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EXPERIMENTAL AND NUMERICAL ANALYSIS OF LATTICE STRUCTURES WITH DIFFERENT HETEROGENEITIES

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Abstract

Lattice structures with optimized material distributions can achieve unique mechanical properties such as high stiffness-to-weight ratio. However, the numerical analysis of the mechanical properties of heterogeneous lattice structures is challenging. In this research, three numerical approaches, including the beam element model, tetrahedral element model, and two-stage homogenization model, were used to predict the stiffness of lattice structures with different heterogeneities. Compression tests were conducted to evaluate the accuracy of the simulation results of each numerical approach. It was found that the accuracy of the numerical model varies with the increasing of heterogeneities. The beam element model significantly underestimated the stiffness. The tetrahedral element model is the most accurate, but the computational cost is extremely higher than others. The results also indicated that, although the homogenization-based numerical model can substantially reduce the computational cost, the accuracy can be compromised due to the heterogeneity of lattice structures.

Keywords: Additive Manufacturing, Finite Element Analysis, Heterogeneous Structure, Lattice Structure

Introduction

Lattice structures in additive manufacturing refer to structures with interconnected struts and nodes in a three-dimensional (3D) space [1]. The manufacturing, design, optimization, and simulation of lattice structures have been studied for over a decade. The lattice structure can be classified into homogeneous and heterogeneous structures based on homogeneity [2], as shown in Figure 1. If the relative density of the structure is uniform across the whole space, it is called homogeneous lattices. If the relative density has a gradient in the space, it is called heterogeneous lattices. Because the material distribution in the lattice structures can be optimized according to the functional requirements, heterogeneous lattice structures can exhibit better mechanical properties than homogeneous lattice structures. Most of the design and optimization methods in the literature for lattice structures are simulation-based. Finite element analysis (FEA) has been widely used to predict the mechanical properties of lattice structures. The FEA methods can be categorized into three types: beam element model, solid element model, and homogenization model.

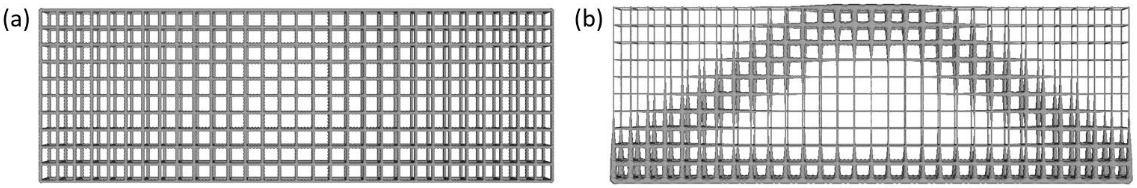

Figure 1 An example of (a) homogeneous lattice and (b) heterogeneous lattice.

Beam elements are prevalently used in FEA of lattice structures because of their low computational costs compared to solid elements [3]. Each lattice strut is meshed with several beam elements. The total degrees of freedom of the beam element model are much smaller than that of the solid element model. Timoshenko beam elements are widely used because the lattice struts are usually stout beams, in which shear deformation is important [4, 5]. However, there are some limitations in the beam element model. It is reported that the beam element cannot accurately model the joint of the lattice structure. It tends to underestimate the stiffness of the whole structure [6]. The beam element needs to be modified to consider the joint stiffening effect in the lattice structure [7]. Furthermore, it is known that the as-built lattice structure varies from the designed geometries [8]. The beam element model cannot reflect the actual geometry of the lattice structure. An alternative way is to vary the diameters of the beam elements according to the as-built strut thickness distribution [9]. Campoli et al. [10] used scanning electron microscopy and Gaussian distribution to determine the diameter of each beam. Then, the FE model was simulated several times to get the mean and standard deviation of the mechanical properties which means the lattice structure may have a range of mechanical properties due to the structural irregularity.

In solid element models, tetrahedral [11] elements are mostly used to model the lattice structure. Because the element size should be smaller than the strut thickness to obtain a high-quality mesh, the number of elements in lattice structures can be extremely large. Therefore, the computational cost of the solid element model is significantly higher than the beam element model. Because solid elements can accurately model the geometry of the joint, the accuracy of the solid element model is higher than the beam element model, especially when the relative density of the lattice structure is high [12]. This is because the joints take more volume ratio, and the struts are stout when the relative density is high. Furthermore, solid elements can be directly built on the X-ray tomography model. The geometrical discrepancy caused by the unmelted powder, defects, and manufacturing errors can be accurately captured by this method. It can more accurately predict the mechanical properties influenced by the poor build quality of AM processes [13].

