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Reconfigurable Lattice of Auxetic Backlash Structures for Shape-Changing Surfaces

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Abstract

Shape-changing structures are used in robotics, wearable technology, and complex dynamic systems. Many of these structures rely on deformation of subcomponents to accomplish conformation to a desired shape within a configuration space. In this work, we present a method for designing many-linked mechanisms for building non-stressed, compliant structures. These lattice structures were printed on a set of FFF systems and demonstrate low cost of complexity with respect to conventional manufacturing methods. Additionally, we propose a method to determine the configuration space and kinematics of a flexible structure with variable degrees of freedom, useful for shape-changing robots. Using this framework, we show examples of shape-changing structures that can be constructed with a Reconfigurable Lattice of Auxetic Backlash Structures (RLABS).

Introduction

Shape-changing structures are used in many fields of engineering, such as robotics, wearable technology, deployable mechanisms, and manufacturing. Structures engineered to modulate their surface and volume are capable of outperforming non-shape-changing structures on strength-to-weight ratio, aerodynamic drag, and stowage size among other metrics. Shape-changing structures are critical to advancing dynamic system performance [1]. Many shape-changing structures are composed of a unit cell repeated along a grid or lattice for stability and uniformity. By selectively deforming unit cells across a lattice, we can drive conformational changes in the structure. Conformal mapping refers to a function that preserves angles locally between structures in the complex plane. Specifically, it is a bijective and holomorphic function that maintains the shape of infinitesimally small figures. Conformal mappings are used in complex analysis, fluid dynamics, and engineering to simplify problems by transforming complex geometries into simpler ones while retaining essential features. Conformational changes can be driven in a structure through the backlash between unit cells of the structure. Backlash is a clearance or gap in a joint of a mechanism. As backlash is increased, the range of free motion for a joint will increase. By linking cells with a relatively high amount of backlash, we can construct lattices of the unit cell mechanism with short ranges of effect in deformation. The distance in a chain of joints under which a fixed cell affects its neighbors can be called the die-off distance. By varying the relative dilation of auxetic cells in a lattice separated by a number of cells greater than or equal to the die-off distance, we can construct shape-changing structures in a structural configuration space that deform along a conformal mapping.

As backlash in joints of a mechanical metamaterial increases, the configuration space of the structure grows while die-off distance shrinks. This works examines these fundamental trade-offs. The configuration space (CS) is a mathematical construct representing all possible states or positions of a system from the generalized coordinates. Each point in this space corresponds to a unique arrangement of the system components. The CS is used to describe and analyze potential states and movements of dynamic systems. As the CS of the lattice grows, a greater range of geometric states can be set. The CS of the lattice can be described by a homeomorphism and is a disconnected space for each discrete cell, which as a whole maintains a connected space. By driving positional shifts of cells within the reach of the mechanistic backlash, the structure is unstressed throughout the CS. For certain types of shape-changing structures, shifts between geometric states within the CS can be mathematically modeled by a conformal map [2]. This mapping preserves angles between points and the shapes of relatively small figures, but not their size or curvature. Through engineering a structure made of mechanisms containing significant backlash to maintain a specific conformal mapping between regions of its CS, we can develop an unstressed, shape-changing system. When a user reconfigures the mapping function for the unit cells of the structure through selective locking, we allow for the entire structure to be reconfigurable. This structure can be locked into place by restricting the degrees of freedom (DoF) present in order to create a static structure with zero degrees of freedom remaining. An RLABS system's cells can be characterized as 'locked' or 'free' respectively. A locked cell has been set by a fixture to remain at a set angle, resulting in a constant dilation factor. A free cell has no constraint on angle explicitly, but its range of rotation (RoR) may be restrained with respect to the fixed angle of a nearby locked cell. The cells are individually contractible within a range determined by the angles of neighboring cells and their individual CS can be described by a 3D connected space.

A lattice structure consisting of relatively high-backlash, auxetic unit cells can maintain a relatively large, unstressed CS. This system can be considered a Reconfigurable Lattice of Auxetic, Backlash Structures (RLABS). The RLABS shown in this work maintain a variable degree of freedom due to backlash between joints. These many-linked structures are capable of maintaining a wide range of shapes and maintain a Jacobian with many solutions across their CS due to the variability of the joints in series and parallel arrangements. Such systems typically have a relatively more voluminous CS than serial mechanisms and it is often necessary to use a computer to calculate their forward kinematics. As the CS for a dynamic structure grows, the range of control in manipulating the state matrix that determines the control of the entire body increases. As the complexity of a dynamic structure grows, the shape-changing needs of the structure may become so complex that serialized methods are insufficient. There may be many solutions to approximate a target shape within error bounds. If the entire area of a control surface could be modulated, the range of control offered by the dynamic structure would be increased. **Figure 1** shows a test model of an RLABS, the auxetic nature of the lattice, a conformational shift in unit cell dilation over the structure, and a range of shapes possible with this structure.

