix for the $i^{th}$ element. Using $K$ and the displacement vector $\mathbf{u}$, the optimal truss can be found through:

$$ \min_{\mathbf{u}} \mathbf{p}^T \mathbf{u} \\
\text{s.t.} \quad \sum_{i=1}^{m} t_i K_i \mathbf{u} = \mathbf{p}, \\
\sum_{i=1}^{m} t_i = V \quad \text{for } t_i \geq 0, \quad i = 1, \ldots, m $$

If the engineer assigns allowable stresses for the given material, $\sigma_T$ being the allowable tensile stress and $\sigma_C$ being the allowable compressive stress, then $p_i \leq \sigma_T a_i$ and $-p_i \leq \sigma_C a_i$ for all values of $i$.

At this point, optimization function is subject to inequalities. The major innovation comes from (Hemp, 1973) adding slack variables developed from $\mathbf{p}$ to eliminate these inequalities, creating a linear programming problem. These are defined as:

$$ a_i = \frac{s_i^+}{\sigma_T} + \frac{s_i^-}{\sigma_C} \quad \& \quad q_i = s_i^+ - s_i^- $$

where $s_i^+$ and $s_i^-$ are slack variables. This changes the optimization problem to:

$$ \min_{s^+, s^-} V = \mathbf{f}^T \left( \frac{s^+}{\sigma_T} + \frac{s^-}{\sigma_C} \right) $$

$$ \text{s.t.} \quad s_i^+ \geq 0, \quad s_i^- \geq 0 $$

$$ B^T(s_i^+ - s_i^-) = q $$

This is the basic formulation for the optimization found in GRAND-3; further information is available in (Zegard & Paulino, 2015). This formulation and GRAND-3 require certain problem parameters: the deformations must be small, the fixed nodes of the structure must be able to create reaction forces to statically hold the load, and the analysis must be performed for a single load case with linear stress constraints and only elastic deformations.

Immediately an issue arises: the analysis of a prosthetic socket is a dynamic problem, typically analyzing the forces as they vary in the walking gait. To address this issue, multiple static analyses are used in socket development. This approach, used by (Faustini et al., 2006), produced a socket that was successfully clinically tested with the patient who is the basis for the surrogate model used in this research. The methods used in this research are similar to quasi-static modeling but have no implied time step, nor do they examine many stances in ambulation. Instead, three stances are selected. These stances represent the peak loads in the sagittal plane.
heel strike when the foot first descends, standing approximated as vertical, and toe-off when the foot is about to lift into the swing phase of the stance. Freebody diagrams of a full prosthesis in these stances are shown in Figure 3. These diagrams also depict the force $F_a$ applied at the top of the residual skeletal structure in each stance.

Optimal trusses are calculated for each extreme stance using GRAND-3 in MATLAB® R2022a (MathWorks, Natick, MA USA) and a Python® (Python Software Foundation, https://www.python.org/) implementation is written to create a non-uniform rational b-spline (NURBS) representation of the truss using the same primitives as (Zegard & Paulino, 2015) (cylinders representing the beams and spheres representing the nodes) in Rhinoceros® Version 6 and Version 7 (Robert McNeel & Associates, Seattle, WA, USA).

The trusses generated for the three stances are combined in an attempt to create a low volume stiff truss that tolerates all positions of ambulation. The original output from the GRAND-3 optimization creates spherical nodes that have the same diameter as the diameter of the member with the largest cross-sectional area. In the NURBS model generated by the Python® implementation, nodes are generated as spheres with 10% larger radii than the largest member they attach to in order to avoid numerical errors inherent in Boolean operation implementations. Members with a diameter less than 2 mm are increased in size to the 2 mm minimum feature size recommended by ProtoLabs®. Note that SLS with nylon-12 has a minimum feature size of 0.5 mm and can reliably print struts with a 1 mm diameter; however, due to this recommendation and the relatively low mesh density used in this optimization (763 nodes and 2253 elements for the domain), the safety factor provided by the 2 mm minimum diameter members is worth a potential increase beyond the necessary volume for manufacturing. This method requires the researcher to perform Boolean operations on many components of the truss by hand, as the vast majority of components encounter errors when Boolean operations are attempted in the Python® script. This is due to seam intersection. While NURBS geometry was selected in order to remain true to the geometry, the Boolean process involved slight variations in size and position, deviating from the original truss structure. This Boolean process and the alteration to the geometry is the largest

![Figure 3: Freebody Diagrams for a full transtibial prosthesis in three critical positions. Left: heel strike, center: standing (vertical), right: toe-off.](image-url)
disadvantage of using existing NURBS software to create a solid truss. Alternative methods for exporting and attaching GSM geometries more efficiently are discussed in future work.