The homogenization model can also be called a two-step FEA model. In the first step, the unit cell in the lattice structure is considered a representative volume element (RVE). The homogenization method is used to calculate the equivalent property of the RVE. In the second step, the lattice structure is considered a bulky part and meshed with hexahedral elements [14, 15]. Each hexahedral element represents one unit cell in the lattice structure. Therefore, the total number of elements in the homogenization model is very small. The advantage of the homogenization model is that it can dramatically reduce the computational cost if the equivalent property of the lattice cell can be found in a database [16]. However, the most obvious limitation of the homogenization model is that it can only work with periodic lattice structures. For randomized lattice structures, the homogenization method is not able to get the equivalent properties of the RVE.

Compared to homogeneous lattice structures, heterogeneous ones bring more challenges to the simulation model. For the beam element model, the joints in the heterogeneous lattice structure are all different. It is difficult to study the mechanical behaviors of all the joints and incorporate them into the beam element model. For the solid element model, the mesh size should be smaller than the minimum thickness of the strut, which will further increase the number of elements and the computational cost. For the homogenization model, the periodic boundary conditions are required for the RVE. It is originally proposed for homogeneous lattices only. Although the homogenization method has been used to model heterogeneous lattice structures [17], the accuracy of the simulation is still unknown. To figure out the accuracy and efficiency of different models in the simulation of heterogeneous lattice structures, in this research, the beam element model, solid element model, and homogenization model were used to simulate the lattice structure with both uniform and non-uniform density distributions. Experiments were conducted to evaluate the simulation models. The results were discussed to explain the discrepancy between the simulation results and the experimental results. Finally, conclusions were wrapped up and future research perspectives were pointed out.

Methodology

2.1 Design of the lattice structures

To study the effect of the heterogeneity on the simulation models, the X-shape lattice structures were generated with the same overall relative density but different heterogeneities. Four types of structures were generated, and the parameters are shown in Table 1. The overall relative density of the lattice structure was set to 20%. The relative density of each unit cell was randomly generated. The upper and lower bounds of the unit cell were different in the lattice structures. L-1 was designed as a homogeneous lattice structure as the relative density of all the unit cells was 20%. L-4 was designed as the lattice structure with the highest heterogeneity.

Table 1 The design parameters for the lattice structures studied in this research.

Name	Overall relative density	Maximum relative density	Minimum relative density	Unit cell size (mm)	Number of unit cells
L-1	20%	20%	20%	5	6
L-2		25%	15%		
L-3		30%	10%		
L-4		35%	5%		

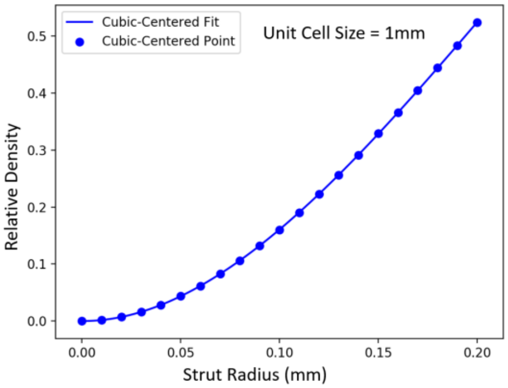

Figure 2 Relationship between the relative density and strut diameters.

A group of lattice unit cells was generated with different radii. The relative density of each unit cell was measured. The relationship between the strut radius and the relative density was obtained by interpolation, as shown in Figure 2. Then, the randomly generated relative density of each unit cell was converted to the radius of the lattice strut. Finally, the geometrical models of four lattice structures were generated, as shown in Figure 3(a). The distribution of the radius of each unit cell is shown in Figure 3(b). L-1 has a uniform relative density, so the radius of unit cells is a constant. L-2 has a radius distribution from 0.48 mm to 0.64 mm. L-3 has a radius of distribution from 0.39 mm to 0.71 mm. L-4 has the maximum heterogeneity. The radius is from 0.28 mm to 0.78 mm. All the radii were randomly generated. The overall relative density of all the lattice structures is kept at 20%.