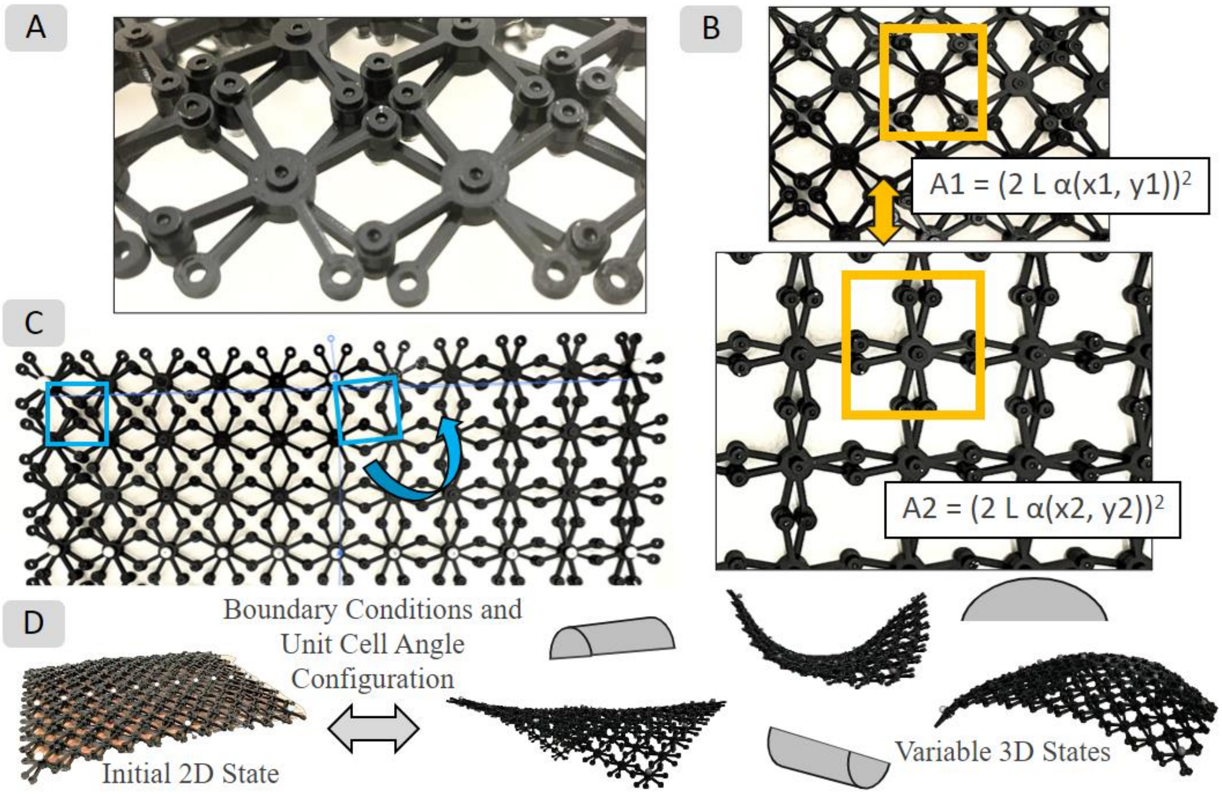

Figure 1. (A) Unit cells of the RLABS structure, repeating pattern outlined in blue. (B) The auxetic cells dilate across the lattice. Dilatation is derived from the rotating squares (RS) mechanism and accommodates loading without shear stress of the unit cells. (C) As grid lines rotate, the angles between them remain fixed. This angle-preserving behavior is a conformal change and arises due to dilation of the RS mechanism, the α(x,y) factor.

Geometry, topology, and actuation drive the development of these auxetic lattices for various domains. The remainder of this paper is organized as follows. The background reviews prior and related work on auxetic lattices. Modeling describes a set of geometric models, helping to understand the relationship between cell shape and CS. Experimentation characterizes the shapes that can be achieved via inflation and boundary conditions in terms of curvature and conformal deformations within a bounded scale factor, α(x, y). Design describes the design of our variable dilation auxetic metamaterial, realized as a rotating squares linkage network. We show how to locally configure expansion by varying the angle and geometry of linkage elements from the flat lattice state. Discussion presents two case studies and physical prototypes that highlight potential applications across domains, from manufacturing forms to airfoil frames. We conclude with a discussion on the limitations of this approach and identify opportunities for future work.

Background

This work demonstrates an auxetic metamaterial. Metamaterials are materials with a property that is occurs rarely in nature. Mechanical metamaterials use patterns of holes, folds, and other transformations to maintain nonlinear, configurable, programmable, or other behaviors [3]. When most materials are placed under compression, they deform in a manner that does not follow a conformal mapping because they expand in the direction orthogonal to the applied stress. Unlike most materials, auxetic materials or structures can deform along a conformal mapping since they maintain a negative Poisson's ratio. Under unidirectional tension, auxetic structures expand in the direction orthogonal to the applied force.

In conventional shape-changing structures made from standard materials, such as a scissor lift or corrugated tube, the DoF present in the structure is constant. There are a set number of joints in the structure that are free to move to a defined extent. The individual DoF are characterized to model the kinematics of the dynamic system. Engineers have discovered that useful systems can be intentionally designed with non-constant or underactuated DoF. For example, certain species of birds have underactuated wings as well as some models of glider aircraft. Underactuation describes mechanical systems that cannot be commanded to follow arbitrary trajectories in their configuration space. This condition can occur for a few reasons, such as when the system is externally blocked by another body or when the system has a lower number of actuators than degrees of freedom.