Once connected, the truss system is combined with the critical components at the proximal and distal end of the socket (the brim and the socket base), and post-processing is executed to create a viable socket model. This process begins with addressing uneven loading of nodes in contact with the residual limb. Loads on the socket interior vary greatly, especially in areas close to the residual skeletal structure. This means that nodes on the socket interior have a range of cross-sectional areas and are not necessarily always in contact with the residual limb as defined in the model. For a successful design, the truss loading needs to be aligned with the static analysis. Furthermore, the primary role of the prosthetic socket is to keep the residual limb stationary relative to the reference frame of the socket. This is a challenge due to the varying node sizes and limited regions of contact. This is addressed by cutting into the nodes with the limb geometry, so the interior evenly contacts the limb. However, this removes volume not only from the nodes but from beams that run along the interior of the socket. Additional volume loss occurs when the file is converted to STL format for printing. The maximum diameter removed from a member due to these changes was determined to be ~2.5 mm. This volume loss is recorded and compensated for in further steps.

Due to aforementioned stress variation, members connected to the same node have large deviations in cross-sectional area. Since node size is based on the largest beam and is further enlarged for Boolean operations, the smallest beam attached to a node approximates a right angle with the node, creating potential stress concentrations. This issue and volume removal problems are resolved by converting the completed truss structure to a subdivision surface model (SubD). This eases the process of smoothing greatly when compared with other mesh methods. Several iterations of quadrangulation and smoothing the truss in Rhinoceros®, followed by mesh repair in Materialise Magics® (Materialise NV, Leuven, Begium), creates a smooth truss model with minimized stress concentrations. The volume loss is compensated for by offsetting the SubD model 1 mm in all directions. Finally, a thin wall is added to the interior creating more points of contact between the socket and the limb and supporting the cut interior nodes and members. A wall that fully compensates for the maximum volume removal is tested, but it is found that the 1 mm offset combined with the 1 mm wall is sufficient for the loading despite the overlap of truss and wall structures.

Once the model is complete, the design is analyzed for both patient and ISO 10328 static proof loads at the three critical stances and at 10% increments between heel strike (0%) and toe-off (100%) to validate that the combined optimization is sound throughout ambulation. While this does not account for dynamic effects in the ambulation process, there has been success in socket design using this methodology and it is considered sufficient for this proof-of-concept (Faustini et al., 2006; Sotola et al., 2020).

**Findings**

Analyses of the surrogate limb used with the surrogate socket and domain indicate that the socket is capable of holding the load when fixed at the socket base for angles of heel strike, standing (vertical), and toe-off. This is the case for both the surrogate patient load (800 N) and
the ISO 10328 static proof load for case P5 condition 1 (2240 N). As a generalization, the domain socket is larger than the surrogate socket model. However, the base of the socket is an exception. It is similar in size and the unsmoothed angles create greater stress concentrations in the area directly proximal to the base. In the final version of this design the brim and base are taken from the surrogate model instead of the domain. While this may protect from stresses in the base, the brim differences have the potential to create inaccuracies within the model. Comparisons of the von-Mises stress of the surrogate socket with quadratic elements, the surrogate socket with linear elements, and the domain are shown in Figure 4 for the ISO 10328 static proof load in the standing (vertical) position.

![Comparison of von-Mises stress on the surrogate socket with quadratic order tetrahedral elements, the surrogate socket with linear order tetrahedral elements, and the domain socket with linear order tetrahedral elements in the vertical (standing) position at the ISO 10328 static proof load. Shown at 1x deformation.](image)

The color scale for each model represents different magnitudes, but stress values in most locations are similar. The primary difference is that the peak stress on the surrogate model is an area of below average stress on the domain. This is a potential issue in the design process and should be examined in the final model analysis to determine if the socket needs further modification.
The domain is sufficient for optimization with the surrogate limb and socket model. Due to large forces at the bottom and a lack of deformation in the initial analysis the bottom, 25% of the socket is considered fixed; no truss is created for this area that is replaced by the base from the surrogate model in the final design. Figure 5 shows the optimal trusses for heel strike, standing, and toe-off, as generated by GRAND-3 and rendered in MATLAB®.

These outputs are imported into Rhinoceros® via Python® with nodes increased by 10% and a minimum cross-sectional area of 2 mm applied to all features. The NURBS truss components modified for assembly and additive manufacturing are shown in Figure 6.
The assembly and post-processing techniques described in the methods section are applied until an acceptable final socket design is created. Images of the truss with the distal and proximal attachments and the final design are shown in Figure 7. The final design is a high-resolution mesh with 691,972 vertices and 1,238,032 faces. The final design has a volume of 6.7681 e-04 m$^3$ compared to the original model volume of 8.2963 e-04 m$^3$, 81.58% of the original volume.

Figure 7: Final design in Rhinoceros®. Top: Truss with distal and proximal components from the surrogate socket, bottom: complete final design with thin wall.