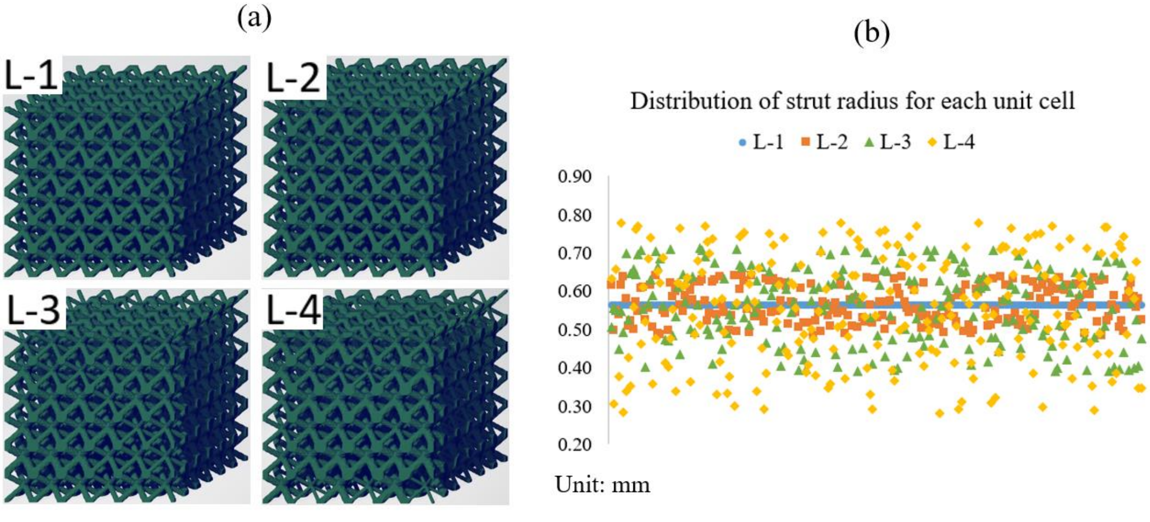

Figure 3 (a) the geometrical model of the designed lattice structures, and (b) the radius distribution of the unit cells in the lattice structures.

2.2 Simulation methods

Three FEA methods were used to simulate the design lattice structures: beam element model, solid element model, and homogenization model. In the beam element model, the diameter of the beam was assigned individually. Two types of beam element models were studied in this research. One did not consider the joint influence of the lattice structure, as shown in Figure 4(a). The other increased the diameter of the beam element at the joint of the structure to consider the joint stiffening effect, as shown in Figure 4(b). In the solid element model, the lattice structure was meshed with coarse and fine tetrahedral elements, as shown in Figure 4(c). nTopology was used to generate the solid mesh model. The fine mesh use edge length 0.25mm and growth rate 2. The coarse mesh use edge length 1mm and growth rate 2. In the homogenization model, the relationship between the equivalent material properties and the relative densities of the X-shape lattice structure was obtained by the homogenization method [18], as shown in Figure 5. Because the unit cell is symmetric, there are only three unknowns (C11, C12, C44) in the homogenized constitutive matrix. Then, the lattice structure was considered as a bulky part meshed by hexahedral elements. Each hexahedral element represents one unit cell. The material property assigned to the hexahedral element is the equivalent material property of the corresponding unit cell, as shown in Figure 4(d). A displacement load along Z-axis was applied on the top of the lattice structure, and the bottom of the lattice structure was fixed. The stiffness of the structure was predicted by the simulation model. It is defined by the ratio of the total reaction force along the Z-axis to the displacement of the top of the lattice. The simulation results would then be evaluated by experimental results.

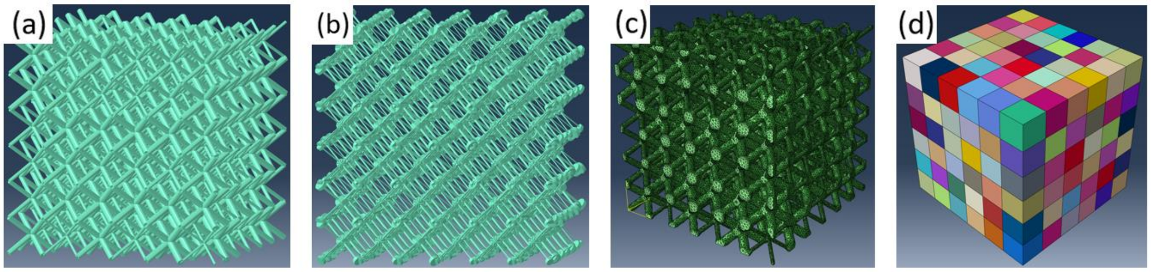

Figure 4 Example of the simulation models used in this research, (a) beam element model without considering the joint influence. (b) beam element model with increased beam diameter at the joint areas, (c) solid element model with tetrahedral elements, and (d) homogenization model with hexahedral elements, of which the material properties are assigned by the equivalent material properties of the corresponding unit cell.