Auxetic structures exhibit a dilation response that can be geometrically varied in space to represent a conformal map [4]. This allows us to generate structures that change shape between arrangement to arrangement along a 2D configuration space as a stress is applied. This was accomplished by fabricating an auxetic material with built-in spatial variation of cell size, thereby limiting the range of expansion when pressure was applied equally over the structure. Others have furthered this work, showing that the conformal mapping along a CS can be provided by changes in the joints of an auxetic structure. By varying the stiffness relationship to deformation of joints across a structure, we can spatially limit the range of dilation and scaling [5]. The total range of dilation is described by an alpha factor that varies cell by cell, $\alpha(x, y)$, by the x and y position in the lattice. Where *x* is the row and *y* is the column of a cell. The alpha factor is also used to quantify the relative dilation of cells in this work.

These prior methods are limited to 2D variation in structural shape across the CS before mapping onto a 3D surface. Beyond 2D variation upon manufacturing, there is a demand for structures that can achieve many shapes in 3D after fabrication for manufacturing. This has been explored in the high-tech interfaces space in recent years [6]. The rotating squares (RS) auxetic pattern consists of an array of square units connected at their corners. When the material is stretched, the squares rotate around their vertices, causing the overall structure to expand laterally. This pattern is particularly effective at distributing stress and maintaining structural integrity. Another common auxetic pattern is the re-entrant honeycomb pattern. It consists of a honeycomb lattice where the cells have inward-bowed (re-entrant) angles. When the material is stretched, these angles open up, causing the structure to expand laterally. This pattern provides high energy absorption and increased toughness compared to the RS pattern, but involves a greater number of features.

By underactuating a lattice of auxetic cells, we can selectively vary a conformal mapping in space with fewer actuators than unit cells. Actuators can be used to drive a sphere of auxetic cells in a lattice such as in single DoF expanding spheres [7]. These individual spheres were shown to generate multiple DoF structures. Auxetic materials are also being used in the mechanical intelligence space. In a material's internal structure, we can discretize small volumes of the material and define the motion of these small volumes relative the motion of adjacent volumes. We may use discretization to consider the stresses and strains of individual elements in an overall volume. Finite element methods in modern analysis require a high number of discretized pieces. This necessitates the use of a computational engine to determine the kinematics and statics of a highly discretized volume.

To address challenges associated with fatiguing and stress in conventional auxetic structures, we have developed a mechanically locked, shape-changing framework that can be composed into independent, modular cells. The RLABS system mimics the expansion found in nature by combining servos with auxetic materials. Auxetic materials are cellular materials that expand across all perpendicular directions at the same time. Each actuator is made up of an auxetic cell and an internal core that translates a servo's rotational movements into the shell's volumetric expansion. In contrast to prior work, this effort seeks to provide an auxetic lattice model where the convergence of the target surface is not constrained to a single conformal mapping function built into the structure at fabrication. Instead, we generalize auxetic lattices for reconfigurable surfaces with a wide range of conformal mappings depending on the configuration applied to the dilation factor and boundary conditions across the lattice.

Modeling and Experimental Methods

To determine the configuration space of the RLABS and compare it to the configuration space of prior work in auxetic lattices, the range of rotation (RoR) and range of motion (RoM) for each RLABS cell must be determined. The RoR is the connected range of angles that a cell can be set and the RoM is the connected 3D space that a cell center can be located. A computer program was developed to determine the RoR and RoM for a cell in a 1D linkage or 2D lattice. In the program, the RoR for each cell in a lattice was tracked and calculated recursively. From the simulated auxetic bilayer of RS cells, we determined the DoF of a structure as well as each cell's die-off distance. The die-off distance varies as a function of the geometry of the cells as well as the relative position of the cells in all three spatial dimensions. Cells beyond the die-off distance of a locked cell are fully free, while some cells inside the die-off distance are partially free. When the RoR of a free cell was determined to be below a set threshold, that cell no longer contributed to the total DoF of the lattice. The simulated geometry and results are shown for cells of L=35 mm and RoR=80^{o} in **Figure 2**. Limits on the geometry of cells and the limits of the dilation factor, $\alpha(x, y)$, are discussed in Section 5 on design. The change in die-off distance from a cell changes non-linearly when a nearby adjacent cell is near the ends of its RoR. This non-linearity arises from the overlapping rotational stiffness models used to describe unit cell rotational range. For each additional lock placed on the simulated lattice, the RoR for each free cell was calculated again and the DoF was determined. Once the RoR and RoM for each cell is calculated, we can describe the complete CS for the lattice as a structure.

