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Reviewed Paper

Criteria-Based Evaluation of Multi-Lattice Structure Design Methods

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Abstract

Design for additive manufacturing (DfAM) can be used to harness the flexibility inherent in additive manufacturing to achieve significant weight reduction. Lattice structures are a highly-effective way to achieve substantial weight reductions; multi-lattice structures compound this benefit by making use of multiple lattice topologies. While various methodologies have been proposed for designing multi-lattice structures, there remains a lack of clarity on how to effectively compare these methods. In this work, we investigate the key design criteria for evaluating multi-lattice structure design methodologies. Our analysis reveals three recurring design criteria across existing literature: 1) lattice connectivity, 2) lattice diversity, and 3) physics-based interpolation of lattices. These criteria are discussed within the context of extant research on multi-lattice structure design.

1. Introduction

Additive manufacturing (AM) has infinitely expanded the design horizon by enabling the production of designs that were once impossible to manufacture. One of the key developments in this space has been the application of lattices, repeating structural elements that can be used to reduce weight all while addressing mechanical performance needs [1–3]. Initially, designers used what are now called *uniform lattices*, as they are a single unit cell uniformly patterned throughout a structure with consistent unit cell size and density [4]. Despite the many benefits of these simple patterns, they are inherently limited to a single unit cell and therefore have limited mechanical properties [1,5,6]. As such, designers continued to improve upon uniform lattices by varying the density of individual unit cells, which have been coined *graded lattices*. However, these too are plagued by a single unit cell, hampering the ability to address energy absorption [7] and going as far as limiting structural connectivity [4]. To address the common inhibitor, researchers have begun to develop *multi-lattice* structures by incorporating multiple unit cells into a single structure [8–10].

On the surface, multi-lattice structures seem more appealing as they should offer more design flexibility to attend to complex loading constraints. However, in reality they pose a challenge as they must be designed inversely at the mesoscale to address connectivity and concurrently optimized at the macroscale to effectively address structural needs [11]. Consequently, designing structures with multiple unit cells has proven to be a complex task, with a myriad of methods attempting it through a combination of shape blending [12–16] and parameterizations techniques [9,11,17–26]. Due to the many approaches to multi-lattice design, it is difficult to effectively compare them during literature review. There is a clear absence of comprehensive methodologies for comparing these various design approaches and collectively analyzing their limitations.

Recognizing this gap, this work aims to establish a framework for systematically evaluating and comparing multi-lattice design methods. Through meticulous analysis, we have identified recurring limitations within existing approaches, which have informed the development of a standardized set of multi-lattice structure design criteria. These criteria will serve as a valuable tool for researchers, facilitating a consistent and objective evaluation of multi-lattice design methodologies. These criteria are poised to evaluate the multi-lattice structures generated from these methods, rather than critiquing every aspect of a design method. Based on our review, we propose the following criteria for comparing existing multi-lattice design methods.

① Lattice Connectivity - maintain connectivity between adjacent unit cells;
② Lattice Diversity - consist of a wide range of unit cell topologies; and
③ Physics-based Interpolation - designed based on the mechanical characteristics of adjacent unit cells

These design criteria provide a common point of comparison for all the models discussed in the current work. The remainder of the paper is organized as follows. Section 2 reviews literature relevant to the three proposed criteria. Section 3 demonstrates how these criteria can be used to meaningfully differentiate between prominent work in this area. Finally, Section 4 concludes this paper with a summary and discussion of future directions.

2. Relevant Literature Review

Many methods for designing multi-lattice structures identify a key goal that the method will address. These goals can be synthesized into design criteria that can be used to describe the success of a multi-lattice design method.

① Lattice Connectivity:

The foremost challenge with developing multi-lattice structures has consistently been how to maintain geometric connectivity throughout the structure, otherwise the structural integrity may be compromised. While some works directly prioritize lattice connectivity within their design method (e.g. [15]), others include discussions on the effects of lattice connectivity (e.g. [11]). Figure 1 visualizes how lattice connectivity could be considered between two adjacent unit cells.