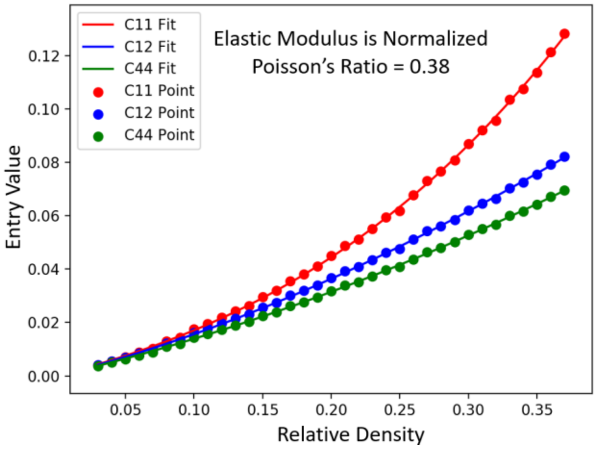

Figure 5 The entries in the homogenized constitutive matrix vs. relative density of X-shape unit cell.

2.3 Experiments

The lattice structures were manufactured by a digital light processing (DLP) printer. The material is FAST abs-like resin provided by Siraya Tech. Each lattice structure was made by three times. A tensile specimen was designed to get the elastic modulus of the bulk material which was used in the simulation model. It was found that the elastic modulus of the material is 846 ± 49 MPa. Compression tests were conducted to get the force-displacement curve of the designed lattice structure, as shown in Figure 6. MTS Insight 30 was used to do the compression test with a 30 kN load cell. The speed of the test was set to 1 mm/min.

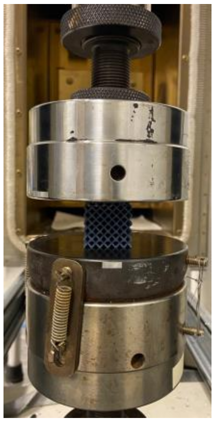

Figure 6 Compression test of the lattice structures.

Results

3.1 Experimental results

The results of the compression test are shown in Figure 7. It can be found from Figure 7(a) that the slope of the curve in the linear region varies among the lattice structures. The stiffness of the lattice structure is defined by the force increment divided by the displacement increment. The L-3 has the maximum stiffness in the linear region, but its force dropped earlier than the other structures. The overall force-displacement curve shown in Figure 6(b) indicates that the force of L-3 in the plateau region is the smallest among all the samples. To understand the relationship between the stiffness and the heterogeneity, the bar plot of the stiffness is shown in Figure 8. The height of the bar shows the average stiffness of the three samples. The error bar indicates the standard deviation of the samples. It was found that the homogeneous lattice structure L-1 has the minimum standard deviation of stiffness. The stiffness slightly dropped from 419 N/mm to 412 N/mm when the relative density changed from 20% (L-1 sample) to a random distribution between 15% and 25% (L-2 sample). However, the stiffness of the structure dramatically increased to 547 N/mm when the relative density distribution was between 10% and 30% (L-3 sample). Then, the stiffness drops again to 532 N/mm when the relative density distribution is between 5% and 35% (L-4 sample). From samples L-1 to L-4, the heterogeneity of the lattice structure was increased. But the stiffness didn't increase monotonously with the heterogeneity. The experimental results will be used to evaluate the accuracy of different simulation models studied in this research.