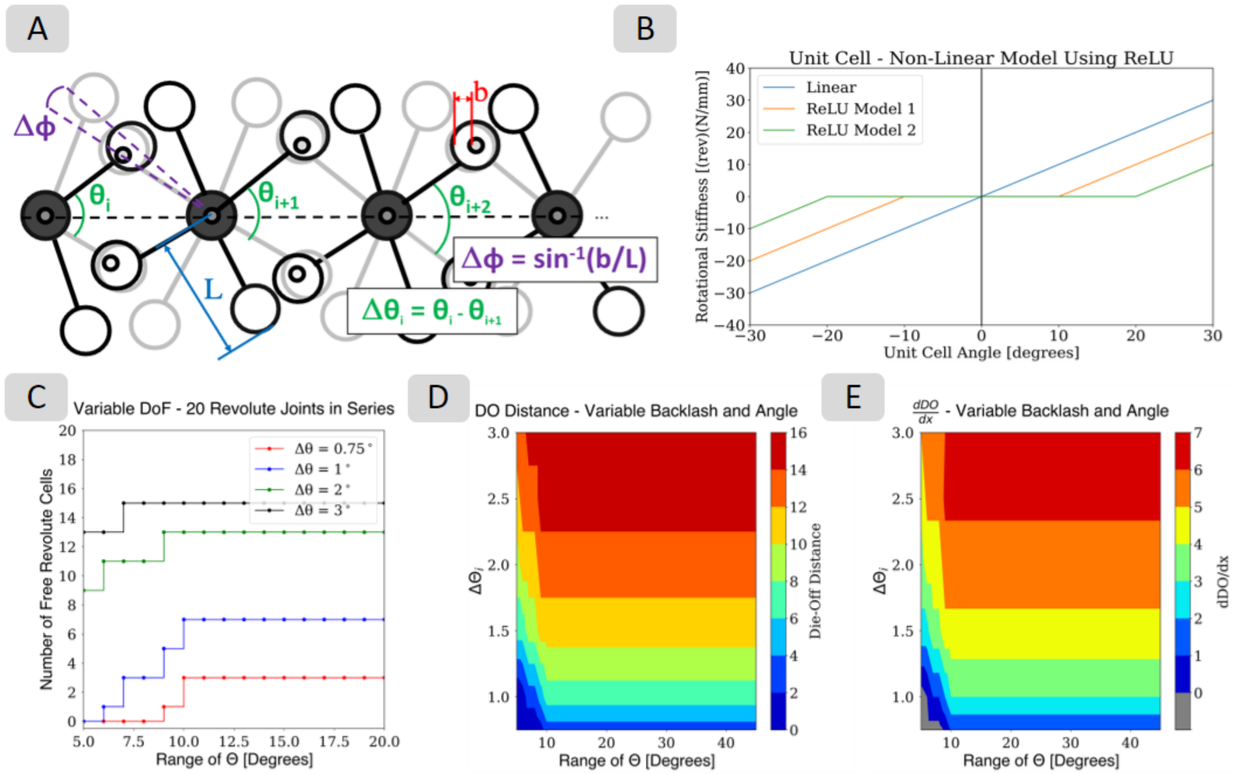

Figure 2. Backlash between joints of an auxetic bilayer material results in a system with non-constant degrees of freedom. As a single revolute joint is rotated in the structure, there is a limited 'die-off distance' over which it affects other joints. (A) Shows the key geometric properties of the structure. Select arms of the auxetic bilayer cell omitted for clarity. (B) Stiffness between joints modeled as the realized linear unit activation function (ReLU). (C) The integer value of DoF present changes in a 1D linkage depending on the fixed angle of the locked cell at one end of the chain. (D) For a line of auxetic bilayer cells, the number of DOF changes non-linearly. (E) Die-off distance and the change in die-off distance over length is a function of cell geometry

Calculating the RoR and the RoM of each cell depends on the rotational stiffness of each cell. If a torque is applied to a free cell near a locked cell, the free cell will undergo deformation to the point where forces between the free and locked cell equalize. For a lattice without any locked cells, a constant torque applied to any cell will drive dilation to its maximum or minimum α(x,y). Adjacent cells will encounter dilation up to this α(x, y) converted to theta minus the backlash angle phi. Complex variable elasticity in auxetic structures in this structure was explored previously in the living hinge RS case [8].

To model the rotational stiffness between adjacent cells, a bidirectional, realized linear unit (ReLU) function was used. The bidirectional, offset, rectified linear unit (ReLU) function is defined as f(x) = max(0, x-b) + min(x+b, 0), which means the outputs equals the input if the absolute value is above a threshold, otherwise, the function outputs zero. The ReLU function is important in neural networks as it introduces non-linearity while maintaining computational efficiency. For analyzing topological flexibility, some prefer to use energy-based frameworks [9]. This is useful in analyzing the solid body and internal stress accurately. The ReLU method is computationally quick and its effectiveness in addressing the vanishing gradient problem makes it a popular stiffness function in this work. If a free cell's neighbors are all locked at a fixed angle, the free cell can rotate within a small range determined by the backlash in the cell. This describes the offset in the bidirectional ReLU model. **Figure 2(A)** shows a simplified image of a 1D linked with the key geometric properties, two bidirectional offset ReLU functions compared to a linear relationship, and figures that describe the DoF, die-off distance, and derivative of the die-off distance with respect to length. **Figure 3** shows the RLABS lattice structure in 2D.