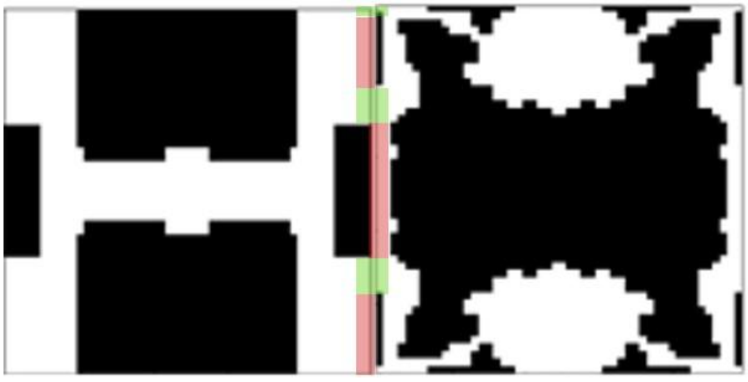

Figure 1: Lattice connectivity example

Sanders et al. focuses on the task of lattice connectivity by implementing a shape blending method to transition between two lattice faces [15]. The method demonstrates the prevalence of developing multi-lattice structures with smooth interfaces in order to ensure manufacturability. Wang et al. discuss many methods for addressing connectivity in their work, and ultimately decide to develop unit cell families that allow efficient multiscale design by guaranteeing connectivity within the family [11]. The task of boundary compatibility motivated the development of their design method that utilized the unit cell families to then map desired material properties into a structure using topology optimization. By guaranteeing the connectivity of unit cells in the family, the topology optimization could freely select unit cells based on their physical properties. This work emphasizes the importance of connectivity by prioritizing it throughout their design method.

Other literature cites the importance of lattice connectivity with respect to their methods. A major review of metamaterial design was conducted by Lee et al. [27], which explores existing methods for designing lattice structures to achieve unique mechanical properties. Multi-lattice structures are a form of metamaterial, as they involve the intricate design of mesostructures to achieve complex mechanical properties. This review considers unit cell compatibility as representative of their ability to possess geometric and physical similarity. Geometric compatibility is consistently discussed with respect to the design methods and often noted as a limitation. Kang et al. explored developing multi-lattice structures using two types of strut-based lattices, however they noted that the interfaces within these lattices controlled the fracture location [9]. They concluded that the poor connectivity between the unit cells simultaneously caused the concentration of stresses and weakened the structure.

Overall, many works either seek to solve lattice connectivity problems directly or have cited these as a primary issue within their multi-lattice structures, which supports incorporating this as a design criterion.

② Lattice Diversity:

Given that the goal of multi-lattice structures is to improve the ability of lattices to address complex loading conditions, it is imperative that design methods are able to design using a plethora of different lattices in order to offer a diverse range of mechanical properties. The importance of lattice diversity is emphasized by works dedicated to improving the diversity of lattice datasets in addition to those introducing it as a task during multi-lattice design. Figure 2 demonstrates the diversity possible even when limited to 2-dimensional lattices.

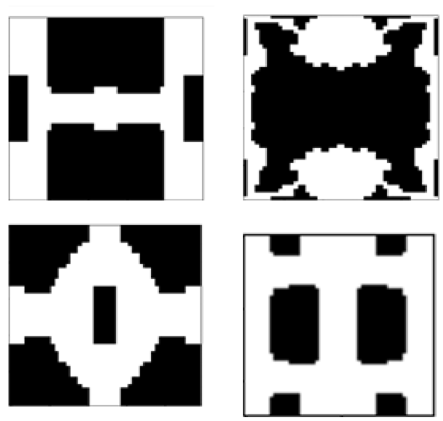

Figure 2: Lattice diversity example

Wang et al. provide a thorough discussion on the importance of unit cell diversity and its role in effective multiscale design [11]. Their work notes that in order to address connectivity, a large dataset is necessary to increase the likelihood of lattice compatibility. Larger datasets provide more options for achieving the same mechanical properties while ensuring geometric compatibility. Additionally, the diversity of the dataset offers better chances of convergence of the optimization as it is more likely to find local minima.

Work by Plocher et al. evaluates the performance of graded lattice structures relative to their uniform counterparts, and the key finding was that the type of lattice had a significant impact on the energy absorption abilities of the lattices [2]. This finding suggests that in order to address complex loading conditions, a wide range of lattices will be necessary to design optimal structures, especially in energy absorption scenarios.

③ Physics-based Interpolation of Lattice Properties:

The similarity of properties between adjacent unit cells is another major factor of multi-lattice design to consider, as geometric connectivity alone does not guarantee that loads will be distributed effectively. Figure 3 demonstrates how the loading of two adjacent lattices could be poorly distributed if their physical characteristics are not considered.