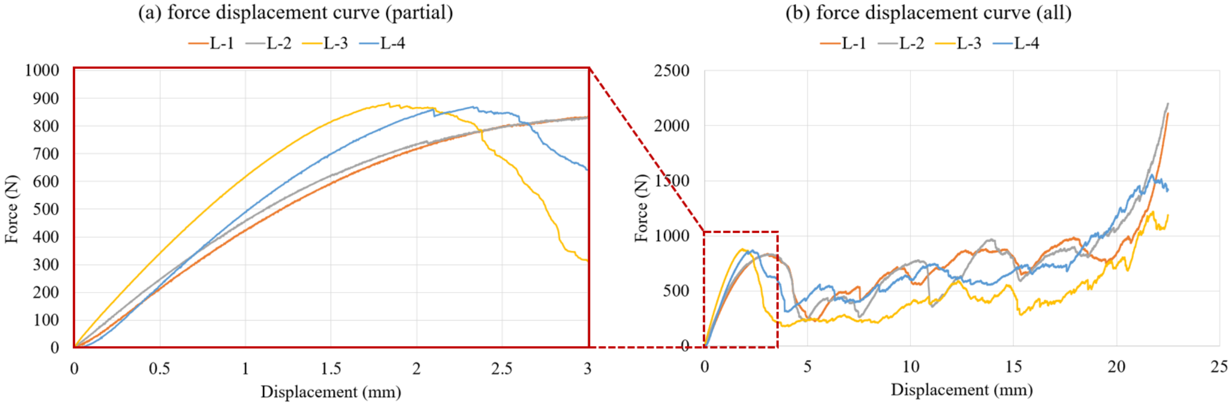

Figure 7 The force-displacement curve of the lattice structures under compression test.

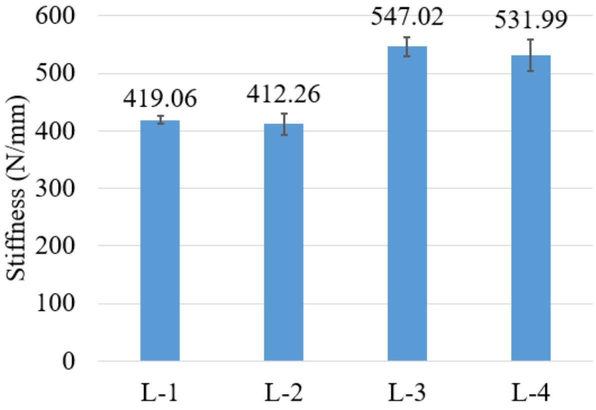

Figure 8 The stiffness of the lattice structures with different heterogeneities.

3.2 Simulation results

The computational costs of the simulation models are related to the number of elements in the model. Table 2 shows the number of elements in each model. It can be found that the homogenization model has the minimum number of elements, which is equal to the number of unit cells in the lattice structure. The beam element model meshed each lattice strut with one element. Thus, the total number of struts in the lattice structure determined the number of beam elements. The joint stiffening effect was considered in a modified beam element model. In this model, each lattice structure was meshed by 10 beam elements. The radius of the beam elements at both ends of the strut was increased 10 times to compensate for the underestimation of the joint stiffness. The total number of the elements was equal to 10 times the number of the strut. Furthermore, two tetrahedral element models were used in this research. The mesh sizes of these two models are different. In the coarse model, the number of elements is around 200,000. In the fine model, the number of elements is around 1 million.

Table 2 Number of elements in each simulation model.

	L-1	L-2	L-3	L-4
Beam Element	1728	1728	1728	1728
Beam Element (Joint stiffened)	17280	17280	17280	17280
Tetrahedral Element (Coarse)	191575	210952	232182	225617
Tetrahedral Element (Fine)	996840	1143053	1166370	1234284
Homogenization Method	216	216	216	216

Table 3 The simulation result of the stiffness of lattice structures with different heterogeneities (unit: N/mm).

	L-1	L-2	L-3	L-4
Beam Element	180.62	180.30	177.85	180.19
Beam Element (Joint stiffened)	261.69	260.68	255.56	256.85
Tetrahedral Element (Coarse)	582.21	586.91	596.31	616.64
Tetrahedral Element (Fine)	447.42	424.44	468.59	489.27
Homogenization Method	382.10	389.08	400.60	423.04

The simulation results are shown in Table 3. It was found that the results in different simulation models vary a lot. The beam element model without the consideration of the joint gave the minimum stiffness prediction. The coarse tetrahedral element model obtained the maximum stiffness prediction. To evaluate the accuracy of the simulation models, the simulation results were compared to experimental results, as shown in Figure 9. The actual stiffness of the structure is around 3 times the prediction by the beam element model without the consideration of the joint. Even if the size of the joint in the beam element model is increased 10 times, the predicted stiffness is only half of the experimental result. Therefore, the accuracy of the beam element model is the lowest among all the simulation models.