A quadratic tetrahedral mesh with 682,566 nodes and 378,048 is used in the simulations. The thin wall and truss are split into separate components and bonded in ANSYS®. This creates meshable geometries and prevents sliding between the two socket elements in the simulation. The same material properties and surrogate limb model are used. The model is tested for the three critical stances and at 10% increments between heel strike (0%) and toe-off (100%). The safety factors from static analysis and from fatigue analysis (1 e+09 loading cycles) for patient metrics (800 N) and the ISO 10328 static proof load (2240 N) are shown in the charts in Figure 8.
Figure 8: Safety factors for a single and cyclical load (1 e+09 cycles) of the final design in 10% increments from heel strike to toe-off.

The simulation shows that the socket design is acceptable for all stances in static analysis. This is promising but does not necessarily mean the socket will survive dynamic effects. Once fabricated, the socket will be mechanically tested to failure, weak points in the design are noted as they indicate where and how the socket may fail in future testing. The von-Mises stress for heel strike, standing, and toe-off for the ISO 10328 static proof load are shown in Figure 9. Due to the density of meshing on the bars in the truss portion of the design, the simulation results are shown without the elements depicted on the model.
Figure 9: Simulation results showing von-Mises stress on the final design in heel strike, standing (vertical), and toe-off using the ISO 10328 static proof load (2240 N). Peak and notable stress areas are indicated for each stance. Shown at 1x deformation.

In this analysis there are some commonalities in the locations of high von-Mises stress for all three stances, specifically in the anterior lateral corner where the tibia is close to the socket wall and in the posterior of the socket near the proximal end of the solid base. Both regions should be observed in further testing and may indicate areas for design improvement. In the vertical stance, high von-Mises stress is also noted in the posterior of the socket distal to the brim.

**Discussion**

In this research a transtibial prosthetic socket is designed using the ground structure method of topology optimization to replace a portion of the socket geometry. This is achieved through the analysis of a surrogate limb and socket model, leading to design decisions to keep the distal and proximal ends of the socket solid. Optimization is run using GRAND-3 (Zegard & Paulino, 2015) on three stances with the surrogate model, each representing peak forces along one of the axes of the sagittal plane. A Python® implementation is written to import the three truss geometries into Rhinoceros® as NURBS primitives where they are assembled into one truss and combined with the distal and proximal components from the surrogate socket. The model is then post-processed, and the interior is modified to evenly contact the limb geometry. Finally, a 1 mm wall is added to
increase the contact area between the limb and the socket interior. Static simulations at 10% increments between heel strike and toe-off are run and the design is found to exceed all required metrics at all stances. The thin wall and truss design may reduce the impact of temperature, which is correlated to wall thickness; decrease shear stress through reduction in socket weight; and provide socket wall compliance, mitigating discomfort due to diurnal volumetric fluctuations (Ghoseiri & Safari, 2014; Pirouzi et al., 2014).

While this design is successful, there is future work that needs to be done to create a prosthetic socket fit for patient use with this method. Perhaps most critical to the viability of the process is to employ a different approach to the assembly of the truss. For printing, the truss does not necessarily need to be one solid model. However, in order to run simulations, this is a necessary step. While NURBS models accurately represent the geometry, the process of applying Boolean operations by hand is too time consuming for multiple socket iterations. A mesh output could be used with more advanced software such as Materialise Magics© to correct intersecting facets in an STL representation of truss primitives. A voxel-based model could also be used as Boolean operations are more successful with voxels. While both these outputs change the geometry, similar tactics employed in post-processing could be applied to replace lost volume. Zegard, the developer of GRAND-3, recommends either using primitives from in X3D format for its simplicity or outputting STL approximations of the tessellated surfaces for GSM export procedure (Zegard & Paulino, 2016).

The method for post-processing using a SubD model is successful and is recommended for both its ease and the resulting quality of the final model. Additions to the primitive outputs, for example cone primitives, could be added to the model to decrease the stress concentrations where smaller bars are attached to large nodes. This could further simplify smoothing portion of the post processing procedure.

Future work should focus on exploring dynamics. The results suggest that the socket is adequate for all stances of the walking gait, but without dynamic testing it is unclear if this method produces a socket that is truly optimal for ambulation. The results indicate that applying GSM for static load cases at loading extremes may be a good starting point for the truss of a complex dynamic design problem. Further meshing studies should be conducted to determine the optimal mesh size for analysis and optimization. Furthermore, applying denser meshing in areas of stress concentrations on the surrogate model could produce better results without creating a prohibitively dense mesh for the entirety of the domain. Different domain spaces could also be used to further remove material from the original geometry and determine if a different over all structure could be used, e.g., a truss that supports a thin wall entirely from the lateral side of the limb to avoid interference with the user’s other leg. Nevertheless, this first proof-of-concept illustrates that socket geometry can be adapted and optimized. Further exploration into optimal structures for stiffness, patient comfort, and other target metrics can and should be conducted to determine how AM and topology optimization can impact the stagnant design of transtibial prosthetic sockets.
References