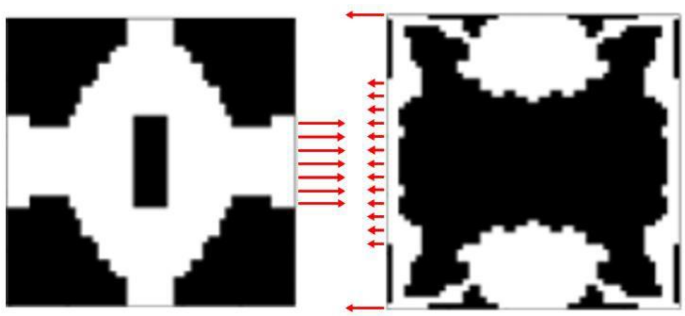

Figure 3: Physical compatibility example

Initially, many works applied density-based topology optimization which proved effective in the development of functionally graded lattice structures. For example, Kang et al. utilized a density mapping approach to multi-lattice design using two lattice types and mapping them within a structure [9]. Although the approach demonstrated the benefits of multi-lattice structures, they recognized that density mapping did not necessarily optimize based on physical properties and suggested that future work should incorporate physics into the optimization scheme.

Wang et al. demonstrate the importance of interpolating physical properties in the latent space by applying topology optimization to organize lattices based on the similarity of their physical characteristics, specifically elasticity tensor data [11]. They do this through the development of unit cell families that have graded physical properties to offer a wide range of properties that can simultaneously reduce stress concentrations. As discussed with respect to design criteria ①, a major review of metamaterial design was conducted by Lee et al. [27], which explores existing methods for designing lattice structures to achieve unique mechanical properties. This review considers unit cell compatibility as representative of their ability to possess geometric and physical similarity. Physical compatibility is mentioned throughout the analysis of the design methods and often serves as a limitation.

Overall, this literature highlights the importance of physical compatibility of unit cells within multi-lattice structures throughout the works reviewed, clearly indicating that this is a design criterion that cannot be overlooked. Although there are many different physical descriptors of lattices, it is evident that physics needs to be considered within multi-lattice design.

3. Application of Criteria

This section summarizes several prominent works and demonstrates how the design criteria are applied based on the multi-lattice structures produced from their corresponding methods. To further categorize the methods, we use design criteria ① to divide the methods based on how they ensure unit cell connectivity. A concise summary of the works based on their ability to address the design criteria can be found in Table 1. The key factor that differentiates the methods is how they address design criteria ① (lattice connectivity), by either shape blending or parameterization. Some methods embrace this constraint directly through shape blending [12–16], which directly interpolates between two geometries to connect them. Alternatively, other methods utilize unit cell parameterizations to interpolate over a series of unit cells to create a transition region between two target unit cells [9,11,17–26]. Of these, some methods are defined parameterizations (meaning that the interpolated unit cells adhere to known mathematical relationships) while others are learned parameterizations (meaning that the interpolated unit cells are generated by machine learning algorithms).

In general, the advantage of using a parameterization is the ability to easily incorporate the method into a topology optimization which inherently addresses design criteria ③, whereas shape blending guarantees design criteria ①. Each of these approaches is discussed in greater detail in the following sections.

Table 1: Example Evaluation Multi-lattice Design Methods

<table>
<tr><th>Methods</th><th>Literature</th><th>Design Criteria 1</th><th>Design Criteria 2</th><th>Design Criteria 3</th></tr>
<tr><td rowspan="3">Shape Blending</td><td>[15]</td><td>Satisfied</td><td>Limited to unit cells composed of struts, bars or plates</td><td>Does not consider physical properties.</td></tr>
<tr><td>[14]</td><td>Satisfied</td><td>Satisfied, but requires blending between every shape to be included in the structure.</td><td>Identifies desirable shapes in a structure, but the blending region properties cannot be controlled.</td></tr>
<tr><td>[13]</td><td>Satisfied</td><td>Limited to equation-based lattices.</td><td>Does not consider physical properties.</td></tr>
<tr><td rowspan="3">Defined Parameterization</td><td>[9]</td><td>Inherently connected boundaries.</td><td>Limited to strut-based lattices.</td><td>Used density-based topology optimization, which does not consider physical properties.</td></tr>
<tr><td>[23]</td><td>Inherently connected boundaries.</td><td>Limited to strut-based lattices.</td><td>Used density-based topology optimization, which does not consider physical properties.</td></tr>
<tr><td>[26]</td><td>Inherently connected boundaries.</td><td>Limited to strut-based lattices.</td><td>Satisfied, used density-based topology optimization, but optimized strut thickness distribution.</td></tr>
<tr><td rowspan="6">Learned Parameterization</td><td rowspan="2">[11]</td><td>Connectivity is not guaranteed since lattices are selected based on their elasticity tensors.</td><td>Satisfied, but limited to the training dataset.</td><td rowspan="2">Satisfied</td></tr>
<tr><td>Satisfied, but unit cells classes are used to guarantee connectivity.</td><td>Limited to unit cell families.</td></tr>
<tr><td>[17]</td><td>Inherently connected boundaries.</td><td>Limited to strut-based lattices.</td><td>Satisfied</td></tr>
<tr><td rowspan="2">[20]</td><td>Optimization does not guarantee connectivity.</td><td rowspan="2">Limited to equation-based lattices.</td><td>Satisfied</td></tr>
<tr><td>Satisfied, but applied a filter to blend the boundaries.</td><td>Satisfied, shape blending reduces the design intent of the original mechanical properties.</td></tr>
<tr><td>[19]</td><td>Satisfied</td><td>Limited to unit cell classes.</td><td>Satisfied</td></tr>
</table>