The tetrahedral element models can more accurately predict the stiffness of the structure. However, the coarse mesh model would overestimate the stiffness of the structure by around 30%. Although the computational cost of the fine tetrahedral element model is significantly higher than other simulation models. It can most accurately predict the stiffness of lattice structures among all the simulation models. The homogenization model also underestimated the stiffness of lattice structures. The difference between the homogenization result and the experimental result is around 10%, which is the second most accurate prediction among all the simulation models. It should be noted that the homogenization model has the lowest computational cost.

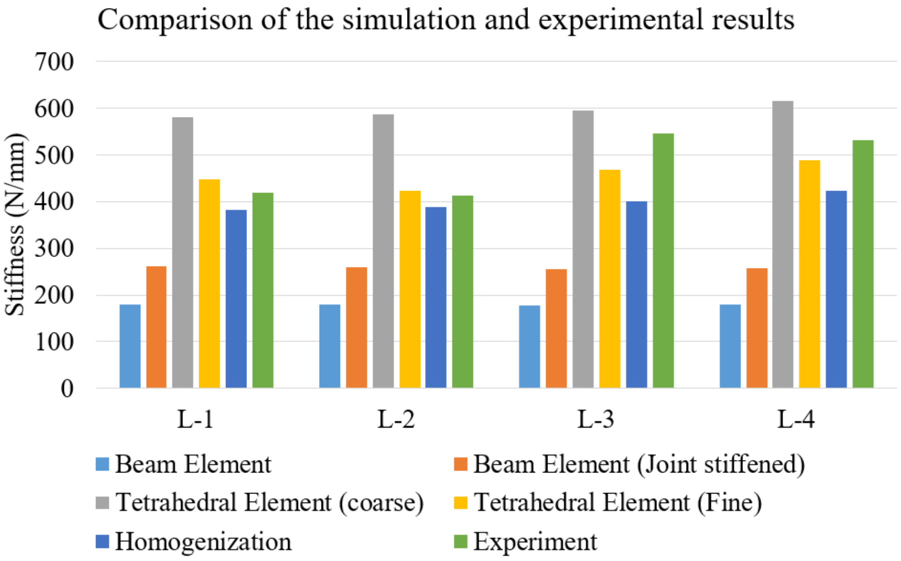

Figure 9 The comparison of stiffness predicted by different simulation models to the experimental result.

Discussion

It is found from the experimental results that the lattice structures with the same weight but different heterogeneities exhibit different mechanical properties. The stiffness changes in the beam element model with joint stiffened, fine tetrahedral element model, homogenization model, and the experimental results are shown in Figure 10. The green line represents the change in the experimental result. From L-1 to L-4, the heterogeneity of the lattice structure was increasing. The stiffness slightly dropped from 419 N/mm to 412 N/mm and then increased to 547 N/mm dramatically. Because the relative density distribution is randomly generated, the change of the mechanical property is also random. The experimental result is a benchmark to evaluate the simulation models.

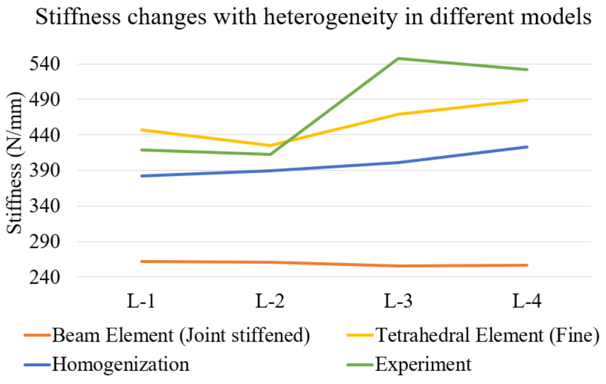

Figure 10 The changes in the stiffness of lattice structures with different heterogeneities in the simulation models and the experimental results

The result of the beam element model didn't change with the increase in heterogeneity. The maximum change of the stiffness in the beam element model is less than 2%. It was also found that the stiffness of L-3 predicted by the beam element model had the minimum value in the four structures. However, the experimental result showed that the stiffness of L-3 is the highest in all the structures, which contradicts the beam element model. It can be concluded that the beam element model cannot capture the change of stiffness with the change of heterogeneity. It also predicted the most inaccurate result. It should be noted that the lattice structure studied in this research is a bending dominant type. The joint influence on the mechanical properties of the bending dominant lattice structure is significant. The beam element cannot accurately model bending dominant lattice structures without precisely considering the joint influence. However, the joints in the heterogeneous lattice structure are not consistent. The joint geometry is determined by the strut thickness connected to the joint. A model that can capture the joint influence in the heterogeneous lattice structure is needed to improve the accuracy of the beam element model.