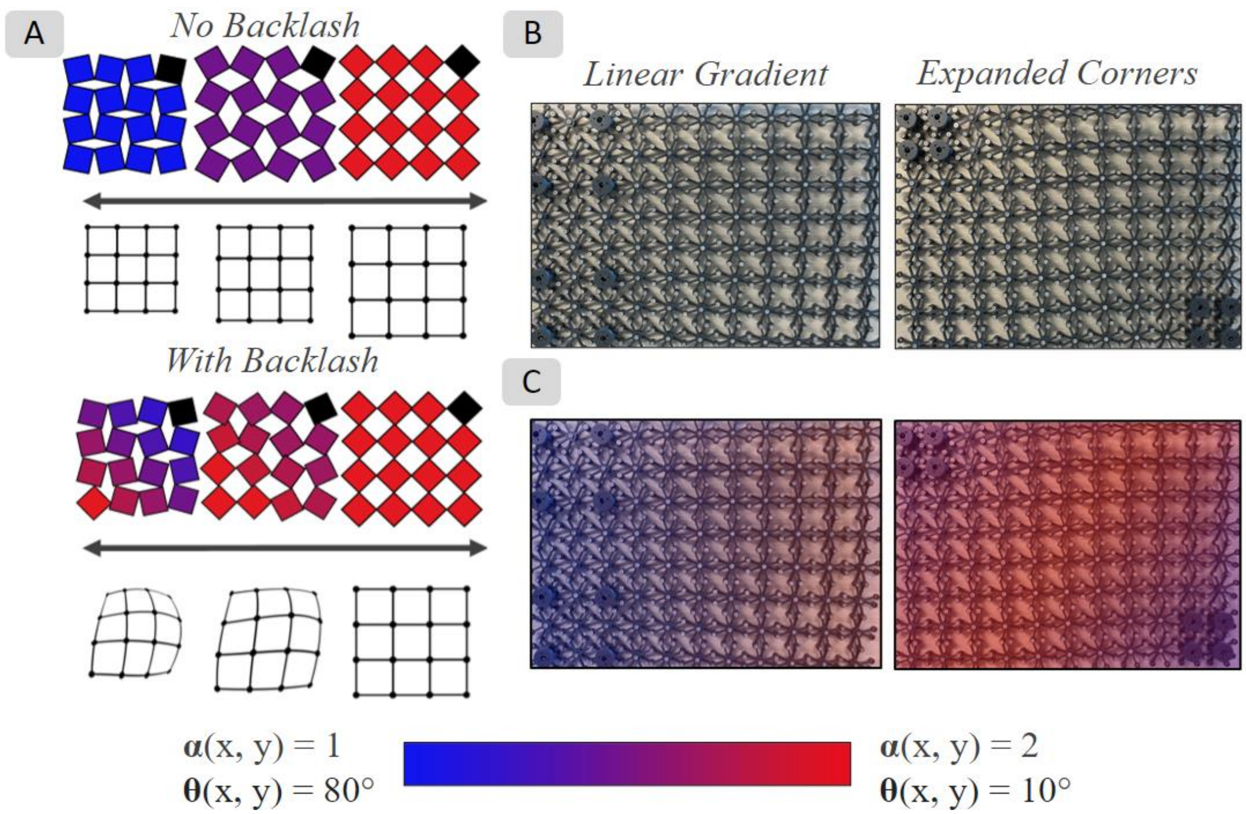

Figure 3. (A) Top lattice depicts 1 DoF while bottom lattice depicts variable DoF. (B) Two examples cases of 2D dilation variation across a planar RLABS. (C) Auxetic dilation colormap continuously applied to the example cases.

By modeling the range of rotation of cells in this mechanical metamaterial with activation functions, we aim to connect methods used in mechanical intelligence to shape-changing structures [10]. Mechanical metamaterials are being integrated into physical representations of machine intelligence by leveraging their unique properties to create programmable materials that can perform complex tasks. These materials can be designed to respond to stimuli in unique ways in contrast to conventional materials, enabling them to act as sensors, actuators, or physical elements of a computational engine within a complex system. This capability allows for the development of smart structures and interfaces that can adapt, learn, and respond to their environment in ways that completely electronic systems cannot, enhancing the versatility and functionality of machine intelligence applications to hardware systems.

Given the computational model of a linkage or lattice, we can determine the range of rotational states that the cells of an RLABS can occupy as specific cells are locked and boundary conditions are applied. This range of positional states must include the target state as well as a connected topological path between the starting state and the target state. The target state (TS) is the desired shape of the entire structure of cells. The TS is the result of boundary conditions as well as each cell's RoR and RoM. Applying boundary conditions and dilation locks to cells is to reduce the configuration space of the lattice to one or a few solutions relative to the completely free state, such that the target state is one of the few or last remaining solutions. For a 1D linkage in 2D space, the configuration space is a rectangular segment of variable length and fixed width. The maximum RoR of a free cell away from and towards a locked cell is equal to the backlash in the joints times the number of cells away.

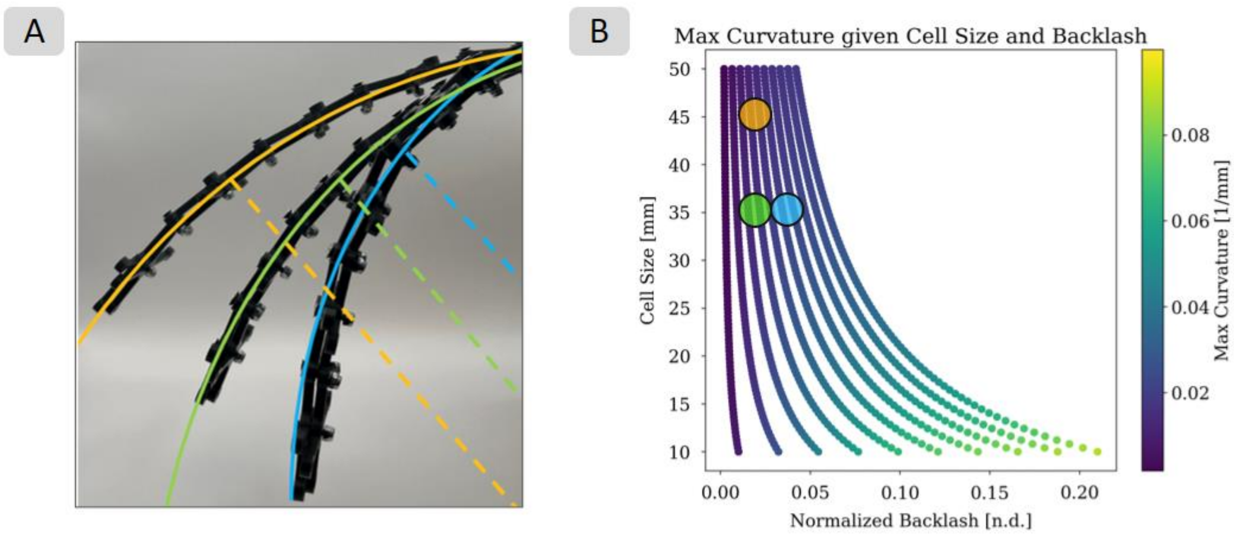

Figure 4. (A) Three RLABS linkages with varying cell length and backlash showing different max curvatures. (B) Plotting the linkage curvature against the modeled result.