Shape Blending:

Shape blending uses the geometry of adjacent unit cells to create a direct interpolation between them in order to guarantee their connectivity, design criteria ① [12–16]. This section will review works that have applied shape blending for developing multi-lattice structures with respect to their ability to address the previously defined design criteria. Sanders et al. developed a shape blending approach that creates an interpolated unit cell using signed distance fields [15]. However, it only operates on unit cells composed of struts, bars or plates, which limits it to only ensuring lattice connectivity, design criteria ①. Other works have implemented shape blending for equation-based unit cells, but again can only ensure connectivity, design criteria ① [12,13].

Another shape blending method developed by Chan et al. demonstrates the ability to blend over a diverse range of geometries [14], design criteria ① and ②. However, their shape blending method does not control the distribution of physical properties throughout the multi-lattice structure. Conversely, a method by Yoo et al. can maintain lattice connectivity and specify physical properties concurrently, design criteria ① and ③, but is also restricted to equation-based unit cells [16]. Additionally, many topology optimization methods for developing multi-lattice structures rely on the cubic assumption to reduce the complexity of the elasticity tensor data by assuming the unit cell is symmetric about all three dimensions [13]. Therefore, shape blending methods are not ideal as they nullify this assumption which increases the complexity during finite element analysis.

Shape blending methods inherently ensure lattice connectivity, design criteria ①, and have demonstrated the ability to be applied over diverse datasets, design criteria ②, but they are limited by the ability to simultaneously ensure desirable mechanical properties over diverse datasets. This supports the use of shape blending methods in design cases where mechanical properties are not a functional requirement of the design. Additionally, shape blending is often a method where computational costs increase exponentially as the number of lattices in the dataset increases. These costs can be due to the number of blending regions to be developed between each pair of unit cells, or due to the increased complexity of finite element analysis caused by the development of asymmetric structures. This motivates the use of methods that transition between unit cells using a series of cells to develop a lattice transition region.

Defined Parameterizations:

Defined parameterizations consist of several methods of addressing connectivity, including designing interpolation regions of multiple unit cells to achieve a smooth transition or defining inherently boundary compatible datasets [9,23–26]. Once a dataset of unit cells is parameterized, then a method for organizing the unit cells is necessary to develop multi-lattice structures. As such, the limiting factor of the defined parameterization method then becomes the method of optimization, as it determines how the unit cells are distributed throughout the structure.

There are many multi-lattice design methods that parameterize strut-based lattices in order to organize unit cells based on density [9,23–25]. However, defined parameterizations only work for some unit cell types and is not generalizable over large datasets, so they cannot address design criteria ②. Additionally, these methods often rely on developing datasets with inherently compatible boundaries, so they are not addressing the challenge of connectivity, design criteria ①. Many defined parameterization multi-lattice design methods utilize density-based topology optimization as it can be used to easily represent unit cells based on their density. However, Kang et al. noted that density-based topology optimization does not truly organize unit cells based on their physical characteristics rather it serves as a cheaper unit cell representations for optimization [9]. As such, many defined parameterizations are unable to actually address design criteria ③.

There are some works that implement modified density-based topology optimization. For example, Liu et al. also optimize using a density-based topology optimization, but they modify the thickness of the struts which serves to better address the physical needs of the structure, design criteria ③ [26].

Generally, models that use a defined parameterization are limited unit cells that have inherently compatible boundaries, design criteria ①, and a reduced dimensional form, which limits their data diversity, design criteria ②. This motivates the use of learned parameterizations, which incorporate a wide range of unit cell topologies using a large dataset. Regardless, these parameterizations offer a low dimensional representation of unit cells that is easily incorporated into topology optimization techniques.