The stiffness predicted by the tetrahedral element model changes the most among the three simulation models shown in Figure 10. It is the only simulation model that predicted a lower stiffness of L-2 than that of L-1. Then, the stiffness predicted by the tetrahedral element model increased when the heterogeneity increased from L-2 to L-3. It means that the tetrahedral element model can indicate the stiffness change with the heterogeneity change. However, the predicted stiffness kept increasing when the heterogeneity increased to L-4, which is different from the experimental result. Although there is little discrepancy between the experimental results and the tetrahedral element prediction, it is the most accurate one among all the simulation models. One of the reasons is that the tetrahedral element considers the lattice structure as a general solid part. There are no other assumptions like the beam element model and the homogenization model. But the complex geometry of the lattice structure leads to a high computational cost of the tetrahedral element model. It also has a high requirement for mesh quality. A coarse mesh model may overestimate the stiffness of the lattice structure. Mesh sensitivity analysis is needed to avoid the inaccuracy caused by the mesh quality.

Because the homogenized material property of the lattice unit cell is obtained from a database. The homogenization model has the lowest computation cost among all the simulation models. It was found in Figure 10 that the stiffness prediction of the homogenization model increased with the increase of the heterogeneity. It didn't reflect the actual stiffness change with the heterogeneity in the experimental result. The possible reason is that, in the homogenization method, the material property is obtained based on a periodic boundary condition on the lattice unit cell. It was developed for homogeneous lattice structures. However, this assumption is not valid for heterogeneous lattice structures. The two-step homogenization FEA model gets inaccurate with the increase of heterogeneity. Nevertheless, the accuracy of the homogenization model is much higher than the beam element model, and the computational cost is significantly lower than the tetrahedral element model. Therefore, this method has been used to model heterogeneous materials and structures in the literature. Future research is needed to modify the homogenization model and increase the accuracy for highly heterogeneous structures.

Heterogeneous lattice structures can be used in many applications that require a high stiffness-to-weight ratio, energy absorption rate, or specific stiffness distribution. Therefore, an accurate simulation model for heterogeneous lattice structures is important to the design process. It is found from this research that there are still gaps between existing strategies and the actual mechanical properties of heterogeneous lattice structures. The computational cost of the fine tetrahedral element model is too high. The homogenization model needs improvements to deal with heterogeneous problems. Furthermore, nonlinear analysis of heterogeneous lattice structures is more challenging for simulation models. It will dramatically increase the computational cost of the tetrahedral element model. Currently, the homogenization model cannot deal with nonlinear problems. The nonlinear mechanical property of the lattice unit cell should be obtained in the first step. The feasibility of the two-step FEA model needs to be validated in future research.

Conclusion

In this research, the stiffness of the lattice structures with different heterogeneities was studied. 3 types of simulation models were used to predict the stiffness. Compression tests were conducted to evaluate the accuracy of each simulation model. It was found from the experimental result that the stiffness does change monotonously with the heterogeneities. It was also found that the results from different simulation models vary a lot from each other. The beam element model has the lowest stiffness prediction, mainly because the joint stiffness is underestimated. Even if the radius of the beam elements at the joint increased 10 times, the prediction was still half of the experimental result. The tetrahedral element model has the highest accuracy, but the computational cost is significantly higher than other simulation models. Furthermore, it is sensitive to the mesh quality. If the mesh is coarse, the tetrahedral element model will overestimate the stiffness by 30%. The homogenization model has a balance between accuracy and computational cost. The stiffness prediction is about 10%-20% lower than the experimental value.

The influence of heterogeneity on the accuracy of the simulation model was also studied. It was found that the beam element model cannot reflect the stiffness change caused by the heterogeneity of the lattice structure. The homogenization model results showed that the stiffness increased with the increase of heterogeneity. However, it was inconsistent with the experimental result. The tetrahedral element model can most accurately predict the stiffness change caused by the increased heterogeneity. But it also lost the accuracy when the heterogeneity kept increasing. Overall, all the simulation models cannot accurately predict the stiffness of lattice structures with high heterogeneity. Future research can focus on the improvement of the homogenization model because of the low computational cost and relatively high accuracy. The nonlinear analysis should also be studied for the homogenization method.

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