To conduct verification experiments of the kinematic and geometric modeling, we 3D printed a 11 x 8 lattice of cells of L=35mm, b=0.4mm and T=4mm. The hinges of thickness l = 0.2mm using a Prusa M4 3D printer and the Overture PLA (30 Shore A). This material is viscoelastic, its Young's modulus is E=3.5 MPa. For imaging purposes, we painted the top of the cells with a matte black paint and a images contrast was adjusted in post-processing. **Figure 5** shows printed RLABS cells with snap-fit or bolted pin joints. **Figure 6** shows examples of RLABS actuated with inflatable bladders, external forces at boundaries, or motors.

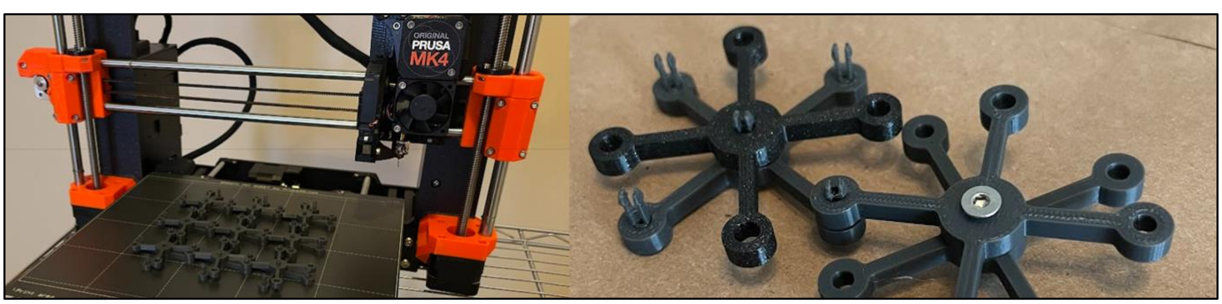

Figure 5. (Left) RLABS 2D lattice section printed directly on build plate. (Right) RLABS cells can be connected with bolts or printed directly with internal snap-fit pin joints.

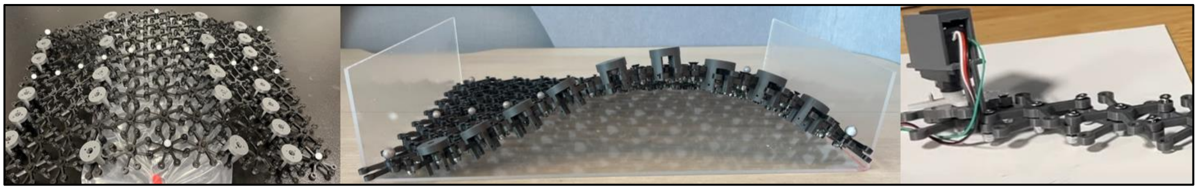

Figure 6. (Left) RLABS 2D lattice with dilation locks and air inflation bladder for deployment. (Center) Boundary conditions can be applied to drive a 3D curvature with dilation locks. (Right) RLABS cells can be connected to servos to programmatically set the dilation factor.

Results

To evaluate the utility of RLABS, two example structures were created - an airfoil frame and a conformal molding structure. These resultant structures demonstrate the model, demonstrate usefulness, and provide verification of the alpha function calculation. Comparing the physical metrology results with the computational model validates the accuracy and reliability of the model, ensuring that it predicts the behavior of RLABS under varying conditions. This comparison helps in refining the computational model, pointing out areas of potential error, and leads to better material designs using auxetic lattices in real-world examples. These results provide confidence in the model's predictive capabilities and help develop it as a tool for engineers using metamaterials.

Foils are streamlined bodies capable of generating more lift than drag while moving through a fluid medium. Wings and sails are examples of airfoils, foils that move through air. These unique shapes are subject to design constraints where the specific curvature across the body impacts the wing performance. By covering the external contour frame of an airfoil with an RLABS, the external profile of the wing can be manipulated by manipulation of the underlying structure. **Figure 7** shows two NACA airfoil profiles generated by selective locking of an RLABS. The NACA airfoil series, developed by the National Advisory Committee for Aeronautics, consists of systematically designed airfoil shapes characterized by specific numerical codes representing their geometric properties. To determine the cell angles to achieve the NACA airfoil profiles, multiple methods were tested. The shape can be approximated with the CS model described in the Methods section. After using the model, some error still remains. The locking angle and boundary condition can be reset using gradient descent to reduce the error between the measured shape and the TS. Gradient descent is an optimization algorithm that iteratively adjusts parameters to minimize a given function, often used in machine learning to minimize the loss function. Starting with an initial guess, the algorithm computes the gradient of the error with respect to each locking angle and cell position, indicating the direction of steepest ascent. It then updates the new cell locks and positions by moving in the opposite direction of the gradient, scaled by a learning rate, until convergence is achieved or further improvements become negligible. Error was determined to be at or lower than 1% across each airfoil frame shape.