Learned Parameterizations:

Learned parameterizations develop a low dimensional representation of data, often using machine learning to implement data reduction methodologies. Like defined parameterizations, learned parameterizations attempt to address the connectivity criteria by using a progressive series of unit cells organized through topology optimization [11,17–22].

A technique using the latent-variable gaussian process can generate new unit cell sets using a learned latent space [17,18]. However, the latent-variable gaussian process is only compatible with strut-based datasets which enable the development of property continuous structures, thereby only addressing design criteria ③. An extension of this uses Laplace-Beltrami spectrum to reduce the data dimensionality of lattices using neural network, which can develop new unit cells [19]. Traveling through this latent space maintains lattice connectivity by establishing classes of unit cells which can then be organized based on their physical properties. These classes restrict the diversity of unit cells that can exist in a multi-lattice structure, which means this method can only address design criteria ① and ③.

Wang et al. developed a learned parameterization that uses an inverse homogenization generative adversarial network to generate an optimized structure by mapping unit cells that meet the mechanical properties defined by the topology optimization scheme [20]. This model uses lattice representations that have low-dimensional formats, in this case equation-based lattices; therefore it cannot be applied over extremely diverse data. The model generates a structure that directly addresses design criteria ③, but it does not guarantee connectivity. Therefore, a shape blending operation is required to ensure connectivity in the structure, design criteria ①, which forfeits some of the mechanical design intent, design criteria ③.

Variational autoencoders (VAEs) are the only dimensionality reduction method that has been able to nearly address all the design criteria of multi-lattice design [11,21,22,28,29]. Wang et al. demonstrate the ability of VAEs to develop a latent space of unit cells, which can be used to generate classes of geometry compatible unit cells [11]. The classes of unit cells ensure connectivity while optimized based on their physical properties through topology optimization, design criteria ① and ③, but significantly reduce the overall diversity in the structure. To combat this, they implemented a method to select unit cells from the training data in the entire latent space based on their elasticity tensors, design criteria ② and ③, but similar elasticity tensors do not guarantee connectivity. Therefore, this approach directly addresses design criteria ③, but only partially addresses design criteria ① and ② depending on which technique they use.

In general, multi-lattice structures designed using learned parameterizations suffer from the difficulty of jointly maintaining connectivity and managing data diversity, design criteria ① and ②. However, they are easily combined with topology optimization, similar to defined parameterizations, which means they can directly organize lattices based on their physical similarity, design criteria ③.

4. Conclusion

Multi-lattice structures enable the design of lightweight structures using multiple lattice topologies to address complex design requirements. However, multi-lattice structures pose a complex multiscale design challenge which has generated a highly-varied range of methodologies that cannot be easily compared. In this work, we proposed a set of design criteria that can be used for categorizing multi-lattice design methodologies based on the structures rendered. These criteria represent thematic elements throughout multi-lattice design literature that can be universally applied to most multi-lattice design methodology:

① Lattice Connectivity - maintain connectivity between adjacent unit cells;
② Lattice Diversity - consist of a wide range of unit cell topologies; and
③ Physics-based Interpolation - designed based on the mechanical characteristics of adjacent unit cells

The application of these design criteria demonstrates common trends in the abilities of multi-lattice design methods. For example, methods typically address design criteria ① through shape blending, defined parameterizations, or learned parameterizations. Within these categories, there are clear trends in the limitations of each method, as the way lattice connectivity is addressed will dictate restrictions on the other design criteria. Shape blending methods were consistently able to address design criteria ①, and sometimes design criteria ②, but forfeited the ability to choose mechanical properties of the lattices. Therefore, shape blending methods serve to benefit applications that do not depend on mechanical functionality. Defined parameterizations often faced the opposite problem, they could often only address design criteria ③, as they were limited to lattices that can be defined using variables and often relied on geometrically compatible lattice types. These traits make defined parameterizations ideal in cases where loading needs to be considered, but addressing complex loading conditions will be challenging due to limited lattice datasets. Learned parameterizations demonstrated the most flexibility, as they can be applied to a large dataset of lattices, design criteria ②, then the problem becomes how to navigate these latent spaces in order to maintain connectivity and encourage compatible physical properties among lattices. As such, learned parameterizations serve to offer the most flexibility while offering lattice diversity.

With these design criteria in mind, there are still other design elements to consider when evaluating a multi-lattice structure. Future work should work to develop a clear set of physical properties that need to be evaluated to determine physical compatibility, as there are many properties that describe an individual unit cell. Additionally, there is a lack of established quantitative evaluations of both physical and geometric compatibility within multi-lattice structures, which could help precisely compare existing methods.

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