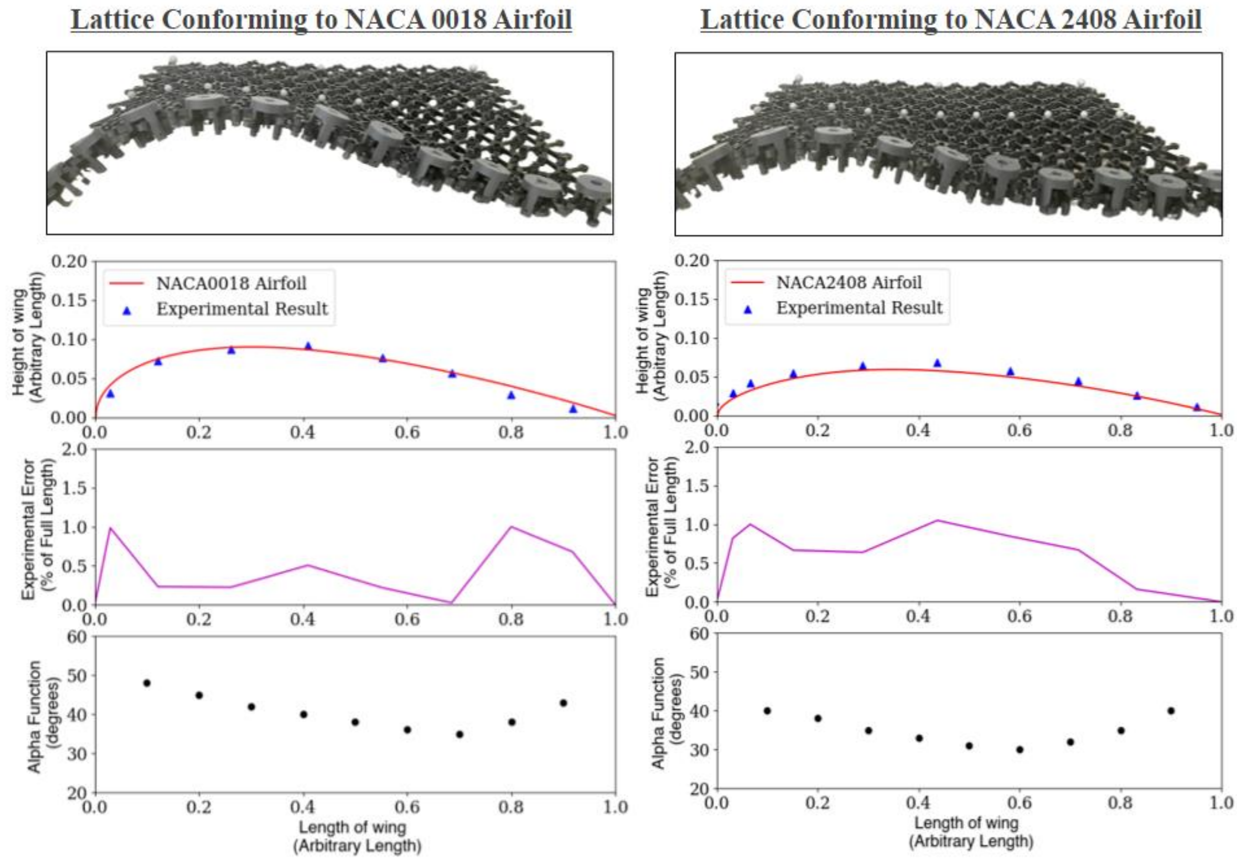

Figure 7. A flat RLABS lattice can be set into a range of desired airfoil forms with boundary conditions and selected dilation factors.

Discussion

The CS of a mechanism with auxetic lattices is the range of possible positions and orientations the mechanism can achieve. Auxetic lattices can be designed to provide greater flexibility and adaptability in this space, allowing for more complex and varied movements. However, optimizing the lattice structure to enhance the CS can sometimes introduce trade-offs in terms of mechanical stability and control. A more flexible lattice might offer a larger CS but could also be more prone to deformations that lead to imprecision or instability. Therefore, engineers must balance the need for a broad and versatile CS with the necessity of maintaining low backlash and high precision, carefully designing the auxetic lattice to achieve optimal performance in both areas. **Figure 8** shows how the RLABS lattice can be covered with a thin rubber covering for non-permeable applications.

For geometric factors, there are physical limits to consider. As backlash increases, the angle between arms of an individual cell decreases. The angle between adjacent arms of the top and bottom layer of a bilayer cell plus the offset angle of the pin joints are complementary. A given linkage has one DoF remaining when a single free cell retains its full RoR. To reduce the configuration space to a single TS, every cell must be within the DO distance of a locked cell. This is accomplished with the highest Manhattan distance between a free cell and a locked cell is less than the DO distance.

Figure 8. The lattice can be covered with a thin rubber skin to give a soft and conformal interpolating surface.

A recursive engineering process may be used to ensure the locked cell angles and boundary conditions for a RLABS result in the TS through reverse kinematics begins with defining the desired RLABS position and orientation in the workspace. The process involves solving the inverse kinematics problem, which translates these desired end-effector coordinates into the necessary joint angles for the RLABS. This can be complex, particularly for RLABS due to the many degrees of freedom, and requires iterative numerical methods. The RLABS $\alpha(x,y)$ solver starts with an initial guess for the joint angles and iteratively adjust them using optimization techniques to minimize the difference between the calculated lattice state and the TS. In this work, a gradient descent algorithm was used. The difference between the TS and the calculated state was found and each cell was expanded or contracted iteratively to reduce the error. Algorithms like the Jacobian transpose, pseudo-inverse methods, or more advanced techniques like Levenberg-Marquardt can be employed in these iterative processes.

To refine the solution further, a feedback loop is integrated into the system. This involves measuring the actual position of the RLABS using metrology sensors, such as Optitrack, and comparing it to the desired position. If discrepancies are found, the error can be put into the RLABS algorithm to calculate the necessary adjustments to the angular locks to correct the RLABS position. This feedback loop ensures that any deviations due to external disturbances, model inaccuracies, or non-linearities may be corrected so that the lowest error surface can be generated from a given cell geometry and lattice size. By iteratively refining the joint angles and incorporating real-time feedback, the system converges on the exact dilation factors needed to achieve the TS of the RLABS. This recursive approach ensures accuracy to the best case and reliability in positioning the RLABS through reverse kinematics.

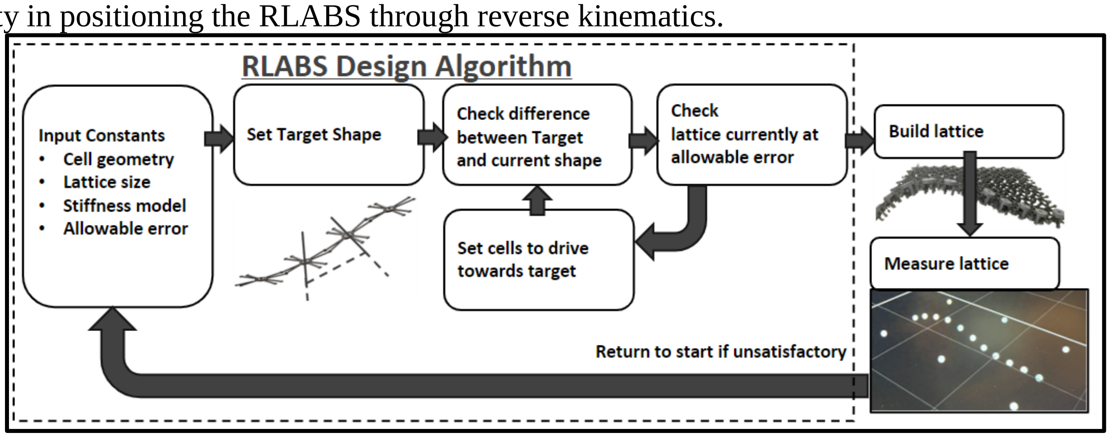

Figure 9. Block diagram for RLABS design algorithm.

Conclusion

This paper demonstrates the potential of reconfigurable and underactuated lattices of auxetic backlash structures for shape-changing systems. By using variable compliance mechanisms that are discretely deformable and mechanically configurable, these backlash structures bridge a design space between parallel manipulators and underactuated systems. The relatively large CS of RLABS allows them to be developed into a wide range of shape-changing surfaces and volumes. At fabrication, the CS can be set by the design of the unit cell that makes up the RLABS. The computational functions and relationships shown in this work is available online for use by other engineers and scientists.

The mechanical characterization of a prototype RLABS demonstrates conformal performance similar to other underactuated schemes – demonstrating great flexibility when used as an airfoil frame or conforming to the complex surfaces. RLABs actuation and fabrication methodology rivals the complexity of other conformal auxetics in a form factor that is more impact resistant, easier to interface with existing electrical systems, and capable of being fabricated quickly with 3D printers. Auxetic materials are being integrated into impact-resistant systems to enhance energy absorption and dispersion upon impact. This significantly reduces the force transmitted to the protected body, providing superior protection compared to conventional materials. Additionally, the flexibility of auxetic materials ensures comfort and mobility, making them ideal for modern protective gear and protective personal equipment applications. Auxetics are also being used in developing reconfigurable tooling for cutting-edge manufacturing technology due to their ability to change shape and adapt under stress, allowing for more versatile and precise machining layup components. This adaptability reduces the need for multiple, specialized tools, streamlining production and enhancing efficiency. Moreover, by minimizing the dependency on specific tooling supplies, auxetic materials help alleviate supply chain concerns, ensuring more resilient and flexible manufacturing operations.

Future work will expand on the basic principles shown in RLABS by creating new cell patterns that can address more specific robotic applications such as gripping and manipulation of inflatable bodies. To make RLABS programmable instead of only configurable, servos can be attached to select cells of the lattice. Each servo actuator will be integrated to an individual auxetic cell that translates a servo's rotational movements directly to cell dilation.

The prototype RLABS shown in this work are stiffer than other conformal actuators, making them less appropriate for extreme deformation contexts, using a different base material than PLA for the RLABS structure will reduce that complication. The revolute joints shown in this work expand the area of a cell up to 1.7x. An optimized pin joint design within the RLABS pattern may enable higher extension ratios for more applications that require even larger differences between compact and expanded CSs. Further mechanical characterization is also needed to better understand how the compliance of the RLABS change upon being extended as well as the optimal control scheme. Extensions of this work may have a significant impact on the performance of surface and volume modulators in industrial and scientific systems.

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